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A puzzle of the $\pi^0\gamma$ transition form factor—resolved?

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By means of QCD sum rules in the limit of "local duality," we analyze the behaviour of the form factors $F_{P\gamma}(Q^2)$ parametrizing the amplitudes for the transitions $\gamma\gamma^* \to P$ of a real photon γ and a virtual photon γ^* to a pseudoscalar meson $P = \pi^0, \eta, \eta', \eta_c$ as functions of the involved spacelike momentum transfer $Q^2 \geq 0$. Except for the findings of the BABAR collaboration for the $\pi^0\gamma$ form factor, the experimental data for all these form factors are compatible with saturation for large Q^2, as predicted by pQCD factorization. For the light pseudoscalar mesons $P = \pi^0, \eta, \eta'$, saturation is observed already at relatively small $Q^2 \geq 10$–$15\,\mathrm{GeV}^2$, whereas for the η_c meson it sets in only at larger $Q^2 \geq 200$–$300\,\mathrm{GeV}^2$. A recent measurement of the $\pi^0\gamma$ transition form factor by the Belle collaboration seems to resolve this disturbing puzzle as its outcome is compatible with saturation for $Q^2 \geq 10$–$15\,\mathrm{GeV}^2$ and with the large-Q^2 behaviour of the $\eta\gamma$ and $\eta'\gamma$ transition form factors.

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1. Introduction

Already a long time ago, it has been realized that the "two-photon fusion" processes $\gamma^*\gamma^*\to P$ to some pseudoscalar meson $P=\pi^0,\eta,\eta',\eta_c$ constitute rather crucial tests for our understanding of quantum chromodynamics (QCD) and of the internal structure of hadrons. Over the years, several experiments have collected impressive amounts of information on these transition processes [1–5].

As far as the theoretical description of such kind of transition of two—in general, off-shell—photons γ^*, with associated polarization four-vectors $\varepsilon_{1,2}$, to a pseudoscalar meson P is concerned, the corresponding amplitude turns out to be parametrizable by just a single form factor $F_{P\gamma\gamma}(q_1^2,q_2^2)$:

$$\langle\gamma^*(q_1)\gamma^*(q_2)|P(p)\rangle = \mathrm{i}\varepsilon_{\varepsilon_1\varepsilon_2 q_1 q_2}F_{P\gamma\gamma}(q_1^2,q_2^2).$$

QCD factorization of short and long distances provides a robust prediction for the behaviour of this form factor at asymptotically large spacelike momentum transfers $q_1^2\equiv -Q_1^2\le 0$, $q_2^2\equiv -Q_2^2\le 0$ [6]:

$$F_{P\gamma\gamma}(Q_1^2,Q_2^2)\to 12e_c^2 f_P\int_0^1\frac{\mathrm{d}\xi\,\xi(1-\xi)}{Q_1^2\xi+Q_2^2(1-\xi)}.$$

For convenience, we henceforth prefer the notation $Q^2\equiv Q_2^2$ and $0\le\beta\equiv Q_1^2/Q_2^2\le 1$ (that is, Q_2^2 is the larger virtuality). For the kinematics of experimental interest, $Q_1^2\approx 0$ and $Q_2^2\equiv Q^2$, for instance, the $\gamma\gamma^*\to\pi$ transition form factor $F_{\pi\gamma}(Q^2)$ asymptotically behaves like $Q^2F_{\pi\gamma}(Q^2)\to\sqrt{2}f_\pi$, where $f_\pi=130$ MeV is the charged-pion decay constant. Similar relations arise for both η and η' mesons.

2. Dispersive QCD sum rule for the form factors of the generic transitions $\gamma^*\gamma^*\to P$

The analysis of the transition $\gamma^*\gamma^*\to P$ within the framework of the QCD sum-rule approach conveniently starts from the transition of two virtual photons γ^* to the vacuum, induced by the quark axial-vector current j_μ^5; its amplitude is found by factorizing off the photon polarization vectors $\varepsilon_{1,2}$:

$$\langle 0|j_\mu^5|\gamma^*(q_1)\gamma^*(q_2)\rangle = e^2T_{\mu\alpha\beta}(p|q_1,q_2)\,\varepsilon_1^\alpha\varepsilon_2^\beta,\qquad p\equiv q_1+q_2. \tag{2.1}$$

We are interested in this amplitude for $-q_1^2\equiv Q_1^2\ge 0$ and $-q_2^2\equiv Q_2^2\ge 0$. The general decomposition of $T_{\mu\alpha\beta}$ contains *four* independent Lorentz structures [7, 8] but in our present study only one enters:

$$T_{\mu\alpha\beta}(p|q_1,q_2)=p_\mu\varepsilon_{\alpha\beta q_1 q_2}\mathrm{i}F(p^2,Q_1^2,Q_2^2)+\cdots.$$

For the related invariant amplitude $F(p^2,Q_1^2,Q_2^2)$, a spectral representation in p^2 for fixed Q_1^2 and Q_2^2 values may be given in terms of its physical spectral density $\Delta(s,Q_1^2,Q_2^2)$ and physical threshold s_{th}:

$$F(p^2,Q_1^2,Q_2^2)=\frac{1}{\pi}\int_{s_{\mathrm{th}}}^\infty\frac{\mathrm{d}s}{s-p^2}\Delta(s,Q_1^2,Q_2^2).$$

Perturbative QCD (pQCD) expresses this spectral density as power series in the strong coupling α_s:

$$\Delta_{\mathrm{pQCD}}(s,Q_1^2,Q_2^2|m)=\Delta^{(0)}_{\mathrm{pQCD}}(s,Q_1^2,Q_2^2|m)+\frac{\alpha_s}{\pi}\Delta^{(1)}_{\mathrm{pQCD}}(s,Q_1^2,Q_2^2|m)+\frac{\alpha_s^2}{\pi^2}\Delta^{(2)}_{\mathrm{pQCD}}(s,Q_1^2,Q_2^2|m)+\cdots,$$

where m is the mass of the quark that propagates in that quark loop to which the two photons couple. The well-known lowest-order term $\Delta^{(0)}_{\mathrm{pQCD}}$ arises from the graph of this one-loop quark triangle with one axial and two vector currents at its vertices [9]. The two-loop $O(\alpha_s)$ correction $\Delta^{(1)}_{\rm pQCD}$ proves to vanish [10]. The three-loop $O(\alpha_s^2)$ correction $\Delta^{(2)}_{\rm pQCD}$ was found to yield a nonzero contribution [11].

In the region of small s, the physical spectral density bears no resemblance to $\Delta_{\rm pQCD}(s,Q_1^2,Q_2^2)$ as it must model both meson pole and hadron continuum. For instance, in the $I=1$ channel one has

$$\Delta(s,Q_1^2,Q_2^2)=\pi\delta(s-m_\pi^2)\sqrt{2}f_\pi F_{\pi\gamma\gamma}(Q_1^2,Q_2^2)+\theta(s-s_{\rm th})\Delta^{(I=1)}_{\rm cont}(s,Q_1^2,Q_2^2).$$

QCD sum rules provide a possibility to relate the properties of hadronic ground states to the spectral densities of QCD correlators. Applying this approach in the conventional way devised by Shifman, Vainshtein, and Zakharov proceeds along a standard routine [12, 13] involving the following steps:

1. Evaluate $F(p^2,Q_1^2,Q_2^2)$ at QCD and hadron level and equate the two resulting representations.
2. In order to suppress effects of the hadron continuum, perform a Borel transformation $p^2\to\tau$.
3. Consider the arising sum rule in the limit of local duality (LD), realized if the Borel parameter τ vanishes [14], to wipe out unwanted nonperturbative power corrections increasing with Q^2.
4. Implement quark–hadron duality by the customary cut of the spectral integral at low energies.

With decay constant f_P, this yields for the transition form factor a sum rule of innocent appearance:

$$\pi f_P F_{P\gamma\gamma}(Q_1^2,Q_2^2)=\int_{4m^2}^{s_{\rm eff}(Q_1^2,Q_2^2)}{\rm d}s\,\Delta_{\rm pQCD}(s,Q_1^2,Q_2^2|m).$$

Therein, all nonperturbative QCD phenomena are encoded in an effective threshold $s_{\rm eff}(Q_1^2,Q_2^2)$; the actual challenge is to design a convincing algorithm for fixing this threshold—a nontrivial task [12].

- For asymptotically large $Q_2^2\equiv Q^2\to\infty$ but fixed ratio $\beta\equiv Q_1^2/Q_2^2$, $s_{\rm eff}(Q^2,\beta)$ may be inferred from pQCD factorization: Generally, for nonzero quark mass $m\neq 0$, $s_{\rm eff}(Q^2\to\infty,\beta)$ depends on β; for $m=0$, the factorization formula is recovered for any β if $s_{\rm eff}(Q^2\to\infty,\beta)=4\pi^2 f_\pi^2$.
- The naïve LD *model* for the transition form factor *assumes* that, also for finite Q^2, $s_{\rm eff}(Q^2,\beta)$ may be sufficiently well approximated by its asymptotic limit: $s_{\rm eff}(Q^2,\beta)=s_{\rm eff}(Q^2\to\infty,\beta)$.

In the LD limit, the form factor $F_{P\gamma}(Q^2)\equiv F_{P\gamma\gamma}(0,Q^2)$ for the transition of a pseudoscalar meson P to a real ($Q_1^2=0$) and a virtual ($Q_2^2\neq 0$) photon reads, for a single massless ($m=0$) quark flavour,

$$F_{P\gamma}(Q^2)=\frac{1}{2\pi^2 f_P}\frac{s_{\rm eff}(Q^2)}{s_{\rm eff}(Q^2)+Q^2}. \qquad (2.2)$$

$F_{P\gamma}(Q^2=0)$ is related to the axial anomaly [7] irrespective of the behaviour of $s_{\rm eff}(Q^2)$ near $Q^2=0$.

3. Form factor for the transition $\gamma^*\gamma^*\to\eta_c$

For bound states composed of heavy quarks, the quark mass can no longer be neglected. Finite quark masses provide an option to exploit not only the correlator $\langle AVV\rangle$ [as in Eq. (2.1)] but also the correlator $\langle PVV\rangle$ [8], with, in each case, an LD model of its own; pQCD factorization then predicts

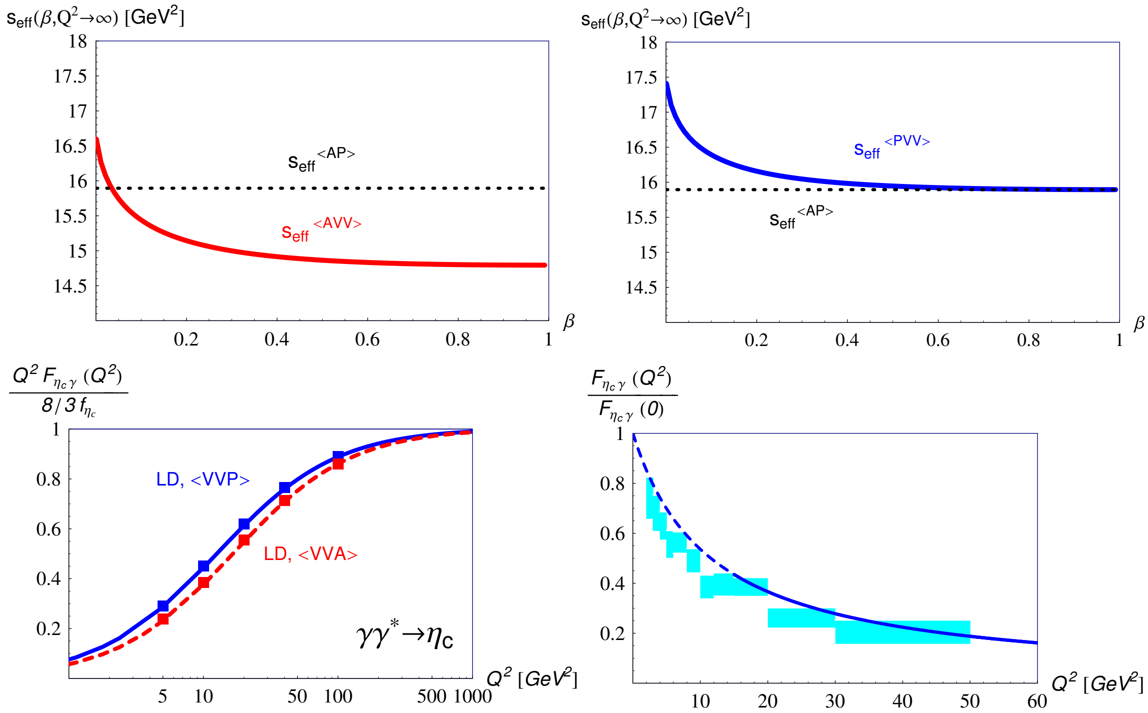

Figure 1: Form factor for the transition $\gamma\gamma^*\to\eta_c$: Exact effective thresholds $s_{\rm eff}^{AVV}(Q^2\to\infty,\beta)$ (top left) and $s_{\rm eff}^{PVV}(Q^2\to\infty,\beta)$ (top right); form factors obtained for finite Q^2 from the LD sum rules for the correlators $\langle AVV\rangle$ and $\langle PVV\rangle$ (bottom left); LD model for the correlator $\langle PVV\rangle$ fitting to BABAR data [3] (bottom right).

$s_{\rm eff}(Q^2\to\infty,\beta)$ for both $\langle AVV\rangle$ and $\langle PVV\rangle$. Figure 1 summarizes our findings: the *exact* effective thresholds $s_{\rm eff}^{AVV}(Q^2\to\infty,\beta)$ and $s_{\rm eff}^{PVV}(Q^2\to\infty,\beta)$ in the $\langle AVV\rangle$ and $\langle PVV\rangle$ sum rules differ in their behaviour from each other as well as from the effective thresholds of relevant two-point correlators. The LD *assumption* $s_{\rm eff}(Q^2,\beta)=s_{\rm eff}(Q^2\to\infty,\beta)$ entails the form-factor behaviour depicted in the bottom row of Fig. 1. For very small Q^2, this simple LD model cannot be expected to be applicable. Interestingly, it yields $F_{\eta_c\gamma}(Q^2=0)=0.067$ GeV^{-1} from $\langle AVV\rangle$ and $F_{\eta_c\gamma}(Q^2=0)=0.086$ GeV^{-1} from $\langle PVV\rangle$, in reasonable agreement with the measured value $F_{\eta_c\gamma}(Q^2=0)=0.08\pm0.01$ GeV^{-1}. Consequently, we feel entitled to conclude that the LD evaluation of, at least, the correlator $\langle PVV\rangle$ provides reliable predictions for a broad Q^2 range starting at very low Q^2 values (see also Ref. [15]).

4. Form factor for the transitions $\gamma\gamma^*\to(\eta,\eta')$

For $\eta^{(\prime)}$ transitions, the form factors $F_{(\eta,\eta')\gamma}(Q^2)$ are mixtures [16] of nonstrange contributions $F_{n\gamma}(Q^2)$, with n abbreviating $(\bar uu+\bar dd)/\sqrt{2}$, and strange contributions $F_{s\gamma}(Q^2)$, with s indicating $\bar ss$:

$$F_{\eta\gamma}(Q^2)=F_{n\gamma}(Q^2)\cos\phi-F_{s\gamma}(Q^2)\sin\phi,\qquad F_{\eta'\gamma}(Q^2)=F_{n\gamma}(Q^2)\sin\phi+F_{s\gamma}(Q^2)\cos\phi,$$

with mixing angle $\phi\approx38^\circ$. Of course, the sum rules in LD limit for the latter form factors involve two separate effective thresholds, $s_{\rm eff}^{(n)}=4\pi^2 f_n^2$ and $s_{\rm eff}^{(s)}=4\pi^2 f_s^2$, with $f_n\approx1.07 f_\pi$ and $f_s\approx1.36 f_\pi$:

$$F_{n\gamma}(Q^2)=\frac{1}{f_n}\int_0^{s_{\rm eff}^{(n)}(Q^2)}ds\,\Delta_n(s,Q^2),\qquad F_{s\gamma}(Q^2)=\frac{1}{f_s}\int_0^{s_{\rm eff}^{(s)}(Q^2)}ds\,\Delta_s(s,Q^2).$$

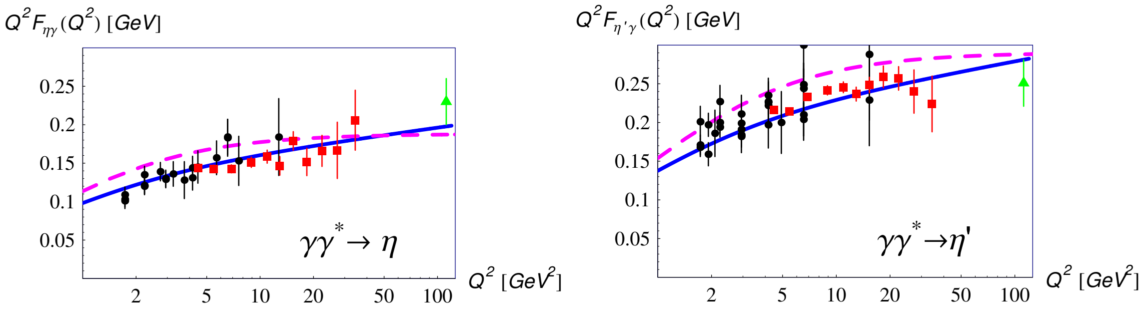

Figure 2: Form factors $F_{(\eta,\eta')\gamma}(Q^2)$ for the transitions $\gamma\gamma^* \to (\eta,\eta')$: LD predictions [7, 8] (dashed lines) and fits [17] (solid lines) to measurements by CELLO and CLEO [1] (black dots) and BABAR (red dots) [4].

Figure 2 reveals that LD sum rules [7, 8] and experiment [1, 4] can live with each other pretty well.

5. Form factor for the transition $\gamma\gamma^* \to \pi^0$

By construction of the sum-rule formalism, the behaviour of any of the π^0, η, and η' transition form factors in the limit of large Q^2 is governed by spectral densities to be deduced by evaluating the

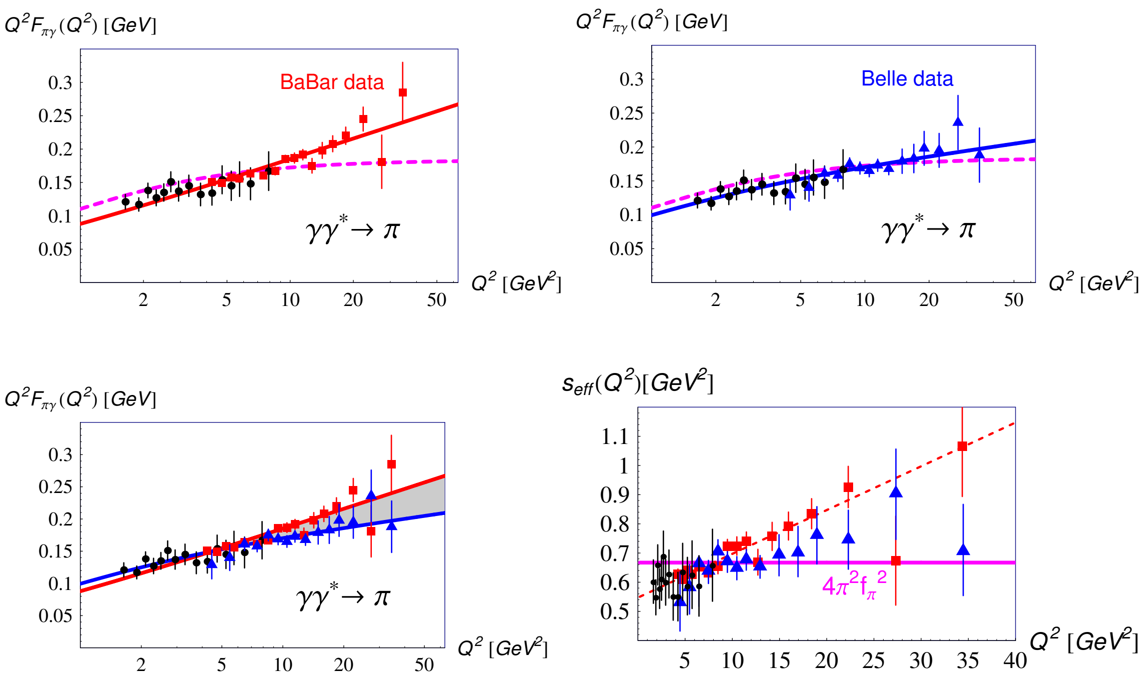

Figure 3: Form factor $F_{\pi\gamma}(Q^2)$ for the transition $\gamma\gamma^* \to \pi^0$: CELLO and CLEO [1] (black dots) vs. BABAR [2] (top left) and Belle [5] (top right) data; discrepancy (grey shaded area) between BABAR [2] and Belle [5] data for larger Q^2 (bottom left); equivalent effective threshold $s_{\rm eff}(Q^2)$ inferred for each data point by means of Eq. (2.2) (bottom right). Magenta lines represent the LD model, red or blue solid lines a fit [17] to the data.

relevant pQCD Feynman diagrams; therefore, it has to be identical for all light pseudoscalar mesons [17]: The sum rule for the correlator $\langle AVV\rangle$ in LD limit is equivalent to the anomaly sum rule [18]

$$F_{\pi\gamma}(Q^2)=\frac{1}{2\sqrt{2}\,\pi^2 f_\pi}\left[1-2\pi\int_{s_{\rm th}}^{\infty}{\rm d}s\,\Delta^{(I=1)}_{\rm cont}(s,Q^2)\right],$$

with similar relations emerging for the $I=0$ and $\bar{s}s$ channels. The behaviour of the spectral densities $\Delta_{\rm cont}(s,Q^2)$ at large s determines that of all form factors $F_{\pi\gamma}(Q^2)$, $F_{\eta\gamma}(Q^2)$, and $F_{\eta'\gamma}(Q^2)$ at large Q^2 [17]. Now, quark–hadron duality assumes all $\Delta_{\rm cont}(s,Q^2)$ to be equal to the associated $\Delta_{\rm pQCD}(s,Q^2)$ in the respective channel; as purely perturbative quantities, all the latter must be equal to each other.

Figure 3 compares several sets of experimental data available for the π^0 transition form factor. The BABAR measurement comes as a great surprise in two respects: On the one hand, it obviously disagrees with the η and η' form factors and with the conventional LD model for Q^2 up to 40 GeV^2. On the other hand, the LD violations claimed to have been found by BABAR rise with Q^2 even near $Q^2\approx 40$ GeV^2, which is in conflict with hints from quantum-mechanical analogues. Our confidence in our precursor studies forces us to conclude that it might be hard to put forward any interpretation of the BABAR results within QCD (see also the related discussions in Refs. [19]). More recent Belle results for $F_{\pi\gamma}(Q^2)$, although within errors compatible with their BABAR counterparts (cf. [20, 21]), strengthen our trust: the Belle π^0 transition form-factor behaviour for large Q^2 resembles the one of η and η' and agrees with the expected onset of the LD regime already in the range $Q^2\geq 5$–10 GeV^2.

6. Conclusions

The form factors parametrizing the amplitudes for the transitions $P\to\gamma\gamma^*$ of the pseudoscalar mesons $P=\pi^0,\eta,\eta',\eta_c$ have been analyzed within the framework of local-duality QCD sum rules; there a single key quantity, the effective continuum threshold, comprises all nonperturbative effects. Aligning form factors and QCD factorization yields a threshold model that we regard as successful:

- For all form factors studied, local duality should perform well for Q^2 larger than a few GeV^2:
  - For the transitions $\eta\to\gamma\gamma^*$, $\eta'\to\gamma\gamma^*$, and $\eta_c\to\gamma\gamma^*$, it indeed works reasonably well.
  - For the transition $\pi^0\to\gamma\gamma^*$, BABAR measures a considerable violation of local duality, manifesting by the effective threshold continuing to rise linearly instead of approaching *asymptotically* a finite constant, whereas the trend observed by Belle fits to local duality.
- As a whole, the existing experimental data on meson–photon transitions point towards a tiny residual *logarithmic* rise of $Q^2F_{P\gamma}(Q^2)$ [17]. If confirmed, this effect may be interpreted by amending the ratio of hadron-level and QCD-level spectral densities by an LD-violating term.
- Quantum-mechanical experience also leads us to suspect that the LD sum-rule prediction for the *elastic* form factor of the *charged* pion improves with Q^2 for $Q^2\gtrapprox 4$–8 GeV^2 and that the corresponding effective threshold approaches its *asymptotic LD limit*, $s_{\rm eff}(Q^2\to\infty)=4\pi^2 f_\pi^2$, already at $Q^2\approx 5$–6 GeV^2 [7], which is verifiable by CLAS12 after the JLab 12 GeV upgrade.

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