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Nucleon Structure from Lattice QCD

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We report on our recent results for iso-singlet and iso-vector matrix elements from Lattice QCD with $N_f = 2$ degenerate light sea quark flavours. The quark contributions to the nucleon spin, Δq, and the sigma terms, σ_q, are calculated including both the connected and disconnected quark line diagrams. The latter are essential for studying strangeness. We find $\Delta s^{\overline{\rm MS}}(\sqrt{7.4}\,{\rm GeV}) = -0.020(10)(4)$ and the fraction $f_{T_s} = \sigma_s/m_N = 0.012(14)^{+10}_{-3}$, suggesting the strange quark only plays a minor role in the nucleon for these quantities. These direct calculations are performed at a single pion mass, $m_\pi \approx 285$ MeV. In order to extrapolate our direct determination of the pion-nucleon σ-term to the physical point, we perform a combined fit with nucleon mass data obtained at different quark masses. We find $\sigma_{\pi N} = 38(8)(6)$ at the physical point. We also present results for the iso-vector average momentum fraction $\langle x\rangle_{u-d}$ at a near physical pion mass, $m_\pi \approx 157$ MeV.

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1. Probing nucleon structure on the lattice

Nucleon structure has been studied for many years both in theory and experiment, with a number of quantities, such as the average momentum fraction of the up and down quarks, being well determined. However, other properties are not known precisely, for example, the strange quark contribution to the nucleon spin or the up, down and strange quark contributions to the nucleon mass. A first principles non-perturbative determination of these quantities can be provided by Lattice QCD. Calculations of well determined quantities are also important in order to demonstrate how well the systematics are under control.

On the lattice one can calculate matrix elements of local operators that are related, via the operator product expansion, to the moments of the polarised and unpolarised structure functions (g_1, g_2 and F_1, F_2, respectively) measured in deep-inelastic scattering (DIS) experiments. The connection to the quark and gluon constituents is made in the parton model which links the structure functions to the unpolarised and polarised (helicity) quark distributions. For example, to leading twist the lowest moment of g_1 is

$$2\int_0^1 \mathrm{d}x\, \mathbb{1}\, g_1(x,Q^2) = \frac{1}{2}\sum_{q=u,d,s} C^{(q)}(\mu,Q^2)\langle \mathbb{1}\rangle_{\Delta q}(\mu) \tag{1.1}$$

$$\langle \mathbb{1}\rangle_{\Delta q}(\mu) = \sum_{q'=u,d,s} Z_{qq'}(\mu)\langle N|\bar{q}'\gamma_5\gamma_\mu q'|N\rangle^{latt} \tag{1.2}$$

where $C^{(q)}$ are Wilson coefficients, Q^2 is the squared virtuality in DIS and μ is the renormalisation scale. Usually one chooses $\mu = Q$. The first moment of the helicity parton distribution function (PDF), $\langle \mathbb{1}\rangle_{\Delta q}$, is the contribution of the quark q to the spin of the nucleon. The renormalisation factors $Z_{qq'}$ relate the lattice matrix elements to a continuum scheme. For F_1 the second moment gives the average quark momentum fraction, $\langle x\rangle_q$ and the corresponding lattice matrix element is $\langle N|\bar{q}\gamma_{\{\mu_1}\overleftrightarrow{D}_{\mu_2\}}q|N\rangle$. One can also study moments of the generalised parton distributions on the lattice. However, investigating transverse momentum dependent distribution functions is more challenging, see for example [1].

The lattice matrix elements, shown pictorially in Figure 1, contain both connected and disconnected contributions. The disconnected diagrams are essential for the study of strangeness in the nucleon, however, they are computationally expensive to calculate as they involve a quark propagator from all space-time points to all points ("all-to-all"). In the past this motivated the study of iso-vector combinations, for example, $\langle x\rangle_{u-d}$ for which the disconnected contributions cancel in the isospin symmetric limit. However, stochastic techniques have been developed which make these diagrams tractable, see for example [2]. In the following we present results for the scalar matrix elements (Section 2) which give the contribution of the quarks to the mass of the nucleon and $\langle \mathbb{1}\rangle_{\Delta q} = \Delta q$ (Section 3), with the disconnected quark line diagrams included. These are quantities for which the statistical errors inherent in the lattice calculation are of a similar order of magnitude to the systematic errors: finite lattice spacing, finite volume, unphysical quark mass and so on. In Section 4 we discuss results for $\langle x\rangle_{u-d}$. As mentioned above, this quantity is well determined, through fits of PDF parameterisations to experimental data, and is a suitable benchmark observable.

The results were generated using configurations with $N_f = 2$ degenerate flavours of dynamical sea quarks using the non-perturbatively improved clover fermion action such that the leading order

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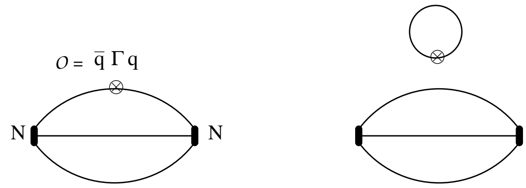

Figure 1: Connected (left) and disconnected (right) quark line diagrams contributing to the matrix element $\langle N|\mathscr{O}|N\rangle^{latt}$.

discretisation errors are $O(a^2)$. The lattice spacing is $a \sim 0.07$ fm. Two values of the quark mass, measured in terms of the pion mass, were used, $m_\pi \approx 285$ MeV and ≈ 157 MeV. Note that simulating at physical or near physical quark masses has only recently become possible due to algorithmic developments. The strange quark mass is treated as "quenched", i.e. it only appears as a valence quark. For the ensembles with $m_\pi \approx 285$ MeV two volumes were studied with $Lm_\pi \sim 3.4$ and ~ 4.2. Finite volume effects are expected to be suppressed for $Lm_\pi \gtrsim 4$ and we found no significant volume dependence in our results. For $m_\pi \approx 157$ MeV only one volume is available at present with $Lm_\pi \sim 2.7$. The disconnected contributions are not calculated as yet for this ensemble and we only present results for $\langle x\rangle_{u-d}$ in this case.

2. Scalar matrix elements

The contribution of the light quarks to the mass of the nucleon can be expressed in terms of the familiar pion-nucleon sigma term ($\sigma_{\pi N}$) and the y parameter: $\sigma_{\pi N} = \sigma_u + \sigma_d$ where $\sigma_q = m_q\langle N|\bar{q}q|N\rangle$ and $y = 2\langle N|\bar{s}s|N\rangle/\langle N|\bar{u}u + \bar{d}d|N\rangle$. Also of interest are the fractions $f_{T_q} = \sigma_q/m_N$, which appear quadratically in the cross-section for the scattering of dark matter candidates with the nucleon. The size of the cross-section determines the likelihood of detecting such candidates in the many direct experimental searches, however, the f_{T_q} cannot be extracted from experiment without model assumptions and are not well known. The strange quark fraction, f_{T_s} is of particular interest as is it expected to give the largest contribution to the cross-section.

Two approaches have been pursued to calculate the scalar matrix elements on the lattice: the direct method involving the evaluation of the corresponding quark line diagrams and the indirect method which employs the Hellmann-Feynman theorem. The latter relates $\sigma_{\pi N}$ to the derivative of the nucleon mass with respect to the quark masses.

$$\sigma_{\pi \mathrm{N}} = m_u \frac{\partial m_\mathrm{N}}{\partial m_u} + m_d \frac{\partial m_\mathrm{N}}{\partial m_d} \approx m_\mathrm{PS}^2 \frac{dm_\mathrm{N}}{dm_\mathrm{PS}^2}\bigg|_{m_\mathrm{PS}=m_\pi} \tag{2.1}$$

where m_PS denotes the pseudoscalar meson mass. Similarly, for strangeness $\sigma_s = m_s\partial m_N/\partial m_s$. By fitting the nucleon mass as a function of the pseudoscalar meson masses with a functional form usually taken from baryon chiral perturbation theory, the sigma terms can be obtained.

We apply a combined approach. From a direct calculation with $m_\pi \approx 285$ MeV we obtain [3]

$$\sigma_{\pi N} = 107(10)\,\mathrm{MeV}, \qquad y = 0.041(37)(49), \qquad f_{T_s} = 0.012(14)^{+10}_{-3} \tag{2.2}$$

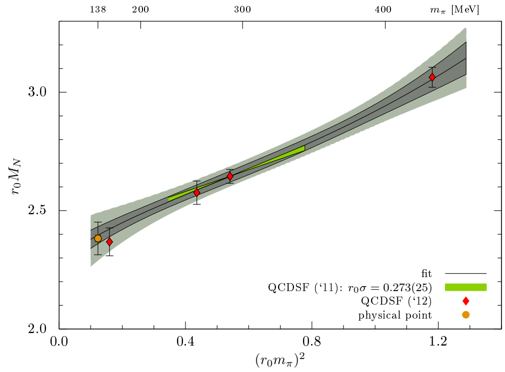

Figure 2: A simultaneous fit to the nucleon mass (as a function of m_π^2) and the direct determination of $\sigma_{\pi N}$. m_N and m_π are given in units of $r_0 = 0.501(10)(11)$ fm. See [5] for details.

where the first error is statistical and the second is from the renormalisation factors. $\sigma_{\pi N} = \sigma_u + \sigma_d$ is a renormalisation group invariant combination. However, the strangeness matrix element mixes with the light quark contributions under renormalisation and this leads to large subtractions for y and f_{T_s}. Performing the calculation at smaller lattice spacings or with an action that has better chiral properties would improve this. Note that in order for the results to be $O(a)$ improved one needs to consider mixing with the gluonic operator aGG. We plan to investigate the gluonic contributions in the future. The ETM Collaboration have also recently performed a direct determination of the sigma term at a heavier quark mass, $m_\pi = 380$ MeV and $\sigma_{\pi N} = 150(1)(10)$ MeV [4]. This is roughly consistent with our result, assuming $m_N \propto m_{PS}^2$.

To obtain a value for $\sigma_{\pi N}$ at the physical point we perform a combined fit with nucleon mass data in the range 160 MeV $\lesssim m_\pi \lesssim$ 430 MeV using a fit function from covariant baryon chiral perturbation theory [5]. The order of the function was varied (up to $O(m_\pi^4)$) along with the values for the low energy constants and the fitting range. Figure 2 displays a typical fit. Note that the direct calculation fixes the slope at $m_\pi = 285$ MeV. We find $\sigma_{\pi N} = 38(8)(6)$ MeV at the physical point, where the first error is statistical and the second is systematic and dominated by the chiral extrapolation. Other recent lattice determinations, shown in Figure 3, are consistent with this value. In the same figure, agreement can also be seen between the different lattice results for f_{T_s}.

3. Quark contributions to the nucleon spin, Δq

The spin of the nucleon can be decomposed into the contributions from its quark and gluonic constituents,

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L + J_g \tag{3.1}$$

where $\Delta\Sigma = \Delta u + \Delta d + \Delta s$ (ignoring heavier quarks) is the quark spin contribution, $L = L_u + L_d + L_s$ is the quark angular momentum contribution and J_g is the gluonic total angular momentum. In our notation Δq represents the combined contribution of the quark and the anti-quark. While in the naïve non-relativistic SU(6) quark model $\Delta\Sigma = 1$ ($\Delta s = L = J_g = 0$) it has been found to be much smaller, for example HERMES reported $\Delta\Sigma = 0.330(11)(25)(28)$ [24]. As discussed above the Δq

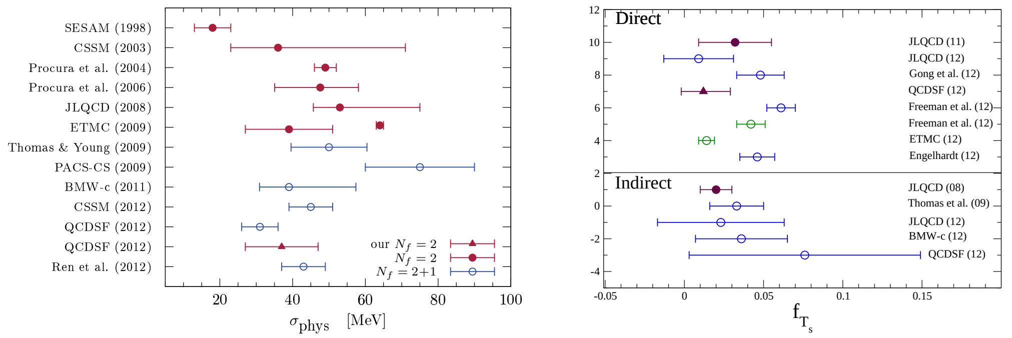

Figure 3: Left: A summary of lattice estimates for $\sigma_{\pi N}$ at the physical point taken from [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Our result is shown as a red triangle. Right: Summary of results for f_{T_s} taken from [4, 10, 12, 14, 16, 18, 19, 20, 21, 22, 23]. The colours and symbols are as in the plot on the left.

are extracted from the first moment of g_1. Fits are made to the experimental data in order to evolve $g_1(x,Q^2)$ in Q^2. There is a minimum accessible x in experiment and assumptions must be made in order to integrate g_1 over the full range of x. The results for the Δs are particularly sensitive to these model assumptions. Figure 4 shows COMPASS results for the strangeness spin contribution for x down to 0.004 from semi-inclusive deep-inelastic scattering (SIDIS) using pion and kaon beams [25]. The results are compatible with zero over the range of x. A naive extrapolation to $x = 0$ gives $\Delta s = -0.02(2)(2)$, while using a parameterisation from De Florian et al (DSSV) [26] gives $\Delta s = -0.10(2)(2)$. However, at present the uncertainties in the experimental results are not well determined due to limited knowledge of the quark fragmentation functions.

Our results for Δu, Δd and Δs for $m_\pi \approx 285$ MeV [27] are displayed in Figure 4 and compared to the values obtained from a DSSV global analysis of different DIS and SIDIS experimental data [28]. Consistency is seen for the u and d quark contributions and there is only a limited variation in the DSSV results with the range of x used. However, we find a very small value for $\Delta s^{\overline{\rm MS}}(\sqrt{7.4}\,{\rm GeV}) = -0.020(10)(4)$ that is only compatible with the truncated DSSV fit. The systematic errors have been inflated to 20% due to the fact that we underestimate $g_A = \Delta u - \Delta d$ by 13%. This provides an indication of the size of the uncertainties due to the lack of continuum limit and unphysical u and d quark masses, however, the statistical error still dominates for Δs. Unlike with the scalar matrix elements, there is very little mixing under renormalisation. Other groups using similar methods, albeit at heavier quark masses and without renormalisation, obtain consistent results, see [29] and [20].

4. The iso-vector quark momentum contribution, $\langle x \rangle_{u-d}$

As mentioned above, $\langle x \rangle_{u-d}$, is well determined from experiment using PDF fits and on the lattice is more straightforward to calculate compared to the other observables considered so far. However, until very recently there was a large discrepancy between the lattice results and the values from PDF parameterisations. This can be seen in Figure 5 which displays results for a variety

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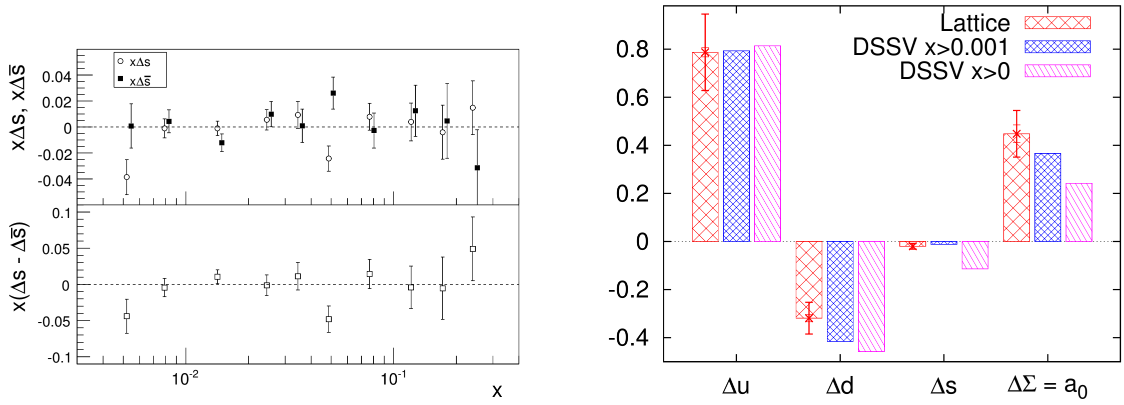

Figure 4: Left: COMPASS results for the strangeness and anti-strangeness spin contribution from SIDIS [25]. Right: Our results for the quark and anti-quark spin contributions compared to the results of DSSV global fits where the full range in x is used (magenta) and where $x > 0.001$ (blue) [28]. The errors include the systematic and statistical uncertainties.

of lattice actions, lattice spacings, volumes and quark masses. For $m_\pi > 250$ MeV the lattice results are consistent but significantly too high. Only new results with near physical pion masses, ours ($\langle x\rangle^{\overline{\rm MS}}_{u-d}(2\,\text{GeV}) = 0.207(16)$ [30]) and those of LHPC [31], are closer to or consistent with the PDF values (albeit within large statistical errors). LHPC use large volumes with $Lm_\pi \sim 3.6-5$ but coarse lattices, $a \sim 0.09-0.12$ fm. In our simulation $a \sim 0.07$ fm, however, the volume is rather small and finite volume effects need to be studied. In addition to the sensitivity to the quark mass, possible contamination from excited states may have an effect and needs to be excluded: to maintain acceptable statistical errors, $t_f - t_i$, where t_i and t_f are the source and sink times, respectively, cannot be made arbitrarily large. However, only in the limit $t_f \gg t \gg t_i$, can the ground state contribution be determined. The unwanted contributions can be reduced, for example, by combining the results from several different $t_f - t_i$ in a summation method [32], as implemented by LHPC. We take the approach of optimising the source and sink interpolating operators and are at present investigating the dependence on $t_f - t_i$. Excited state contamination may explain the lack of a smooth approach to the physical limit from heavier quark masses. Dinter et al. find a 10% reduction in $\langle x\rangle_{u-d}$ at $m_\pi = 380$ MeV when excited states are better accounted for [33].

5. Conclusions

We presented results for iso-singlet quantities which require the calculation of disconnected diagrams. For the quark contributions to the nucleon mass we find $\sigma_{\pi N} = 38(8)(6)$ MeV and $f_{T_s} = 0.012(14)^{+10}_{-3}$. The latter value is much smaller than that often used in phenomenology (see for example [40] which uses $f_{T_s} \sim 1/3$). We find the light and strange quark contributions are of a similar magnitude and in total correspond to between 3% and 8% of the nucleon mass. For the nucleon spin, the strangeness contribution is smaller than expected from some polarised PDF fits: $\Delta s^{\overline{\rm MS}}(\sqrt{7.4}\,\text{GeV}) = -0.020(10)(4)$. The assumption of SU(3) flavour symmetry in hyperon decays used in the fits can be tested on the lattice to help understand this disagreement.

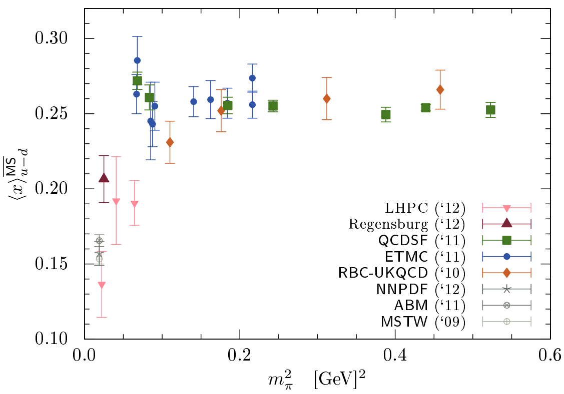

Figure 5: Recent lattice results for $\langle x \rangle_{u-d}$ from [30, 34, 35, 36, 31] compared to the values derived from PDF parameterisations NNPDF [37], ABM [38] and MSTW [39]. All the results are renormalised to the $\overline{\rm MS}$ scheme at 2 GeV.

We also presented results for $\langle x \rangle_{u-d}$ at $m_\pi \approx 157$ MeV. Our findings, together with those of other groups, suggest that with near physical quark masses and control over the excited state contamination a large part of the discrepancy with the PDF values is accounted for. In the future improved statistics can be expected close to or at the physical point. At that point lattice spacing and finite volume effects may also become apparent. Ideally, a comparison would also be made with the individual $\langle x \rangle_q$ rather than the iso-vector combination. First steps in this direction have been made, see [41].

Acknowledgments

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