PROCEEDINGS OF SCIENCE

Some recent progress in understanding exclusive double charmonium production at B factories

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We review some recent progress in understanding various exclusive double charmonium production processes at B factories, within the nonrelativistic QCD factorization framework. First we investigate the impact of the joint perturbative and relativistic correction on the process that has attracted a great amount of attention in the past decade, $e^+e^- \to J/\psi + \eta_c$. We then briefly discuss the phenomenological implication of the next-to-leading order perturbative correction to the processes $e^+e^- \to J/\psi + \chi_{c0,1,2}(\eta_{c2})$. We further emphasize a novel theoretical challenge, which is recently discovered by applying the NRQCD factorization approach to the helicity-suppressed hard exclusive reactions involving heavy quarkonium.

Xth Quark Confinement and the Hadron Spectrum
8-12 October 2012
TUM Campus Garching, Munich, Germany

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1. Introduction

The discovery of a number of double-charmonium production processes at B factories about a decade ago [1, 2, 3] has triggered the long-lasting interest. Experimentally, this production mechanism offers a unique environment to search for the new C-even charmonium states, particularly those X, Y, Z states, by fitting the recoil mass spectrum against the J/ψ (ψ'). The famous examples are the establishing of the $X(3940)$ and $X(4160)$ states along this avenue.

On the theoretical side, the double-charmonium production provides a powerful arsenal to strengthen our understanding toward perturbative QCD, especially toward the application of the light-cone approach [4, 5] and the nonrelativistic QCD (NRQCD) factorization approach [6] to hard exclusive reactions involving heavy quarkonium.

Thanks to the very clean $J/\psi(\psi') \to l^+l^-$ signals, experimentally it is most favorable to reconstruct those double-charmonium events that involve a J/ψ (ψ') meson. Thus far, the most intensively-studied double charmonium production process is $e^+e^- \to J/\psi + \eta_c$. The lowest-order (LO) NRQCD predictions to this process [7, 8, 9] was about one order of magnitude smaller than the original Belle measurement [1]. This disquieting discrepancy has spurred a great amount of theoretical investigations in both NRQCD and light-cone approaches [10]. One crucial ingredient in alleviating the discrepancy between the NRQCD prediction and the data is the *positive* and *substantial* next-to-leading order (NLO) perturbative corrections [11, 12] [1]. By contrast, owing to some long-standing theoretical obstacles for the helicity-flipped process, *e.g.* the "endpoint singularity problem", the $O(\alpha_s)$ correction to this process has never been worked out in the light-cone approach. Therefore, despite some shortcomings, the NRQCD factorization approach remains to be the *only* viable formalism to tackle double-charmonium production which is both based on the first principles of QCD and also amenable to the systematical improvement.

In this talk, we review some recent investigations on the higher order corrections to the double-charmonium production processes $e^+e^- \to J/\psi + \chi_{c0,1,2}(\eta_{c2})$ at B factories [15, 16, 17, 18, 19]. In particular, we address the impact of the $O(\alpha_s)$ [or $O(\alpha_s v^2)$, where v denotes the characteristic velocity of the charm quark inside charmonium] corrections on these processes in NRQCD factorization approach. The theoretical predictions will be confronted with the measurements whenever possible. We also remark on a novel theoretical phenomenon, *i.e.*, the occurrence of the double logarithms in the NRQCD short-distance coefficients at NLO in α_s.

The rest of the paper is structured as follows. In Sec. 2, we recall the helicity selection rule relevant for the exclusive double charmonium production processes considered in this work. In Sec. 3, we sketch some key techniques underlying the $O(\alpha_s)$ calculation. In Sec. 4, we consider the $O(\alpha_s v^2)$ correction to the process $e^+e^- \to J/\psi + \eta_c$, presenting the asymptotic expressions of the $O(\alpha_s)$ NRQCD short-distance coefficients through relative order-v^2. The phenomenological impact of this new correction is also explored. In Sec. 5, we briefly discuss the effects of the $O(\alpha_s)$ corrections to the processes $e^+e^- \to J/\psi + \chi_{c0,1,2}(\eta_{c2})$, and propose that these types of processes may be used to unravel the quantum number of the famous $X(3872)$ meson. Finally in Sec. 6, we present our personal perspective on those most important problems remaining in this field, which urgently await the exploration.

[1] The tree-level relativistic corrections to this process have also been investigated [7, 13, 14]. Including these corrections appears to be helpful to further reduce the gap between the NRQCD prediction and the data.

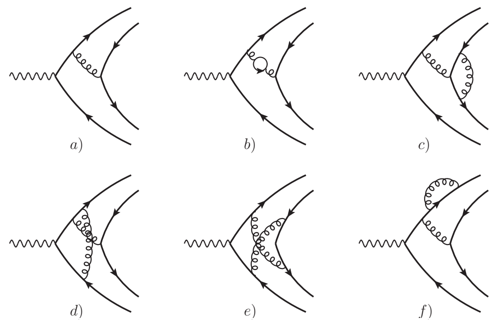

Figure 1: One sample LO diagram and five sample NLO diagrams that contribute to $\gamma^* \to J/\psi + H$, where H is a C-even charmonium state such as η_c, $\chi_{c0,1,2}$ or η_{c2}.

2. Helicity selection rule for double-charmonium production

For the exclusive double-charmonium production processes, it is most informative to look into the polarized cross section, where the power-law scaling in each helicity configuration is governed by the famous *helicity selection rule* (HSR) [20]. In the hard-scattering limit $\sqrt{s} \gg m_c \gg \Lambda_{\rm QCD}$ ($\sqrt{s}$ stands for the center-of-mass energy of the e^+e^- collider, m_c for the charm quark mass, and $\Lambda_{\rm QCD}$ for the intrinsic QCD scale), the HSR implies that the asymptotic behavior for the rate of producing J/ψ together with a C-even charmonium H (with the leading $c\bar{c}$ Fock component possessing the quantum number ${}^{2S+1}L_J^{(1)}$) in a definite helicity configuration is [7]

$$\frac{\sigma[e^+e^- \to J/\psi(\lambda_1) + H(\lambda_2)]}{\sigma[e^+e^- \to \mu^+\mu^-]} \sim v^{6+2L} \left(\frac{m_c^2}{s}\right)^{2+|\lambda_1+\lambda_2|}, \tag{2.1}$$

where λ_1, λ_2 represent the helicities carried by the J/ψ and H, respectively. Eq. (2.1) implies that the helicity state which exhibits the slowest asymptotic decrease, thus constitutes the "leading-twist" contribution, *i.e.*, $\sigma \sim 1/s^3$, is $(\lambda_1, \lambda_2) = (0,0)$. For the processes exemplified by $e^+e^- \to J/\psi + \eta_c(\chi_{c1}, \eta_{c2})$, parity invariance forbids the occurrence of the $(0,0)$ configuration, therefore they are entirely of the "higher twist" (*helicity-flipped*) nature.

3. Techniques in calculating the $O(\alpha_s)$ corrections

The NRQCD short-distance coefficient can be most easily obtained by computing the on-shell quark amplitude $\gamma^* \to c\bar{c}(P_1, {}^3S_1^{(1)}) + c\bar{c}(P_2, {}^{2S+1}L_J^{(1)})$, with the aid of the covariant spin projectors [21, 7]. At LO in α_s, there are 4 diagrams for this parton process; while at NLO in α_s, there are 20 two-point, 20 three-point, 18 four-point, and 6 five-point one-loop diagrams. Some typical diagrams are shown in Fig. 1.

In spite of resorting to perturbative matching method, a much more economic way is to directly extract the short-distance coefficients following the philosophy of *threshold expansion* [22]. That is, when projecting each $c\bar{c}$ pair onto the desired orbital-angular-momentum state, one first expands the amplitude in powers of the relative quark momentum q prior to performing the loop integration.

Consequently, a technical complication arises, that one inevitably encounters some unusual one-loop integrals that contain the propagators of (up to) cubic power, due to taking the derivative over q. The MATHEMATICA packages FIRE [23] and the code APART [24] are utilized to reduce these unconventional higher-point one-loop tensor integrals into a minimal set of masters integrals. Thanks to the integration-by-part algorithm and partial fraction technique built into these codes, it turns out that all the encountered master integrals are nothing but the ordinary 2-point and 3-point one-loop scalar integrals, whose analytic expressions can be found in Appendix of Ref. [12].

When adding the contributions of all the diagrams, and after renormalizing the charm quark mass and the QCD coupling constant, we end up with both UV and IR finite NLO expressions for the $O(\alpha_s)$ amplitude associated with $\gamma^* \to J/\psi + \chi_{c0,1,2}(\eta_{c2})$. In comparison, the amplitude for $\gamma^* \to J/\psi + \eta_c$ contains an uncanceled IR pole at $O(\alpha_s v^2)$. Fortunately, it can be factored into the relative order-v^2 NRQCD matrix element via the pull-up mechanism. In either case, one ends up with the IR-finite $O(\alpha_s)$ NRQCD short-distance coefficients for all the helicity amplitudes affiliated with $e^+e^- \to J/\psi + H$. Our calculation explicitly confirms the assertion made in Ref. [25], that the NRQCD factorization holds beyond tree level for the exclusive production of a S-wave quarkonium plus any higher orbital-angular-momentum quarkonium in e^+e^- annihilation.

4. $O(\alpha_s v^2)$ correction to $e^+e^- \to J/\psi + \eta_c$

The production rate for $e^+e^- \to J/\psi + \eta_c$ can be expressed as

$$\sigma[e^+e^- \to J/\psi + \eta_c] = \frac{4\pi\alpha^2}{3}\left(\frac{|\mathbf{P}|}{\sqrt{s}}\right)^3 |G(s)|^2, \tag{4.1}$$

where $|\mathbf{P}|$ signifies the magnitude of the momentum carried by the J/ψ (η_c) in the center-of-mass frame. $G(s)$ is the $J/\psi + \eta_c$ time-like electromagnetic form factor, defined through $\langle J/\psi(P_1,\lambda) + \eta_c(P_2)|J^\mu_{\rm EM}|0\rangle = i\,G(s)\,\varepsilon^{\mu\nu\rho\sigma}P_{1\nu}P_{2\rho}\varepsilon^*_\sigma(\lambda)$, where $J^\mu_{\rm EM}$ is the electromagnetic current, P_1 ($\varepsilon^\sigma(\lambda)$) denote the momentum (polarization vector) of the J/ψ, and P_2 the momentum of the η_c, respectively.

4.1 NRQCD factorization formula

NRQCD factorization allows one to factorize the $J/\psi + \eta_c$ electromagnetic form factor as

$$G(s) = \sqrt{4M_{J/\psi}M_{\eta_c}}\langle J/\psi|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\chi|0\rangle\langle\eta_c|\psi^\dagger\chi|0\rangle\left[c_0 + c_{2,1}\langle v^2\rangle_{J/\psi} + c_{2,2}\langle v^2\rangle_{\eta_c} + \cdots\right], \tag{4.2}$$

where c_0 and $c_{2,i}$ are the dimensionless short-distance coefficients that depend on m_c^2/s. For simplicity, we have also introduced the following dimensionless ratios of NRQCD matrix elements to signify the $O(v^2)$ corrections: $\langle v^2\rangle_{J/\psi} = \langle J/\psi(\lambda)|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}(\lambda)\chi|0\rangle/(m_c^2\langle J/\psi(\lambda)|\psi^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}(\lambda)\chi|0\rangle)$, $\langle v^2\rangle_{\eta_c} = \langle\eta_c|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\chi|0\rangle/(m_c^2\langle\eta_c|\psi^\dagger\chi|0\rangle)$, where $\psi^\dagger\overleftrightarrow{\mathbf{D}}\chi \equiv \psi^\dagger\mathbf{D}\chi - (\mathbf{D}\psi)^\dagger\chi$.

Inserting Eq. (4.2) into (4.1), one can decompose the cross section into the $O(v^0)$ and $O(v^2)$ pieces, $\sigma[e^+e^- \to J/\psi + \eta_c] = \sigma_0 + \sigma_2 + O(\sigma v^4)$, where

$$\sigma_0 = \frac{8\pi\alpha^2 m_c^2(1-4r)^{3/2}}{3}\langle\mathcal{O}_1\rangle_{J/\psi}\langle\mathcal{O}_1\rangle_{\eta_c}|c_0|^2, \tag{4.3a}$$

$$\sigma_2 = \frac{4\pi\alpha^2 m_c^2(1-4r)^{3/2}}{3}\langle\mathcal{O}_1\rangle_{J/\psi}\langle\mathcal{O}_1\rangle_{\eta_c} \tag{4.3b}$$

$$\left\{\left(\frac{1-10r}{1-4r}|c_0|^2 + 4\,\mathrm{Re}[c_0c^*_{2,1}]\right)\langle v^2\rangle_{J/\psi} + \left(\frac{1-10r}{1-4r}|c_0|^2 + 4\,\mathrm{Re}[c_0c^*_{2,2}]\right)\langle v^2\rangle_{\eta_c}\right\}.$$

We have employed the Gremm-Kapustin relation [26] to eliminate the explicit occurrences of $M_{J/\psi}$ and M_{η_c} in (4.3). To condense the notation, we have introduced the following symbols: $r = 4m_c^2/s$, $\langle \mathscr{O}_1 \rangle_{J/\psi} = \left| \langle J/\psi(\boldsymbol{\varepsilon}) | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\varepsilon} \chi | 0 \rangle \right|^2$, and $\langle \mathscr{O}_1 \rangle_{\eta_c} = \left| \langle \eta_c | \psi^\dagger \chi | 0 \rangle \right|^2$.

4.2 Various NRQCD short-distance coefficients

We organize the coefficients c_i $(i = 0,2)$ in power series of the strong coupling constant, *i.e.*, $c_i = c_i^{(0)} + \frac{\alpha_s}{\pi} c_i^{(1)} + \cdots$. Accordingly, one may decompose the cross section σ_i into $\sigma_i^{(0)} + \sigma_i^{(1)}$ $(i = 0,2)$ as well. Thus far, the only missing piece is $\sigma_2^{(1)}$. The tree-level short-distance coefficients through $O(v^2)$ have been available long ago [7]:

$$c_0^{(0)} = \frac{32\pi C_F e_c \alpha_s}{N_c m_c s^2}, \qquad c_{2,1}^{(0)} = \frac{3-10r}{6} c_0^{(0)}, \qquad c_{2,2}^{(0)} = \frac{2-5r}{3} c_0^{(0)}, \tag{4.4}$$

where $e_c = \frac{2}{3}$ is the electric charge of the charm quark, and $C_F = \frac{N_c^2-1}{2N_c} = \frac{4}{3}$.

The $O(\alpha_s)$ NRQCD short-distance coefficients $c_0^{(1)}$ and $c_{2,i}^{(1)}$ are generally complex-valued, and cumbersomely-looking. Nevertheless, it is much more illuminating to look at their asymptotic expressions in the limit $\sqrt{s} \gg m_c$:

$$\begin{aligned} c_0^{(1)}\left(r, \frac{\mu_r^2}{s}\right)_{\text{asym}} &= c_0^{(0)} \times \left\{ \beta_0 \left(-\frac{1}{4} \ln \frac{s}{4\mu_r^2} + \frac{5}{12} \right) + \left(\frac{13}{24} \ln^2 r + \frac{5}{4} \ln 2 \ln r - \frac{41}{24} \ln r \right.\right. \\ &\left.\left. - \frac{53}{24} \ln^2 2 + \frac{65}{8} \ln 2 - \frac{1}{36}\pi^2 - \frac{19}{4} \right) + i\pi \left(\frac{1}{4}\beta_0 + \frac{13}{12} \ln r + \frac{5}{4} \ln 2 - \frac{41}{24} \right) \right\}, \end{aligned} \tag{4.5a}$$

$$\begin{aligned} c_{2,1}^{(1)}\left(r, \frac{\mu_r^2}{s}, \frac{\mu_f^2}{m_c^2}\right)_{\text{asym}} &= \frac{1}{2} c_0^{(0)} \times \left\{ \frac{16}{9} \ln \frac{\mu_f^2}{m_c^2} + \beta_0 \left(-\frac{1}{4} \ln \frac{s}{4\mu_r^2} + \frac{11}{12} \right) + \left(\frac{3}{8} \ln^2 r + \frac{19}{12} \ln 2 \ln r \right.\right. \\ &\left.\left. + \frac{31}{24} \ln r - \frac{1}{24} \ln^2 2 + \frac{893}{216} \ln 2 - \frac{5}{36}\pi^2 - \frac{497}{72} \right) + i\pi \left(\frac{1}{4}\beta_0 + \frac{3}{4} \ln r + \frac{19}{12} \ln 2 + \frac{9}{8} \right) \right\}, \end{aligned} \tag{4.5b}$$

$$\begin{aligned} c_{2,2}^{(1)}\left(r, \frac{\mu_r^2}{s}, \frac{\mu_f^2}{m_c^2}\right)_{\text{asym}} &= \frac{2}{3} c_0^{(0)} \times \left\{ \frac{4}{3} \ln \frac{\mu_f^2}{m_c^2} + \beta_0 \left(-\frac{1}{4} \ln \frac{s}{4\mu_r^2} + \frac{2}{3} \right) + \left(\frac{1}{12} \ln^2 r + \frac{11}{12} \ln 2 \ln r \right.\right. \\ &\left.\left. - \frac{1}{24} \ln r - \frac{11}{8} \ln^2 2 + \frac{241}{144} \ln 2 - \frac{1}{8}\pi^2 - \frac{99}{16} \right) + i\pi \left(\frac{1}{4}\beta_0 + \frac{1}{6} \ln r + \frac{11}{12} \ln 2 - \frac{1}{24} \right) \right\}, \end{aligned} \tag{4.5c}$$

where $\beta_0 = \frac{11}{3} C_A - \frac{2}{3} n_f$ is the one-loop coefficient of the QCD β function, and $n_f = 4$ denotes the number of active quark flavors. μ_r denotes the renormalization scale, and μ_f signifies the NRQCD factorization scale in the $\overline{\rm MS}$ scheme, which naturally ranges from $m_c v$ to m_c.

As first pointed out in Ref. [27], a peculiar double-logarithmic correction $\propto \ln^2 r$ arises in $c_0^{(1)}$ for this helicity-flipped process, and our (4.5a) exactly agrees with the corresponding expression there [2]. Eqs. (4.5) imply that the double logarithms survive at $O(\alpha_s v^2)$ as well. With $\sqrt{s} \gg m_c$, one presumably needs to sum these types of logarithms to all orders in α_s to warrant a reliable prediction. Such a resummation may even be mandatory at the B-factory energy [27]. At present, how to fulfill this goal remains to be a thorny challenge.

[2] Our result slightly differs from Ref. [12] on the imaginary part of $c_0^{(1)}$, though it does not affect the phenomenology.

Table 1: The individual contribution to the predicted $\sigma[e^+e^- \to J/\psi + \eta_c]$ at $\sqrt{s} = 10.58$ GeV, as specified by the powers of α_s and v^2. We take $\alpha(\sqrt{s}) = 1/130.9$, $\Lambda^{(4)}_{\overline{\rm MS}} = 0.338$ GeV, $m_c = 1.4$ GeV, and $\mu_f = m_c$. The LO NRQCD matrix elements are $\langle \mathcal{O}_1 \rangle_{J/\psi} \approx \langle \mathcal{O}_1 \rangle_{\eta_c} = 0.387\ \text{GeV}^3$. The Gremm-Kapustin relation is used to obtain $\langle v^2 \rangle_{J/\psi} = 0.223$ and $\langle v^2 \rangle_{\eta_c} = 0.133$. The cross sections are in units of fb.

$\alpha_s(\mu_r)$	$\sigma_0^{(0)}$	$\sigma_0^{(1)}$	$\sigma_2^{(0)}$	$\sigma_2^{(1)}$
$\alpha_s(\frac{\sqrt{s}}{2}) = 0.211$	4.40	5.22	1.72	0.73
$\alpha_s(2m_c) = 0.267$	7.00	7.34	2.73	0.24

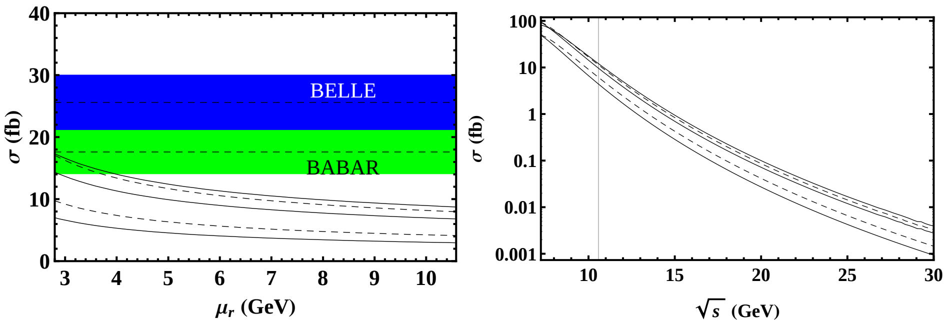

Figure 2: The μ_r- and $\sqrt{s}$-dependence of the cross section for $e^+e^- \to J/\psi + \eta_c$. The 5 curves from bottom to top are $\sigma_0^{(0)}$ (solid line), $\sigma_0^{(0)} + \sigma_2^{(0)}$ (dashed line), $\sigma_0^{(0)} + \sigma_0^{(1)}$ (solid line), $\sigma_0^{(0)} + \sigma_2^{(0)} + \sigma_0^{(1)}$ (dashed line), and $\sigma_0^{(0)} + \sigma_2^{(0)} + \sigma_0^{(1)} + \sigma_2^{(1)}$ (solid line), respectively. In the left panel, we fix $\sqrt{s} = 10.58$ GeV, and the blue and green bands represent the measured cross sections by the Belle and BaBar experiments, with respective systematic and statistical errors added in quadrature.

4.3 Phenomenological impact of the $O(\alpha_s v^2)$ correction

Apart from the ambiguity in the values of m_c and NRQCD matrix elements, the freedom of choosing the scale entering the strong coupling constant leads to a large uncertainty for our prediction to $\sigma[e^+e^- \to J/\psi + \eta_c]$. This is a serious drawback of the NRQCD approach [28]. For simplicity we assign all the occurring α_s with a common scale, μ_r, and choose $\mu_r = \sqrt{s}/2$ and $\mu_r = 2m_c$, respectively, hoping that the less biased results interpolate somewhere in between.

Table 1 lists the predicted $\sigma[e^+e^- \to J/\psi + \eta_c]$ with two sets of μ_r, organized in double expansions of α_s and v. We reproduce the well-known results, i.e., the positive and substantial $O(\alpha_s)$ correction [11, 12], and the positive but less pronounced $O(v^2)$ correction [7, 13, 14]. Taking $r = 0.07$, which is relevant for the B factory energy, $\text{Re}[c_{2,i}^{(1)}/c_{2,i}^{(0)}]$ $(i = 1, 2)$ turn out to be both large and negative. One might naively expect that including the new $O(\alpha_s v^2)$ correction would largely dilute the existing $O(v^2)$ term. However, the new correction to the cross section, $\sigma_2^{(1)}$, is actually positive and modest. This may be attributed to the accidental cancelation between the two terms in the prefactor of $\langle v^2 \rangle_H$ in (4.3b), which represent two difference sources of relativistic correction.

In Fig. 2, we plot the $\sigma[e^+e^- \to J/\psi + \eta_c]$ as a function of μ_r and $\sqrt{s}$. At the B factory energy, incorporating the new correction $\sigma_2^{(1)}$ appears not to make a big difference. When μ_r is relatively small, the state-of-the-art NRQCD prediction converges to the BaBar measurement within errors.

Had we taken somewhat larger values of the NRQCD matrix elements $\langle \mathcal{O}_1 \rangle_H$ as in [13, 14], the agreement with the B factory measurements would improve.

As shown in the right panel of Fig. 2, $\sigma[e^+e^- \to J/\psi + \eta_c]$ drops steeply as $\sqrt{s}$ increases, reflecting the helicity-flipped nature of this process, $\sigma \sim 1/s^4$. In contrast to its minor impact at B factory, the $O(\alpha_s v^2)$ correction turns out to be much more relevant at higher $\sqrt{s}$.

5. $O(\alpha_s)$ correction to $e^+e^- \to J/\psi + \chi_{c0,1,2}(\eta_{c2})$

The NLO perturbative corrections to the double-charmonium production processes $e^+e^- \to J/\psi + \chi_{c0,1,2}(\eta_{c2})$ have also been investigated recently [15, 16, 19]. The calculational technique resembles that for the $O(\alpha_s v^2)$ correction to $e^+e^- \to J/\psi + \eta_c$, and has been reviewed in Sec. 3. The processes $e^+e^- \to J/\psi + \chi_{c0,1,2}$ have been independently studied by two groups, and they agree with each other [15, 16].

The $O(\alpha_s)$ correction to $e^+e^- \to J/\psi + \chi_{c0}$ appears to be substantial. With some reasonable input parameters, and the renormalization scale μ_r set as $\frac{\sqrt{s}}{2}$, including the $O(\alpha_s)$ correction enhances the LO cross section from 4.8 fb to 8.6 fb. This value is compatible with the Belle and BaBar measurements within errors. However, the NLO perturbative prediction to the $\psi' + \chi_{c0}$ production rate is still significantly below the central value of the Belle measurement [15, 16].

The situation for $e^+e^- \to J/\psi + \chi_{c1,2}$ is less clear. The impact of the NLO perturbative corrections to these processes seems to be modest, even with the sign uncertain, depending on the different choices of μ_r. The predicted cross sections for both processes are around 1 fb, which are almost one order of magnitude smaller than that for $e^+e^- \to J/\psi + \chi_{c0}$. Encouragingly, the recent Belle experiment [29] did observe a considerable number of $e^+e^- \to J/\psi + \chi_{c1,2}$ events, from which one may roughly estimate the corresponding production cross sections. They appear to be qualitatively consistent with our expectations.

Recently there has arisen some controversy about the canonical charmonium option of the $X(3872)$ meson, whether it being η_{c2} or χ'_{c1} [30]. Motivated by this concern, we have also performed a comparative study for $e^+e^- \to J/\psi + \eta_{c2}$ and $e^+e^- \to J/\psi + \chi'_{c1}$ at B-factory energy, hoping that it may provide some guidance to unravel the quantum number of the $X(3872)$ meson in the future experiments [19]. The NLO perturbative correction to the former process is of medium size. With μ_r taken as $\sqrt{s}/2$, implementing the $O(\alpha_s)$ correction enhances the LO cross section from 0.22 fb to 0.29 fb. The production rate of the latter process is about 6-7 times greater than the former, thereby it seems realistic to observe the $J/\psi + \chi'_{c1}$ signals based on the current 1 ab^{-1} BELLE data sample, if the χ'_{c1} is indeed the narrow $X(3872)$ meson.

Refs. [16, 19] also conduct a comprehensive study on the polarized cross sections for $e^+e^- \to J/\psi + \chi_{c0,1,2}(\eta_{c2})$. It is found that the bulk of the total cross section comes from the $(0, \pm 1)$ helicity channels for $e^+e^- \to J/\psi + \chi_{c1}$, from the $(0,0)$ and $(\pm 1, 0)$ helicity states for $e^+e^- \to J/\psi + \chi_{c2}$, and from the $(\pm 1, 0)$ states for $e^+e^- \to J/\psi + \eta_{c2}$. The hierarchy among the various helicity channels appears to often conflict with what is expected from the HSR. It will be interesting for the future experiments to concretely test these polarization patterns.

By working out the asymptotic expressions of the various helicity amplitudes for the processes $\gamma^* \to J/\psi + \chi_{c0,1,2}(\eta_{c2})$, we firmly confirm the pattern speculated in Ref. [27]: The hard exclusive reaction involving double charmonium at leading twist can only host the single collinear logarithm

$\ln s/m_c^2$ at NLO in α_s, while the double logarithms of form $\ln^2 s/m_c^2$ are always associated with those helicity-suppressed channels.

6. Outlook

After a decade of intensive study, our understanding of exclusive double charmonium production has gradually matured. The most notable lesson is perhaps that, the NRQCD factorization approach has proved to be a successful and indispensable tool in dealing with hard exclusive reactions involving heavy quarkonium. However, in our opinion, this research area is still far from being closed, and there remain some important questions to be answered. In the following, we enumerate two topics which may urgently beg for the exploration.

We have reviewed some recent advances in the NLO perturbative correction to double charmonium production processes. Aside from $e^+e^- \to J/\psi + \eta_c$, the relativistic correction has hardly been investigated for any double-charmonium production process involving the P, D-wave charmonium. There is no ground to believe their effects are less important than the perturbative corrections. A practical difficulty to assess the relativistic correction for these processes is that, in general more NRQCD matrix elements than for the S-wave charmonium will come into play, about whose values we have almost no any clue. Hopefully, the lattice NRQCD simulation will eventually provide some useful information for those long-distance matrix elements.

A great theoretical challenge is to tame the double logarithms of form $\ln^2(s/m_c^2)$ in the $O(\alpha_s)$ NRQCD short-distance coefficients which are always affiliated with the helicity-suppressed exclusive quarkonium production channels. The occurrence of these process-dependent, (positive) double logarithms severely jeopardizes the reliability of the fix-order perturbation theory prediction. We note that some important progress has been made recently in tracing the origin of these double logarithms at one-loop order (differentiating the harmless Sudakov double logarithm from the problematic endpoint double logarithm) [31]. Nevertheless, there is still a long way to go to finally develop a systematic control over these endpoint double logarithms, *e.g.*, to resum them to all orders in α_s. In our perspective, the occurrence of these double logarithms is likely intertwined with the long-standing failure in applying the light-cone approach to the hard exclusive reactions beyond tree level. Looking on the bright side, with explicit expressions of the $O(\alpha_s)$ NRQCD short-distance coefficients for many channels at our disposal, one may view the exclusive double quarkonium production as a fertile theoretical laboratory, from which some fresh insight may be gained by reexamining those old problems of the light-cone approach.

Acknowledgments

This research was supported in part by the National Natural Science Foundation of China under Grant Nos. 10935012, 11125525, DFG and NSFC (CRC 110), and by the Ministry of Science and Technology of China under Contract No. 2009CB825200.

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PoS(Confinement X)132