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Order-v^4 Relativistic Corrections to Gluon Fragmentation into $^3S_1$ Quarkonium

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We compute the relativistic corrections to the color-singlet contribution to gluon fragmentation into a J/ψ at relative order v^4, making use of the nonrelativistic QCD (NRQCD) factorization approach. The corresponding full-QCD process exhibits infrared divergences that manifest themselves as single and double poles in ε in $4-2\varepsilon$ dimensions. We isolate the infrared-divergent contributions and treat them analytically. In the matching of full QCD to NRQCD, the pole contributions are absorbed into long-distance NRQCD matrix elements. The renormalizations of the ultraviolet divergences of the long-distance NRQCD matrix elements involve Born and one-loop single-pole counterterm contributions and Born double-pole counterterm contributions. While the order-v^4 contribution enhances the J/ψ hadroproduction rate for the color-singlet channel substantially, this contribution is not important numerically in comparison with the color-octet contributions. We also find that the ^3P_J color-octet channel in the gluon fragmentation function contributes to J/ψ hadroproduction significantly in comparison with the complete contribution of next-to-leading order in α_s in that channel.

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1. Introduction

In this proceedings contribution, we summarize the computation of the fragmentation of a gluon into a spin-triplet S-wave quarkonium via the $^3S_1^{[1]}$ channel in relative order v^4, where v is the velocity of the heavy quark Q or heavy antiquark $\bar{Q}$ in the quarkonium rest frame. We refer the reader to Ref. [1] for details of this calculation. The expression $^{2s+1}l_j^{[c]}$ is the standard spectroscopic notation for the spin s, orbital angular momentum l, total angular momentum j, and color $c = 1$ (singlet) or 8 (octet) of the $Q\bar{Q}$ pair that is created at short distances and that evolves into the heavy quarkonium.

Our computation make use of the nonrelativistic QCD (NRQCD) factorization approach [2]. That is, we express the fragmentation functions as sums of products of NRQCD long-distance matrix elements (LDMEs) and short-distance coefficients. The focus of our calculation is the short-distance coefficient that appears in the order v^4 contribution to the fragmentation function for the $^3S_1^{[1]}$ channel. The short-distance coefficients for gluon fragmentation in this channel in relative orders v^0 and v^2 have already been computed in Refs. [3, 4] and [5], respectively. The contribution of relative order v^4 is interesting theoretically because it is at this order that the $^3S_1^{[1]}$ fragmentation channel first develops soft divergences. These soft divergences are ultimately absorbed into the NRQCD LDME for the evolution of a $^3S_1^{[8]}$ or $^3P_J^{[8]}$ $Q\bar{Q}$ pair into a spin-triplet S-wave quarkonium state. Consequently, the $^3S_1^{[1]}$ contribution in order v^4 and the $^3S_1^{[8]}$ and $^3P_J^{[8]}$ contributions at the leading nontrivial order in v are related by logarithms of the factorization scale. Since the $^3S_1^{[8]}$ contribution to the fragmentation of a gluon into a J/ψ is known to be significant phenomenologically, it is important to compute the $^3S_1^{[1]}$ contribution in order v^4, as well.

An important technical issue in this calculation is the appearance of both single and double soft poles in the dimensional-regularization parameter ε. This is the first NRQCD factorization calculation in which double soft poles have appeared. In order to effect the matching between NRQCD and full QCD that removes these poles, it is necessary to work out, for the first time, two-loop corrections to NRQCD operators and the corresponding ultraviolet renormalizations.

We employ the Collins-Soper definition [6] of the fragmentation function for a gluon fragmenting into a quarkonium. We compute the full-QCD fragmentation functions for a gluon fragmenting into free $Q\bar{Q}$ states with various quantum numbers. We then determine the NRQCD short-distance coefficients by comparing the full-QCD fragmentation functions with the corresponding NRQCD expressions, making use of the NRQCD factorization formulas.

The remainder of this paper is organized as follows. In Sec. 2, we present the NRQCD factorization formulas through relative order v^4 for the fragmentation functions for a gluon fragmenting into a J/ψ. The results of the full-QCD calculation for the corresponding fragmentation functions for free $Q\bar{Q}$ states are given in Sec. 3. In Sec. 4, we compute the relevant NRQCD LDMEs for free $Q\bar{Q}$ states analytically in dimensional regularization, and we determine the evolution equations for the LDMEs. In Sec. 5, we compute the short-distance coefficients by matching the NRQCD and full-QCD results. Sec. 6 contains estimates of the relative sizes of the various fragmentation contributions to the J/ψ hadroproduction cross section. We give a summary of our results in Sec. 7.

2. Factorization formulas

We denote by $D[g \to J/\psi](z,\mu_\Lambda)$ the fragmentation function for a gluon fragmenting into a J/ψ. Here, μ_Λ is the factorization scale, $z = P^+/k^+$, and P and k are the momenta of the J/ψ and the fragmenting gluon, respectively. We define light-cone coordinates for a four-vector $V = (V^+, V^-, \boldsymbol{V}_\perp)$ by $V^\pm \equiv (V^0 \pm V^3)/\sqrt{2}$. We make use of the Collins-Soper definition of $D[g \to J/\psi](z,\mu_\Lambda)$ [6], which is gauge invariant, and we regularize soft and ultraviolet divergences dimensionally, taking $d = 4-2\varepsilon$ space-time dimensions.

The NRQCD factorization formula for $D[g \to J/\psi](z,\mu_\Lambda)$ is given by

$$D[g \to J/\psi](z) = \sum_n d_n(z) \langle 0|\mathscr{O}_n^{J/\psi}|0\rangle, \tag{2.1}$$

where the $\langle 0|\mathscr{O}_n^{J/\psi}|0\rangle$ are NRQCD LDMEs, the $d_n(z)$ are short-distance coefficients, and we have suppressed the μ_Λ dependences in $d_n(z)$ and $\langle 0|\mathscr{O}_n^{J/\psi}|0\rangle$. Since the $d_n(z)$ are independent of the hadronic final state, we can write the fragmentation function for a gluon fragmenting into a free $Q\bar{Q}$ state as

$$D[g \to Q\bar{Q}](z) = \sum_n d_n(z) \langle 0|\mathscr{O}_n^{Q\bar{Q}}|0\rangle. \tag{2.2}$$

By computing the left side of Eq. (2.2) in full QCD and comparing it with the right side, we can determine the $d_n(z)$. In the remainder of this paper, we denote the order-v^k contribution to $D[g \to H]$ by $D_k[g \to H]$, where H can be either a quarkonium or a free-$Q\bar{Q}$ final state.

The operator LDMEs that we use in this paper are

$$\langle 0|\mathscr{O}_0^H(^3S_1^{[1]})|0\rangle = \langle 0|\chi^\dagger \sigma^i \psi \, \mathscr{P}_H \, \psi^\dagger \sigma^i \chi|0\rangle, \tag{2.3}$$

$$\langle 0|\mathscr{O}_2^H(^3S_1^{[1]})|0\rangle = \frac{1}{2}\langle 0|\chi^\dagger \sigma^i (-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2 \psi \, \mathscr{P}_H \, \psi^\dagger \sigma^i \chi + \text{H. c.}|0\rangle, \tag{2.4}$$

$$\langle 0|\mathscr{O}_0^H(^1S_0^{[8]})|0\rangle = \langle 0|\chi^\dagger T^a \psi \, \mathscr{P}_H \, \psi^\dagger T^a \chi|0\rangle, \tag{2.5}$$

$$\langle 0|\mathscr{O}_{4,1}^H(^3S_1^{[1]})|0\rangle = \langle 0|\chi^\dagger \sigma^i (-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2 \psi \, \mathscr{P}_H \, \psi^\dagger \sigma^i (-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2 \chi|0\rangle, \tag{2.6}$$

$$\langle 0|\mathscr{O}_{4,2}^H(^3S_1^{[1]})|0\rangle = \frac{1}{2}\langle 0|\chi^\dagger \sigma^i (-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^4 \psi \, \mathscr{P}_H \, \psi^\dagger \sigma^i \chi + \text{H. c.}|0\rangle, \tag{2.7}$$

$$\begin{aligned}\langle 0|\mathscr{O}_{4,3}^H(^3S_1^{[1]})|0\rangle = \frac{1}{2}\langle 0|&\chi^\dagger \sigma^i \psi \, \mathscr{P}_H \, \psi^\dagger \sigma^i (\overleftrightarrow{\boldsymbol{D}} \cdot g_s \boldsymbol{E} + g_s \boldsymbol{E} \cdot \overleftrightarrow{\boldsymbol{D}}) \chi \\ &- \chi^\dagger \sigma^i (\overleftrightarrow{\boldsymbol{D}} \cdot g_s \boldsymbol{E} + g_s \boldsymbol{E} \cdot \overleftrightarrow{\boldsymbol{D}}) \psi \, \mathscr{P}_H \, \psi^\dagger \sigma^i \chi|0\rangle,\end{aligned} \tag{2.8}$$

$$\langle 0|\mathscr{O}_0^H(^3S_1^{[8]})|0\rangle = \langle 0|\chi^\dagger \sigma^i T^a \psi \, \mathscr{P}_H \, \psi^\dagger \sigma^i T^a \chi|0\rangle, \tag{2.9}$$

$$\langle 0|\mathscr{O}_0^H(^3P^{[8]})|0\rangle = \langle 0|\chi^\dagger (-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^r \sigma^n T^a \psi \, \mathscr{P}_H \, \psi^\dagger (-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^r \sigma^n T^a \chi|0\rangle, \tag{2.10}$$

where $g_s = \sqrt{4\pi\alpha_s}$, $\psi^\dagger$ and χ are two-component (Pauli) fields that create a heavy quark and a heavy antiquark, $\boldsymbol{D}$ is the gauge-covariant derivative, and $\boldsymbol{E}$ is the chromoelectric field operator. $\mathscr{P}_{H(P)} = \sum_X |H(P)+X\rangle\langle H(P)+X|$ is a projection onto a state consisting of a quarkonium H, with four-momentum P, plus anything. $\mathscr{P}_{H(P)}$ contains a sum over any quarkonium polarization quantum numbers that are not specified explicitly. The symmetric traceless product is defined by $A^{(i}B^{j)} = \frac{1}{2}(A^iB^j + A^jB^i) - \frac{1}{d-1}\delta^{ij}A^kB^k$, and the antisymmetric product is defined by $A^{[i}B^{j]} = \frac{1}{2}(A^iB^j - A^jB^i)$.

We now give the NRQCD factorization formulas for gluon fragmentation into a J/ψ through relative order v^4. The contributions to $D[g \to J/\psi]$ in relative orders v^0, v^2, and v^3 are

$$D_0[g \to J/\psi] = d_0[g \to Q\bar{Q}(^3S_1^{[1]})]\langle 0|\mathcal{O}_0^{J/\psi}(^3S_1^{[1]})|0\rangle, \tag{2.11}$$
$$D_2[g \to J/\psi] = d_2[g \to Q\bar{Q}(^3S_1^{[1]})]\langle 0|\mathcal{O}_2^{J/\psi}(^3S^{[1]})|0\rangle, \tag{2.12}$$
$$D_3[g \to J/\psi] = d_0[g \to Q\bar{Q}(^1S_0^{[8]})]\langle 0|\mathcal{O}_0^{J/\psi}(^1S_0^{[8]})|0\rangle. \tag{2.13}$$

The short-distance coefficients $d_0[g \to Q\bar{Q}(^3S_1^{[1]})]$ and $d_2[g \to Q\bar{Q}(^3S_1^{[1]})]$ were calculated previously in Refs. [3, 4] and Ref. [5], respectively. The short-distance coefficient $d_0[g \to Q\bar{Q}(^1S_0^{[8]})]$ is related by an overall color factor to $d_0[g \to Q\bar{Q}(^1S_0^{[1]})]$, which was calculated in Ref. [7]. The contribution to $D[g \to J/\psi]$ in relative order v^4 is

$$\begin{aligned} D_4[g \to J/\psi] = &\left\{ d_{4,1}[g \to Q\bar{Q}(^3S_1^{[1]})] + d_{4,2}[g \to Q\bar{Q}(^3S_1^{[1]})] \right\} \langle 0|\mathcal{O}_4^{J/\psi}(^3S_1^{[1]})|0\rangle \\ &+ d_0[g \to Q\bar{Q}(^3P^{[8]})]\langle 0|\mathcal{O}_0^{J/\psi}(^3P^{[8]})|0\rangle \\ &+ d_0[g \to Q\bar{Q}(^3S_1^{[8]})]\langle 0|\mathcal{O}_0^{J/\psi}(^3S_1^{[8]})|0\rangle. \end{aligned} \tag{2.14}$$

The short-distance coefficient $d_0[g \to Q\bar{Q}(^3S_1^{[8]})]$ was calculated previously in Refs. [5, 7, 8, 9]. The short-distance coefficient $d_0[g \to Q\bar{Q}(^3P^{[8]})]$ is related by an overall color factor to the sum over J of the short-distance coefficients $d_0[g \to Q\bar{Q}(^3P_J^{[1]})]$ that were calculated in Ref. [7]. The computation of the combination of short-distance coefficients $d_{4,1}[g \to Q\bar{Q}(^3S_1^{[1]})] + d_{4,2}[g \to Q\bar{Q}(^3S_1^{[1]})]$ is the main goal of this work. [1]

3. Full-QCD calculations

We compute the fragmentation functions for free $Q\bar{Q}$ states in full QCD in $d = 4 - 2\varepsilon$ dimensions, multiplying the square of the dimensional-regularization scale μ by the factor $e^{\gamma_E}/(4\pi)$ that is appropriate to the modified-minimal-subtraction ($\overline{\rm MS}$) scheme. Here, γ_E is the Euler-Mascheroni constant.

The results for $D_0[g \to Q\bar{Q}(^3S_1^{[8]})]$ and $D_2[g \to Q\bar{Q}(^3P^{[8]})]$ at leading order (LO) in α_s and v are

$$D_0[g \to Q\bar{Q}(^3S_1^{[8]})] = \frac{\pi\alpha_s}{m^3}\left(\frac{\mu^2}{4\pi}e^{\gamma_E}\right)^{\varepsilon}\delta(1-z), \tag{3.1}$$
$$\begin{aligned} D_2[g \to Q\bar{Q}(^3P^{[8]})] = &\frac{8\alpha_s^2\boldsymbol{q}^2}{(d-1)m^5}\frac{N_c^2-4}{4N_c}(1-\varepsilon)\Gamma(1+\varepsilon)\left(\frac{\mu^2}{4\pi}e^{\gamma_E}\right)^{\varepsilon}\left(\frac{\mu^2}{4m^2}e^{\gamma_E}\right)^{\varepsilon} \\ &\times\left[-\frac{1}{2\varepsilon_{\rm IR}}\delta(1-z) + f(z)\right], \end{aligned} \tag{3.2}$$

[1] In fact, the color-singlet contribution to $D_4[g \to J/\psi]$ is a linear combination of the LDMEs $\langle 0|\mathcal{O}_{4,i}^{J/\psi}(^3S_1^{[1]})|0\rangle$, for $i =$ 1–3. We express the contribution of $\langle 0|\mathcal{O}_{4,3}^{J/\psi}(^3S_1^{[1]})|0\rangle$ in terms of the other two LDMEs by making use of the NRQCD equations of motion, and we use the vacuum-saturation approximation to replace the $\langle 0|\mathcal{O}_{4,i}^{J/\psi}(^3S_1^{[1]})|0\rangle$ with a single LDME, $\langle 0|\mathcal{O}_4^{J/\psi}(^3S_1^{[1]})|0\rangle$, making an error of relative order v^2 [10].

where N_c is the number of colors, $\boldsymbol{q}$ is half the relative momentum of the Q and $\bar{Q}$ in the $Q\bar{Q}$ rest frame, and the finite function $f(z)$ is defined in Eq. (5.20) of Ref.[1]. The subscript "IR" in $\varepsilon_{\rm IR}^{-1}$ indicates that the pole is infrared in origin.

The result for $D_4[g \to Q\bar{Q}(^3S_1^{[1]})]$ at LO in α_s is

$$D_4[g \to Q\bar{Q}(^3S_1^{[1]})] = D_4[g \to Q\bar{Q}(^3S_1^{[1]})]^{\rm finite} + I[S_{12}] + I[S_1] + I[S_2], \tag{3.3}$$

where $I[S_{12}]$, $I[S_1]$, and $I[S_2]$ contain the double- and single-pole contributions:

$$\begin{aligned} I[S_{12}] \,=\, & \left\{ \frac{1}{8\varepsilon_{\rm IR}^2}\,\delta(1-z) - \frac{1}{2\varepsilon_{\rm IR}} \left[\frac{1}{(1-z)^{1+4\varepsilon}}\right]_+ + \frac{1-z^{1+2\varepsilon}}{2\varepsilon_{\rm IR}(1-z)^{1+4\varepsilon}} \right\} \\ & \times \left(\frac{8\alpha_s}{3\pi m^2}\right)^2 \frac{N_c^2-4}{16N_c^2}\,\frac{\pi\alpha_s}{(d-1)m^3} \left(\frac{\mu^2}{4\pi}e^{\gamma_{\rm E}}\right)^{\varepsilon} \frac{\boldsymbol{q}^4}{d-1} \\ & \times \left(\frac{\mu^2}{4m^2}e^{\gamma_{\rm E}}\right)^{2\varepsilon} \frac{\Gamma^2(1+\varepsilon)\Gamma^2(1-2\varepsilon)}{\Gamma(1-4\varepsilon)} (1-\varepsilon)(6-2\varepsilon-\varepsilon^2-2\varepsilon^3), \end{aligned} \tag{3.4}$$

$$\begin{aligned} I[S_1] \,=\, I[S_2] = & \left(-\frac{\tau_1}{2\varepsilon_{\rm IR}} + \tau_0\right) \left(\frac{\mu^2}{4m^2}e^{\gamma_{\rm E}}\right)^{2\varepsilon} \frac{z^{-2+2\varepsilon}(1-z)^{-4\varepsilon}\Gamma^2(1+\varepsilon)}{48(1-\varepsilon)} \\ & \times \left(\frac{8\alpha_s}{3\pi m^2}\right)^2 \frac{N_c^2-4}{16N_c^2} \left(\frac{\mu^2}{4\pi}e^{\gamma_{\rm E}}\right)^{\varepsilon} \frac{\pi\alpha_s}{(d-1)m^3}\,\frac{\boldsymbol{q}^4}{d-1} + O(\varepsilon). \end{aligned} \tag{3.5}$$

Here, τ_1 and τ_0 are functions of z that are defined in Eq. (5.29) of Ref.[1]. $D_4[g \to Q\bar{Q}(^3S_1^{[1]})]^{\rm finite}$ in Eq. (3.3) is a finite contribution, which we evaluate numerically in $d = 4$ dimensions.

4. NRQCD LDMEs

In this section we tabulate our results for the NRQCD LDMEs for free $Q\bar{Q}(^3S_1^{[1]})$ states that are relevant through relative order v^4.

The free $Q\bar{Q}$ matrix elements at order α_s^0 are normalized as

$$\langle 0|\mathscr{O}_0^{Q\bar{Q}(^1S_0^{[8]})}(^1S_0^{[8]})|0\rangle^{(0)} \,=\, (N_c^2-1), \tag{4.1}$$

$$\langle 0|\mathscr{O}_0^{Q\bar{Q}(^3S_1^{[1]})}(^3S_1^{[1]})|0\rangle^{(0)} \,=\, 2(d-1)N_c, \tag{4.2}$$

$$\langle 0|\mathscr{O}_0^{Q\bar{Q}(^3S_1^{[8]})}(^3S_1^{[8]})|0\rangle^{(0)} \,=\, (d-1)(N_c^2-1), \tag{4.3}$$

$$\langle 0|\mathscr{O}_2^{Q\bar{Q}(^3S_1^{[n]})}(^3S_1^{[n]})|0\rangle^{(0)} \,=\, \boldsymbol{q}^2\langle 0|\mathscr{O}_0^{Q\bar{Q}(^3S_1^{[n]})}(^3S_1^{[n]})|0\rangle^{(0)}, \tag{4.4}$$

$$\langle 0|\mathscr{O}_4^{Q\bar{Q}(^3S_1^{[n]})}(^3S_1^{[n]})|0\rangle^{(0)} \,=\, \boldsymbol{q}^4\langle 0|\mathscr{O}_0^{Q\bar{Q}(^3S_1^{[n]})}(^3S_1^{[n]})|0\rangle^{(0)}, \tag{4.5}$$

$$\langle 0|\mathscr{O}_0^{Q\bar{Q}(^3P^{[8]})}(^3P^{[8]})|0\rangle^{(0)} \,=\, \boldsymbol{q}^2(d-1)(N_c^2-1), \tag{4.6}$$

where there is an implied sum over final-state polarizations, and the superscript (k) indicates the order in α_s.

In order α_s and order α_s^2, the relevant LDMEs mix:

$$\langle 0|\mathscr{O}_0^{Q\bar{Q}(^3S_1^{[1]})}(^3P^{[8]})|0\rangle^{(1)}_{\overline{\rm MS}} \,=\, \frac{8\alpha_s}{3\pi m^2}\left(\frac{-1}{2\varepsilon_{\rm IR}}\right)\frac{N_c^2-1}{4N_c^2}\,\frac{1}{d-1}\langle 0|\mathscr{O}_{4,1}^{Q\bar{Q}(^3S_1^{[1]})}(^3S_1^{[1]})|0\rangle^{(0)}, \tag{4.7}$$

$$\langle 0|\mathscr{O}_0^{Q\bar{Q}(^3P^{[8]})}(^3S_1^{[8]})|0\rangle^{(1)}_{\overline{\rm MS}} = \frac{8\alpha_s}{3\pi m^2}\left(\frac{-1}{2\varepsilon_{\rm IR}}\right)\frac{N_c^2-4}{4N_c}\langle 0|\mathscr{O}_0^{Q\bar{Q}(^3P^{[8]})}(^3P^{[8]})|0\rangle^{(0)}, \tag{4.8}$$

$$\langle 0|\mathscr{O}_0^{Q\bar{Q}(^3S_1^{[1]})}(^3S_1^{[8]})|0\rangle^{(2)}_{\overline{\rm MS}} = \frac{1}{8\varepsilon_{\rm IR}^2}\left(\frac{8\alpha_s}{3\pi m^2}\right)^2\frac{(N_c^2-1)(N_c^2-4)}{16N_c^3}\frac{1}{d-1}\langle 0|\mathscr{O}_{4,1}^{Q\bar{Q}(^3S_1^{[1]})}(^3S_1^{[1]})|0\rangle^{(0)}. \tag{4.9}$$

Here, the subscript $\overline{\rm MS}$ indicates that we have removed the ultraviolet poles in ε by using the minimal-subtraction procedure, with the choice of scale that is appropriate to $\overline{\rm MS}$ subtraction. The renormalization subtractions for $\langle 0|\mathscr{O}_0^{Q\bar{Q}(^3S_1^{[1]})}(^3S_1^{[8]})|0\rangle^{(2)}$ involve both Born diagrams for the double-pole counterterm and one-loop diagrams for the single-pole counterterm. In the minimal subtraction procedure, one subtracts only pure pole contributions in the ultraviolet-divergent subdiagrams. It is essential for the consistency of the minimal-subtraction program to treat factors that are external to those divergent subdiagrams, such as angular-momentum projections involving external momenta, exactly in $d=4-2\varepsilon$ dimensions. A failure to follow this procedure in the presence of poles in $d=4-2\varepsilon$ of order two or higher can result in the appearance of uncanceled poles in ε in the NRQCD short-distance coefficients. We refer the reader to Ref. [1] for details.

The dimensional-regularization scale μ can be identified with the NRQCD factorization scale μ_Λ. It then follows from $d\alpha_s/d\log(\mu_\Lambda) = -2\varepsilon\alpha_s + O(\alpha_s^2)$ that the renormalization-group evolution equations for the LDMEs are

$$\frac{d}{d\log\mu_\Lambda}\langle 0|\mathscr{O}_0^{Q\bar{Q}(^3S_1^{[1]})}(^3P^{[8]})|0\rangle^{(1)}_{\overline{\rm MS}} = \frac{8\alpha_s}{3\pi m^2}\frac{N_c^2-1}{4N_c^2}\frac{1}{d-1}\langle 0|\mathscr{O}_{4,1}^{Q\bar{Q}(^3S_1^{[1]})}(^3S_1^{[1]})|0\rangle^{(0)}, \tag{4.10}$$

$$\frac{d}{d\log\mu_\Lambda}\langle 0|\mathscr{O}_0^{Q\bar{Q}(^3P^{[8]})}(^3S_1^{[8]})|0\rangle^{(1)}_{\overline{\rm MS}} = \frac{8\alpha_s}{3\pi m^2}\frac{N_c^2-4}{4N_c}\langle 0|\mathscr{O}_0^{Q\bar{Q}(^3P^{[8]})}(^3P^{[8]})|0\rangle^{(0)}, \tag{4.11}$$

$$\frac{d}{d\log\mu_\Lambda}\langle 0|\mathscr{O}_0^{Q\bar{Q}(^3S_1^{[1]})}(^3S_1^{[8]})|0\rangle^{(2)}_{\overline{\rm MS}} = \frac{8\alpha_s}{3\pi m^2}\frac{N_c^2-4}{4N_c}\langle 0|\mathscr{O}_0^{Q\bar{Q}(^3S_1^{[1]})}(^3P^{[8]})|0\rangle^{(1)}_{\overline{\rm MS}}. \tag{4.12}$$

The result in Eq. (4.10) agrees with the corresponding results of Refs. [11, 12]. Equations (4.11) and (4.12) agree with the result in Eq. (B19b) of Ref. [2] at the leading nontrivial order in v and with the corresponding result in Ref [12], but disagree with the corresponding result in Ref. [11].

5. Short-distance coefficients

By making use of the results of Secs. 3 and 4 and the free-$Q\bar{Q}$ versions of the NRQCD factorization (matching) equations (2.11), (2.12), and (2.14) in Sec. 2, we obtain

$$\begin{aligned}
&d_{4,1}[g\to Q\bar{Q}(^3S_1^{[1]})]^{(3)} + d_{4,2}[g\to Q\bar{Q}(^3S_1^{[1]})]^{(3)} \\
&\quad = d_4[g\to Q\bar{Q}(^3S_1^{[1]})]^{\rm finite} + \frac{2\alpha_s^3(N_c^2-4)}{3\pi(d-1)^3N_c^3m^7}\left\{\delta(1-z)\left(\frac{1}{24}-\frac{\pi^2}{6}\right.\right. \\
&\quad\quad \left. -\frac{1}{3}\log\frac{\mu_\Lambda}{2m}+\log^2\frac{\mu_\Lambda}{2m}\right) + \left(\frac{1}{1-z}\right)_+\left(\frac{1}{3}-2\log\frac{\mu_\Lambda}{2m}\right)+2\left[\frac{\log(1-z)}{1-z}\right]_+ \\
&\quad\quad -\frac{104-29z-10z^2}{24}+\frac{7[z+(1+z)\log(1-z)]}{2z^2}+\frac{(1-2z)(8-5z)}{4}\log\frac{\mu_\Lambda}{2m} \\
&\quad\quad +\frac{1+z}{4}\left(31-6z-\frac{36}{z}\right)\log(1-z)-\frac{z}{4}\left(39-6z+\frac{8}{1-z}\right)\log z
\end{aligned}$$

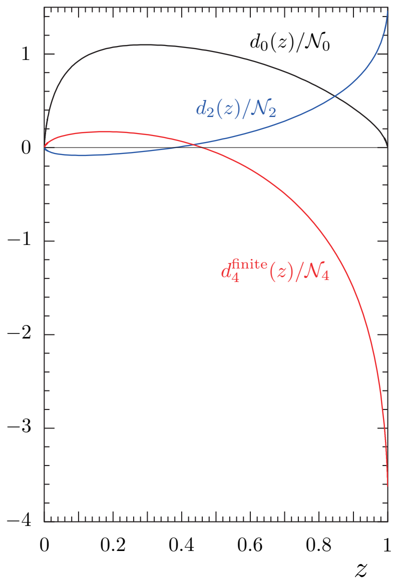

Figure 1: The color-singlet short-distance coefficients $d_0[g \to Q\bar{Q}(^3S_1^{[1]})]^{(3)}(z)$, $d_2[g \to Q\bar{Q}(^3S_1^{[1]})]^{(3)}(z)$ and $d_4[g \to Q\bar{Q}(^3S_1^{[1]})]^{\text{finite}}(z)$, which are defined in Eqs. (2.11), (2.12) and (5.1), respectively, as functions of z. The scaling factors are $(\mathcal{N}_0, \mathcal{N}_2, \mathcal{N}_4) = (10^{-3} \times \alpha_s^3/m^3, 10^{-2} \times \alpha_s^3/m^5, 10^{-2} \times \alpha_s^3/m^7)$.

$$+\frac{13-7z}{2}\left[\left(\log\frac{1-z}{z^2} - \log\frac{\mu_\Lambda}{2m}\right)\log(1-z) - \text{Li}_2(z)\right]\Bigg\} + O(\varepsilon). \tag{5.1}$$

Here, $d_4[g \to Q\bar{Q}(^3S_1^{[1]})]^{\text{finite}} = D_4[g \to Q\bar{Q}(^3S_1^{[1]})]^{\text{finite}}/\langle 0|\mathcal{O}_4^{Q\bar{Q}(^3S_1^{[1]})}(^3S_1^{[1]})|0\rangle^{(0)}$ is evaluated by carrying out the integrations over the phase space numerically. The results of the numerical integration are shown in Fig. 1. One can find details of the matching procedure in Sec. 7 of Ref. [1].

6. Relative sizes of the fragmentation contributions

We now make estimates of the relative sizes of the contributions of the various fragmentation functions to the cross section for J/ψ production in hadron-hadron collisions. At large p_T, the differential cross section for a gluon can be approximated as $d\sigma_g/dp_T \propto 1/p_T^\kappa$, where κ is a fixed power. Then, the fragmentation contribution to the J/ψ production rate becomes $d\sigma_{J/\psi}^{\text{frag}}/dp_T \propto I_\kappa(D)$, where $I_\kappa(D) = \int_0^1 dz z^\kappa D(z)$ [1]. Hence, we can obtain a rough estimate of the relative contribution of a fragmentation process to the cross section by computing $I_\kappa(D)$. At large p_T, the cross section at LO in α_s is dominated by fragmentation of a gluon into a J/ψ with $z = 1$ [13]. The next-to-leading order (NLO) k factor for this channel is essentially independent of p_T for the cross sections that were measured at the Tevatron [14]. Making use of these facts and the result for the $^3S_1^{[8]}$ contribution to $d\sigma/dp_T$ at NLO in α_s that appears in Fig. 1(c) of Ref. [15], we have determined that $\kappa \approx 5.2$.

We can compare the relative contributions to $d\sigma_{J/\psi}^{\text{frag}}$ of each of the three $Q\bar{Q}$ channels that contribute to $D[g \to J/\psi]$ in order v^4 [Eq. (2.14)]. For each channel, we compute $I_{5.2}(D)$. Then, we

Table 1: Relative contributions to $d\sigma^{\rm frag}_{J/\psi}$ in order v^4. The first and second rows give $I_{5.2}(d)$ times the LDMEs that are given in the text. We take $m = m_c = 1.5\,$GeV. For compatibility with Ref. [15], we take $\alpha_s = \alpha_s(m_T)$, where $m_T = \sqrt{p_T^2 + 4m_c^2}$. We choose the point $p_T = 20$ GeV, which implies that $\alpha_s(m_T) = 0.154$.

$I_\kappa(d)\setminus$channel	$\mathcal{O}_0^{J/\psi}(^1S_0^{[8]})$	$\mathcal{O}_0^{J/\psi}(^3S_1^{[8]})$	$\mathcal{O}_0^{J/\psi}(^3P^{[8]})$	$\mathcal{O}_4^{J/\psi}(^3S_1^{[1]})$
$I_{5.2}(d)\|_{\mu_\Lambda=m} \times 10^6\,$LDME	1.65	18.6	32.7	0.192
$I_{5.2}(d)\|_{\mu_\Lambda=2m} \times 10^6\,$LDME	1.65	18.6	43.6	0.455

multiply by the following LDMEs: $\langle 0|\mathcal{O}_4^{J/\psi}(^3S_1^{[1]})|0\rangle = m^4\langle v^2\rangle^2 \times 1.32\,\text{GeV}^3$, $\langle 0|\mathcal{O}_0^{J/\psi}(^3P^{[8]})|0\rangle = -0.109\,\text{GeV}^5$, $\langle 0|\mathcal{O}_0^{J/\psi}(^3S_1^{[8]})|0\rangle = 3.12\times 10^{-3}\,\text{GeV}^3$, $\langle 0|\mathcal{O}_0^{J/\psi}(^1S_0^{[8]})|0\rangle = 4.50\times 10^{-2}\,\text{GeV}^3$. We have obtained these LDMEs from Ref. [15]. In the case of the first LDME, we have used the generalized Gremm-Kapustin relation [16] $\langle 0|\mathcal{O}_4^{J/\psi}(^3S_1^{[1]})|0\rangle = m^4\langle v^2\rangle^2\langle 0|\mathcal{O}_0^{J/\psi}(^3S_1^{[1]})|0\rangle$ to obtain the matrix element of order v^4 from the matrix element of order v^0 that appears in Ref. [15]. We use the value of $m^2\langle v^2\rangle$ from Table I of Ref. [16]: $m^2\langle v^2\rangle = 0.437\,\text{GeV}^2$ at $m = 1.5\,$GeV. The results of this computation are shown in Table 1.

We see from the second row of Table 1 that the $Q\bar{Q}(^3S_1^{[1]})$ channel makes a small contribution to $D_4[g\to J/\psi]$ at $p_T = 20$ GeV, confirming that this channel is not important phenomenologically at the current level of precision. We also see that the $Q\bar{Q}(^3S_1^{[8]})$ and $Q\bar{Q}(^3P^{[8]})$ channels give comparable contributions to $D_4[g\to J/\psi]$ at $p_T = 20$ GeV. We estimate that the fragmentation contribution to $d\sigma/dp_T \times B(J/\psi\to\mu\mu)$ at $p_T = 20$ GeV from the $Q\bar{Q}(^3P^{[8]})$ channel is about 6×10^{-3} nb/GeV [1], which is comparable to (about a factor of 2 larger than) the total NLO contribution in the $Q\bar{Q}(^3P^{[8]})$ channel in Fig. 1(c) of Ref. [15]. A more precise calculation of the fragmentation contribution in the $Q\bar{Q}(^3P^{[8]})$ channel will be necessary in order to determine whether it is the dominant contribution in that channel at NLO in α_s at high p_T.

7. Summary

We have computed NRQCD short-distance coefficients for gluon fragmentation into a J/ψ through relative order v^4. Our main result is the expression in Eq. (5.1) for $d_{4,1}[g\to Q\bar{Q}(^3S_1^{[1]})]^{(3)} + d_{4,2}[g\to Q\bar{Q}(^3S_1^{[1]})]^{(3)}$, which is the sum of short-distance coefficients for the $^3S_1^{[1]}$ NRQCD LDMEs of relative order v^4. This is the first time that double soft divergences, as well as single soft divergences, have appeared in intermediate steps in the calculation of NRQCD short-distance coefficients. We have also computed the free $Q\bar{Q}$ NRQCD LDMEs that appear when one uses the matching equations between full QCD and NRQCD to compute the NRQCD short-distance coefficients. Our perturbative calculations of these LDMEs involve both one-loop and two-loop corrections. The ultraviolet renormalization-subtractions for these LDMEs include Born and one-loop single-pole counterterm contributions and Born double-pole counterterm contributions. We have worked out the renormalization-group evolution of the renormalized LDMEs and confirm some previous results in Refs. [2, 11, 12], but disagree with a result in Ref. [11].

We have estimated the relative sizes of the contributions of the various channels to gluon fragmentation into a J/ψ through relative order v^4. The contribution in order v^4 of the $^3S_1^{[1]}$ channel to the cross section at $p_T = 20$ GeV is about a factor of 2 (4) larger than the contribution in order v^0 when $\mu_\Lambda = m_c$ ($2m_c$). In spite of this large enhancement of the fragmentation contribution in order v^4, the corresponding contribution to the J/ψ production cross section at the Tevatron or the LHC is not important at the current level of precision.

In the $^3S_1^{[8]}$ channel, most of the contribution to the J/ψ production cross section at hadron-hadron colliders at large p_T arises from the fragmentation contribution. This is true at LO in α_s [13] and at NLO in α_s [14]. Our estimate of the fragmentation contribution to the $^3P_J^{[8]}$ channel indicates that it is an important part of the contribution in that channel to the high-p_T J/ψ production cross section at hadron-hadron colliders at NLO in α_s. However, it will be necessary to carry out a more precise calculation of the fragmentation contribution in the $^3P_J^{[8]}$ channel in order to see whether it is actually dominant at high p_T.

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