

A nonrelativistic quark model evaluation of exclusive $b \rightarrow c$ semileptonic decay of triply heavy baryons and $c \rightarrow s, d$ semileptonic decay of cb baryons

C. Albertus

Departamento de Física Fundamental e IUFFyM, Universidad de Salamanca, E-37008 Salamanca, Spain
E-mail: albertus@usal.es

J.M. Flynn

School of Physics and Astronomy, University of Southampton, Highfield, Southampton SO17 1BJ, United Kingdom
E-mail: flynn@phys.soton.ac.uk

E. Hernández^{*†}

Departamento de Física Fundamental e IUFFyM, Universidad de Salamanca, E-37008 Salamanca, Spain
E-mail: gajateee@usal.es

J. Nieves

Instituto de Física Corpuscular (IFIC), Centro Mixto CSIC-Universidad de Valencia, Institutos de Investigación de Paterna, Apartado 22085, E-46071 Valencia, Spain
E-mail: jmnieves@ific.uv.es

We present results for exclusive $b \rightarrow c$ semileptonic decays of ground state triply-heavy baryons and for semileptonic $c \rightarrow s, d$ decays of doubly heavy ground state cb baryons. In both cases, we have derived for the first time heavy quark spin symmetry relations for the hadronic amplitudes near zero recoil. Though strictly valid in the limit of very large heavy quark masses and near zero recoil, they turn out to be reasonable accurate for the whole available phase space in these decays and for the actual heavy quark masses we use. With these relations we have made approximate, but model independent, predictions for ratios of decay widths. In the case of spin-1/2 cb baryons, we find that hyperfine mixing in the wave function has a great impact on their $c \rightarrow s, d$ decay widths.

*Xth Quark Confinement and the Hadron Spectrum,
October 8-12, 2012
TUM Campus Garching, Munich, Germany*

^{*}Speaker.

[†]This research was supported by DGI and FEDER funds, under contracts FIS2011-28853-C02-02, FPA2010-21750-C02-02, and the Spanish Consolider-Ingenio 2010 Programme CPAN (CSD2007-00042), by Generalitat Valenciana under contract PROMETEO/20090090 and by the EU HadronPhysics3 Grant no. 283286. C. A. thanks a Juan de la Cierva contract from the Spanish Ministerio de Educación y Ciencia.

1. Introduction

In this contribution we report on two recent calculations we have done concerning exclusive $b \rightarrow c$ semileptonic decay of triply heavy baryons [1] and $c \rightarrow s, d$ semileptonic decay of doubly heavy cb baryons [2].

The analysis of triply heavy baryons allows to study the interaction among heavy quarks in an environment free of valence light quarks. With no experimental information available on these systems so far, previous studies have concentrated on their spectrum [3, 4, 5, 6, 7, 8, 9]. However, it is likely that triply heavy baryons would be discovered at LHC [10] so that the study of their properties beyond spectroscopy seems timely. As for exclusive semileptonic $c \rightarrow s, d$ decays of doubly heavy ground state cb baryons, previous studies [11, 12, 13] are very limited. This is in contrast to their corresponding $b \rightarrow c$ driven decays which have been more extensively studied [11, 14, 15, 16, 17, 18]. However, the analysis of the $c \rightarrow s, d$ decays of cb baryons could also give relevant information on heavy quark physics complementary to the one obtained from the study of their $b \rightarrow c$ decays.

In both calculations we derive for the first time heavy quark spin symmetry (HQSS) approximate expressions for the hadronic matrix elements. From these we predict approximate, but model independent, relations among different decay widths.

The calculations are done in a nonrelativistic quark model framework. We use the AL1 potential of Refs. [19, 5] which contains $1/r$ and hyperfine terms, that can be understood as originating from a one-gluon exchange potential, together with a linear confining term. All the parameters of this potential have been adjusted to describe light and heavy meson spectra.

2. $b \rightarrow c$ semileptonic decays of triply heavy baryons

The wave functions we use to describe triply heavy baryons have the general form

$$\Psi_{\alpha_1 \alpha_2 \alpha_3} = \delta_{f_1 h} \delta_{f_2 h} \delta_{f_3 h'} \frac{\epsilon_{c_1 c_2 c_3}}{\sqrt{3!}} \Phi(r_1, r_2, r_{12})(1/2, 1/2, 1; s_1, s_2, s_1 + s_2)(1, 1/2, J; s_1 + s_2, s_3, M),$$

where α_j represents the spin (s), flavor (f) and color (c) quantum numbers of the j -th quark. As we are interested only in spin $J = 1/2$ or $J = 3/2$ ground state baryons, the total orbital angular momentum is $L = 0$. To solve the three-body problem we shall use a variational ansatz for the orbital part of the wave function. We write the orbital wave functions as the product of three functions, $\Phi(r_1, r_2, r_{12}) = \phi_{hh'}(r_1) \phi_{hh'}(r_2) \phi_{hh}(r_{12})$, each one depending on just one of the three variables r_1, r_2, r_{12} , where r_1, r_2 are the relative distances between quark three and quarks one and two respectively, and r_{12} is the relative distance between the first two quarks. For each of the ϕ functions above we take an expression consisting in the sum of displaced gaussians of the form $\phi(r) = \sum_{j=1}^4 a_j e^{-b_j^2(r+d_j)^2}$. We fix the variational parameters by minimizing the energy while the overall normalization is fixed at the end of the calculation.

In Table 1 we show the calculated masses of the triply heavy baryons. Our results agree nicely with the Faddeev evaluation in Ref. [5] using the same interquark potential. For comparison we also show results obtained in lattice QCD (LQCD) [9], the bag model (BM) [3], relativistic three quark model (RTQM) [6], QCD sum rules (QCDSR) [8] and the next to next to leading order

	This work	[5]	[9]	[3]	[6]	[8]	[20]
	Variational	Faddeev	LQCD	BM	RTQM	QCDSR	NNLO pNRQCD
$m_{\Omega_{bbb}^*}$	14398	14398	14371 ± 12	14300	14569	13280 ± 100	14700 ± 300
$m_{\Xi_{bbc}^*}$	11245	–	–	11200	11287	10540 ± 110	11400 ± 300
$m_{\Xi_{bbc}}$	11214	11217	–	–	11280	10300 ± 100	11400 ± 300
$m_{\Xi_{ccb}^*}$	8046	–	–	8030	8025	7450 ± 160	8150 ± 300
$m_{\Xi_{ccb}}$	8018	8019	–	–	8018	7410 ± 130	8150 ± 300
$m_{\Omega_{ccc}^*}$	4799	4799	–	4790	4803	4670 ± 150	4900 ± 250

Table 1: Triply heavy baryon masses (in MeV) obtained with the AL1 potential of Refs. [19, 5] using our variational approach. For comparison we also show the results from the Faddeev calculation performed in Ref. [5] using the same potential. Predictions within other approaches are also compiled. The Ω^* and Ξ^* baryons have total spin 3/2 while the Ξ ones have total spin 1/2.

calculation using potential nonrelativistic QCD (NNLO pNRQCD) [20]. The agreement with the LQCD result for the Ω_{bbb}^* baryon is good. We also agree with the results in the BM and RTQM calculations. On the other hand the QCDSR results are much smaller while the NNLO pNRQCD calculation predicts larger masses.

In the limit of very large heavy quark masses, HQSS predicts for the hadronic transition matrix elements near zero recoil [1]

$$\begin{aligned}
\Xi_{ccb} &\rightarrow \Omega_{ccc}^* & 2\eta \bar{u}'^\mu u, \\
\Xi_{ccb}^* &\rightarrow \Omega_{ccc}^* & -\sqrt{3}\eta \bar{u}'^\lambda \gamma^\mu (1 - \gamma_5) u_\lambda, \\
\Xi_{bbc} &\rightarrow \Xi_{ccb} & -\chi \bar{u}' (\gamma^\mu - \frac{5}{3} \gamma^\mu \gamma_5) u, \\
\Xi_{bbc} &\rightarrow \Xi_{ccb}^* & -\frac{2}{\sqrt{3}} \chi \bar{u}'^\mu u, \\
\Xi_{bbc}^* &\rightarrow \Xi_{ccb} & -\frac{2}{\sqrt{3}} \chi \bar{u}' u^\mu, \\
\Xi_{bbc}^* &\rightarrow \Xi_{ccb}^* & -2\chi \bar{u}'^\lambda \gamma^\mu (1 - \gamma_5) u_\lambda, \\
\Omega_{bbb}^* &\rightarrow \Xi_{bbc} & 2\xi \bar{u}' u^\mu, \\
\Omega_{bbb}^* &\rightarrow \Xi_{bbc}^* & -\sqrt{3}\xi \bar{u}'^\lambda \gamma^\mu (1 - \gamma_5) u_\lambda,
\end{aligned}$$

where the factors η , χ and ξ are the Isgur-Wise functions that depend on the product of four velocities of the two baryons $w = v \cdot v'$. We evaluate those Isgur-Wise functions in our model and we see that, as predicted by the HQSS relations above, they reduce to only three independent ones in very good approximation. With these functions we get estimates of the $b \rightarrow c$ semileptonic decay widths that we give in Table 2.

As the $d\Gamma/dw$ differential decay width peaks at w values very close to 1 [1], one can make further approximations valid in that region. The lepton tensor is approximately given by $\mathcal{L}^{\alpha\beta}(q) \approx \frac{-\pi q^2}{6} (g^{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2})$, where q is the total four-momentum of the leptonic system. Besides in the product of lepton and hadron tensors we can approximate $w \approx 1$ and $\frac{(v \cdot q)^2}{q^2} \approx \frac{(v \cdot q)(v' \cdot q)}{q^2} \approx \frac{(v' \cdot q)^2}{q^2}$.

$B \rightarrow B' e \bar{\nu}_e$	$\Gamma [\text{ps}^{-1}]$
$\Xi_{ccb} \rightarrow \Omega_{ccc}^* e \bar{\nu}_e$	8.01×10^{-2}
$\Xi_{ccb}^* \rightarrow \Omega_{ccc}^* e \bar{\nu}_e$	6.28×10^{-2}
$\Xi_{bbc} \rightarrow \Xi_{ccb} e \bar{\nu}_e$	7.98×10^{-2}
$\Xi_{bbc} \rightarrow \Xi_{ccb}^* e \bar{\nu}_e$	2.42×10^{-2}
$\Xi_{bbc}^* \rightarrow \Xi_{ccb} e \bar{\nu}_e$	1.17×10^{-2}
$\Xi_{bbc}^* \rightarrow \Xi_{ccb}^* e \bar{\nu}_e$	7.74×10^{-2}
$\Omega_{bbb}^* \rightarrow \Xi_{bbc} e \bar{\nu}_e$	3.95×10^{-2}
$\Omega_{bbb}^* \rightarrow \Xi_{bbc}^* e \bar{\nu}_e$	6.34×10^{-2}

Table 2: Estimated decay widths in units of ps^{-1} . We use $|V_{bc}| = 0.0410$.

Further assuming $m_{B_{bbc}} \approx m_{B_{bbc}}^*$; $m_{B_{ccb}} \approx m_{B_{ccb}}^*$, we predict the approximate ratios

$$\frac{2\Gamma(\Xi_{bbc}^* \rightarrow \Xi_{ccb})}{\Gamma(\Xi_{bbc} \rightarrow \Xi_{ccb}^*)} \approx 1,$$

$$\frac{\Gamma(\Xi_{bbc}^* \rightarrow \Xi_{ccb}^*)}{4\Gamma(\Xi_{bbc} \rightarrow \Xi_{ccb}) - 10\Gamma(\Xi_{bbc} \rightarrow \Xi_{ccb}^*)} \approx 1.$$

These approximate, but model independent, predictions are satisfied in our own calculation at the 3.4% and 0.25% level respectively and we expect them to hold in other approaches as well.

3. $c \rightarrow s, d$ semileptonic decays of doubly heavy cb baryons

The baryons involved in the present calculation are given in Table 3. The quark model masses quoted have been taken from our previous works in Refs. [21, 18], where they were obtained using the AL1 potential of Refs. [19, 5]. All the details on the wave functions and how they are evaluated can be found in Refs. [2, 21, 22]. Experimental masses shown in Table 3 are isospin averaged over the values reported by the particle data group [23]. For the actual calculation of the decay widths we use experimental masses whenever possible.

The classification scheme shown in Table 3 assumes that the two heavy quarks or the two light quarks have well defined total spin S . This is not correct for spin-1/2 states. Due to the finite value of the heavy quark masses, the hyperfine interaction between a light quark and a heavy quark can admix both $S=0$ and 1 components into the wave function. We neglect these effects for the Ξ_b and Ξ_b' states as the hyperfine matrix elements linking the two states are proportional to the inverse of the m_b quark mass. On the other hand, for the Ξ_{cb} , Ξ_{cb}' (Ω_{cb} , Ω_{cb}') the effect is only suppressed by the c quark mass and it is relevant. As a result, the actual physical spin-1/2 cb baryons are admixtures of the Ξ_{cb} , Ξ_{cb}' (Ω_{cb} , Ω_{cb}') states. The physical states that we obtain in our model are given by [18]¹

$$\Xi_{cb}^{(1)} = -0.902 \Xi_{cb}' + 0.431 \Xi_{cb} ; M_{\Xi_{cb}^{(1)}} = 6967 \text{ MeV},$$

¹Note that here we use the order cb , whereas in Ref. [18], we used bc . Thus our Ξ_{cb}' and Ω_{cb}' states, where the heavy quark subsystem is coupled to spin zero, differ in sign with those used in Ref. [18].

Baryon	J^π	I	S^π	Quark content	Mass [MeV]	
					Quark model [21, 18]	Experiment [23]
Ξ_{cb}	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	cbn	6928	–
Ξ'_{cb}	$\frac{1}{2}^+$	$\frac{1}{2}$	0^+	cbn	6958	–
Ξ^*_{cb}	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	cbn	6996	–
Ω_{cb}	$\frac{1}{2}^+$	0	1^+	cbs	7013	–
Ω'_{cb}	$\frac{1}{2}^+$	0	0^+	cbs	7038	–
Ω^*_{cb}	$\frac{3}{2}^+$	0	1^+	cbs	7075	–
Λ_b	$\frac{1}{2}^+$	0	0^+	udb	5643	5620.2 ± 1.6
Σ_b	$\frac{1}{2}^+$	1	1^+	nmb	5851	5811.5 ± 2.4
Σ^*_b	$\frac{3}{2}^+$	1	1^+	nmb	5882	5832.7 ± 3.1
Ξ_b	$\frac{1}{2}^+$	$\frac{1}{2}$	0^+	nsb	5808	5790.5 ± 2.7
Ξ'_b	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	nsb	5946	–
Ξ^*_b	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	nsb	5975	–
Ω_b	$\frac{1}{2}^+$	0	1^+	ssc	6033	6071 ± 40
Ω^*_b	$\frac{3}{2}^+$	0	1^+	ssc	6063	–

Table 3: Quantum numbers of the baryons involved in this study. The usual classification scheme in which the two heavy quarks or the two light quarks have well defined total spin is used. J^π and I are the spin-parity and isospin of the baryon, S^π is the spin-parity of the two heavy or the two light quark subsystem. n denotes a u or d quark.

$$\begin{aligned}\Xi_{cb}^{(2)} &= 0.431 \Xi'_{cb} + 0.902 \Xi_{cb} ; M_{\Xi_{cb}^{(2)}} = 6919 \text{ MeV}, \\ \Omega_{cb}^{(1)} &= -0.899 \Omega'_{cb} + 0.437 \Omega_{cb} ; M_{\Omega_{cb}^{(1)}} = 7046 \text{ MeV}, \\ \Omega_{cb}^{(2)} &= 0.437 \Omega'_{cb} + 0.899 \Omega_{cb} ; M_{\Omega_{cb}^{(2)}} = 7005 \text{ MeV}.\end{aligned}$$

These physical spin-1/2 cb baryon states turn out to be very close to the states (here B stands for Ξ or Ω)

$$\begin{aligned}\widehat{B}_{cb} &= -\frac{\sqrt{3}}{2} B'_{cb} + \frac{1}{2} B_{cb}, \\ \widehat{B}'_{cb} &= \frac{1}{2} B'_{cb} + \frac{\sqrt{3}}{2} B_{cb}.\end{aligned}$$

in which the c and the light q quark couple to well defined spin $S_{cq} = 1$ (\widehat{B}_{cb}) or 0 (\widehat{B}'_{cb}), and then the b quark couples to that state to make the baryon with total spin 1/2. Hyperfine mixing for the $\widehat{B}_{cb}, \widehat{B}'_{cb}$ states is much less important since it is inversely proportional to the b quark mass.

While masses are not very sensitive to hyperfine mixing, it was pointed out in Ref. [24] that hyperfine mixing could greatly affect the decay widths of doubly heavy spin-1/2 cb baryons. This assertion was confirmed for $b \rightarrow c$ semileptonic decay in Refs. [16, 18] and for electromagnetic transitions in Refs. [25, 26]. We expected configuration mixing to also play an important role for $c \rightarrow s, d$ semileptonic decay of cb baryons.

The decay widths we evaluate appear in Tables 4 and 5. We show our full results and, in between parentheses, the results where configuration mixing is not considered. In all cases we find a good agreement with the few other previous calculations. We also see that configuration mixing effects are very important for transitions to final states where the two light quarks couple to spin 1, where we find enhancements or reductions as large as a factor of 2.

	$\Gamma [10^{-14} \text{ GeV}]$			$\Gamma [10^{-14} \text{ GeV}]$
	This work	Others		
$\Xi_{cbu}^{(1)+} \rightarrow \Xi_b^0 e^+ \nu_e$	3.74 (3.45)	(3.4) [11]	$\Omega_{cbs}^{(1)0} \rightarrow \Omega_b^- e^+ \nu_e$	7.21 (3.12)
$\Xi_{cbu}^{(2)+} \rightarrow \Xi_b^0 e^+ \nu_e$	2.65 (2.87)		$\Omega_{cbs}^{(2)0} \rightarrow \Omega_b^- e^+ \nu_e$	3.49 (7.12)
$\Xi_{cbu}^{(1)+} \rightarrow \Xi_b^{*0} e^+ \nu_e$	3.88 (1.66)	$2.44 \div 3.28^\dagger$ [12]	$\Omega_{cbs}^{(1)0} \rightarrow \Omega_b^{*-} e^+ \nu_e$	2.98 (6.90)
$\Xi_{cbu}^{(2)+} \rightarrow \Xi_b^{*0} e^+ \nu_e$	1.95 (3.91)		$\Omega_{cbs}^{(2)0} \rightarrow \Omega_b^{*-} e^+ \nu_e$	5.50 (2.07)
$\Xi_{cbu}^{(1)+} \rightarrow \Xi_b^{*0} e^+ \nu_e$	1.52 (3.45)		$\Omega_{cbs}^{*0} \rightarrow \Omega_b^- e^+ \nu_e$	1.35
$\Xi_{cbu}^{(2)+} \rightarrow \Xi_b^{*0} e^+ \nu_e$	2.67 (1.02)		$\Omega_{cbs}^{*0} \rightarrow \Omega_b^{*-} e^+ \nu_e$	10.2
$\Xi_{cbu}^{(2)+} \rightarrow \Xi_b^0 e^+ \nu_e + \Xi_b^{*0} e^+ \nu_e + \Xi_b^{*0} e^+ \nu_e$	7.27 (7.80)	$(9.7 \pm 1.3)^\ddagger$ [13]		
$\Xi_{cbu}^{*+} \rightarrow \Xi_b^0 e^+ \nu_e$	4.08			
$\Xi_{cbu}^{*+} \rightarrow \Xi_b^{*0} e^+ \nu_e$	0.747			
$\Xi_{cbu}^{*+} \rightarrow \Xi_b^{*0} e^+ \nu_e$	5.03			

Table 4: Γ decay widths for $c \rightarrow s$ decays. We use $|V_{cs}| = 0.973$. Results where configuration mixing is not considered are shown in between parentheses. The result with a \dagger corresponds to the decay of the $\widehat{\Xi}_{cb}$ state. The result with an \ddagger is our estimate from the total decay width and the branching ratio given in [13].

	$\Gamma [10^{-14} \text{ GeV}]$		$\Gamma [10^{-14} \text{ GeV}]$
$\Xi_{cbu}^{(1)+} \rightarrow \Lambda_b^0 e^+ \nu_e$	0.219 (0.196)	$\Omega_{cbs}^{(1)0} \rightarrow \Xi_b^- e^+ \nu_e$	0.179 (0.164)
$\Xi_{cbu}^{(2)+} \rightarrow \Lambda_b^0 e^+ \nu_e$	0.136 (0.154)	$\Omega_{cbs}^{(2)0} \rightarrow \Xi_b^- e^+ \nu_e$	0.120 (0.133)
$\Xi_{cbu}^{(1)+} \rightarrow \Sigma_b^0 e^+ \nu_e$	0.198 (0.0814)	$\Omega_{cbs}^{(1)0} \rightarrow \Xi_b^{\prime-} e^+ \nu_e$	0.169 (0.0702)
$\Xi_{cbu}^{(2)+} \rightarrow \Sigma_b^0 e^+ \nu_e$	0.110 (0.217)	$\Omega_{cbs}^{(2)0} \rightarrow \Xi_b^{\prime-} e^+ \nu_e$	0.0908 (0.182)
$\Xi_{cbu}^{(1)+} \rightarrow \Sigma_b^{*0} e^+ \nu_e$	0.0807 (0.184)	$\Omega_{cbs}^{(1)0} \rightarrow \Xi_b^{*-} e^+ \nu_e$	0.0690 (0.160)
$\Xi_{cbu}^{(2)+} \rightarrow \Sigma_b^{*0} e^+ \nu_e$	0.147 (0.0556)	$\Omega_{cbs}^{(2)0} \rightarrow \Xi_b^{*-} e^+ \nu_e$	0.130 (0.0487)
$\Xi_{cbu}^{*+} \rightarrow \Lambda_b^0 e^+ \nu_e$	0.235	$\Omega_{cbs}^{*0} \rightarrow \Xi_b^- e^+ \nu_e$	0.196
$\Xi_{cbu}^{*+} \rightarrow \Sigma_b^0 e^+ \nu_e$	0.0399	$\Omega_{cbs}^{*0} \rightarrow \Xi_b^{\prime-} e^+ \nu_e$	0.0336
$\Xi_{cbu}^{*+} \rightarrow \Sigma_b^{*0} e^+ \nu_e$	0.246	$\Omega_{cbs}^{*0} \rightarrow \Xi_b^{*-} e^+ \nu_e$	0.223

Table 5: Γ decay widths for $c \rightarrow d$ decays. We use $|V_{cd}| = 0.225$. In between parentheses we show the results without configuration mixing.

Now, in the limit of very large heavy quark masses we can use HQSS to approximately evaluate the hadronic matrix elements for semileptonic transitions between hatted states (S_{cq} well defined). Close to zero recoil those matrix elements are given by [2]

- $\widehat{B}_{cb} \rightarrow \Lambda_b, \Xi_b \quad \frac{1}{\sqrt{3}} \eta \bar{u}' (-\gamma^\mu \gamma_5) u,$
- $\widehat{B}'_{cb} \rightarrow \Lambda_b, \Xi_b \quad \eta \bar{u}' \gamma^\mu u,$
- $\widehat{B}_{cb}^* \rightarrow \Lambda_b, \Xi_b \quad -\eta \bar{u}' u^\mu,$
- $\widehat{B}_{cb} \rightarrow \Sigma_b, \Xi_b', \Omega_b \quad \beta \bar{u}' (\gamma^\mu - \frac{2}{3} \gamma^\mu \gamma_5) u,$

- $\widehat{B}'_{cb} \rightarrow \Sigma_b, \Xi'_b, \Omega_b \quad \frac{1}{\sqrt{3}}\beta \bar{u}'(-\gamma^\mu \gamma_5)u,$
- $\widehat{B}^*_{cb} \rightarrow \Sigma_b, \Xi'_b, \Omega_b \quad \frac{1}{\sqrt{3}}\beta \bar{u}'u^\mu,$
- $\widehat{B}_{cb} \rightarrow \Sigma_b^*, \Xi_b^*, \Omega_b^* \quad \frac{1}{\sqrt{3}}\beta \bar{u}'^\mu u,$
- $\widehat{B}'_{cb} \rightarrow \Sigma_b^*, \Xi_b^*, \Omega_b^* \quad -\beta \bar{u}'^\mu u,$
- $\widehat{B}^*_{cb} \rightarrow \Sigma_b^*, \Xi_b^*, \Omega_b^* \quad -\beta \bar{u}'^\lambda \gamma^\mu (1 - \gamma_5)u_\lambda.$

These relations impose restrictions on the form factors for the different decays that are well satisfied within our model [2] over the whole w range accessible for the decays and for the actual heavy quark masses. With the use of the above HQSS relations, and the approximations (exact a zero recoil) similar to the ones described in the previous section, we are able to predict approximate, but model independent, relations among decay widths for hatted states. Those are given in the following where we also show the results of our full calculation using physical (close to hatted) states.

$$\begin{aligned} \Gamma(\widehat{\Xi}_{cb} \rightarrow \Lambda_b) &\approx \Gamma(\widehat{\Xi}_{cb}^* \rightarrow \Lambda_b) & 0.219 &\approx 0.235, \\ \Gamma(\widehat{B}_{cb} \rightarrow \Xi_b) &\approx \Gamma(\widehat{B}_{cb}^* \rightarrow \Xi_b) & 0.179 &\approx 0.196 \quad (B = \Omega) & 3.73 &\approx 4.08, \quad (B = \Xi) \end{aligned}$$

$$\begin{aligned} \Gamma(\widehat{\Xi}'_{cb} \rightarrow \Sigma_b) &\approx 3\Gamma(\widehat{\Xi}'_{cb} \rightarrow \Sigma_b) \approx \frac{3}{2}\Gamma(\widehat{\Xi}_{cb} \rightarrow \Sigma_b^*) \approx \frac{1}{2}\Gamma(\widehat{\Xi}'_{cb} \rightarrow \Sigma_b^*) \\ &0.110 \approx 0.120 \approx 0.121 \approx 0.074, \end{aligned}$$

$$\begin{aligned} \Gamma(\widehat{B}'_{cb} \rightarrow \Xi'_b) &\approx 3\Gamma(\widehat{B}'_{cb} \rightarrow \Xi'_b) \approx \frac{3}{2}\Gamma(\widehat{B}_{cb} \rightarrow \Xi_b^*) \approx \frac{1}{2}\Gamma(\widehat{B}'_{cb} \rightarrow \Xi_b^*) \\ &0.097 \approx 0.101 \approx 0.104 \approx 0.065 \quad (B = \Omega) \\ &1.95 \approx 2.24 \approx 2.29 \approx 1.34 \quad (B = \Xi), \end{aligned}$$

$$\begin{aligned} \Gamma(\widehat{\Omega}'_{cb} \rightarrow \Omega_b) &\approx 3\Gamma(\widehat{\Omega}'_{cb} \rightarrow \Omega_b) \approx \frac{3}{2}\Gamma(\widehat{\Omega}_{cb} \rightarrow \Omega_b^*) \approx \frac{1}{2}\Gamma(\widehat{\Omega}'_{cb} \rightarrow \Omega_b^*) \\ &3.49 \approx 4.05 \approx 4.48 \approx 2.75, \end{aligned}$$

$$\begin{aligned} \Gamma(\widehat{\Xi}_{cb}^* \rightarrow \Sigma_b^*) &\approx \Gamma(\widehat{\Xi}_{cb}^* \rightarrow \Sigma_b) + \Gamma(\widehat{\Xi}_{cb} \rightarrow \Sigma_b) & 0.246 &\approx 0.238, \\ \Gamma(\widehat{B}_{cb}^* \rightarrow \Xi_b^*) &\approx \Gamma(\widehat{B}_{cb}^* \rightarrow \Xi'_b) + \Gamma(\widehat{B}_{cb} \rightarrow \Xi'_b) & 0.223 &\approx 0.203 \quad (B = \Omega) & 5.03 &\approx 4.62 \quad (B = \Xi), \\ \Gamma(\widehat{\Omega}_{cb}^* \rightarrow \Omega_b^*) &\approx \Gamma(\widehat{\Omega}_{cb}^* \rightarrow \Omega_b) + \Gamma(\widehat{\Omega}_{cb} \rightarrow \Omega_b) & 10.2 &\approx 8.56, \end{aligned}$$

Our results agree with the HQSS based predictions at the 10% level in most cases. The large discrepancies present in a few notable cases are mainly due to the different phase space as a result of baryon mass differences [2]. We expect the above relations to hold in other approaches to the same level of accuracy.

References

- [1] J.M. Flynn, E. Hernández and J. Nieves, Phys. Rev. D **85**, 014012 (2012).
- [2] C. Albertus, E. Hernández and J. Nieves, Phys. Rev. D **85**, 094035 (2012).
- [3] P. Hasenfratz, R. R. Horgan, J. Kuti, J. M. Richard, Phys. Lett. **B94**, 401 (1980).
- [4] J.D. Bjorken, Preprint FERMILAB-Conf-85/69.
- [5] B. Silvestre-Brac, Few-Body Systems **20**, 1 (1996).
- [6] A. P. Martynenko, Phys. Lett. **B663**, 317 (2008).
- [7] W. Roberts and M. Pervin, Int. J. Mod. Phys. A **23**, 2817 (2008).
- [8] J. R. Zhang and M. Q. Huang, Phys. Lett. **B674** (2009) 28.
- [9] S. Meinel, Phys. Rev. D **82**, 114514 (2010).
- [10] Y. -Q. Chen, S. -Z. Wu, JHEP **1108**, 144 (2011); Erratum-ibid. **1109**, 089 (2011).
- [11] M. A. Sanchis-Lozano, Nucl. Phys. B **440**, 251 (1995).
- [12] A. Faessler et al., Phys. Lett. **B518**, 55 (2001).
- [13] V. V. Kiselev and A. K. Likhoded, Phys. Usp. **45**, 455 (2002) [Usp. Fiz. Nauk **172**, 497 (2002)].
- [14] D. Ebert, R. N. Faustov, V. O. Galkin and A. P. Martynenko, Phys. Rev. D **70**, 014018 (2004); Erratum-ibid. D **77**, 079903 (2008).
- [15] E. Hernandez, J. Nieves, J. M. Verde-Velasco, Phys. Lett. **B663**, 234 (2008).
- [16] W. Roberts and M. Pervin, Int. J. Mod. Phys. A **24**, 2401 (2009).
- [17] A. Faessler et al, Phys. Rev. D **80**, 034025 (2009).
- [18] C. Albertus, E. Hernandez and J. Nieves, Phys. Lett. **B683**, 21 (2010).
- [19] C. Semay, and B. Silvestre-Brac, Z. Phys. C **61**, 271 (1994).
- [20] F.J. Llanes-Estrada, O.I. Pavlova, R. Williams, Eur. Phys. J. C **72**, 2019 (2012).
- [21] C. Albertus, J. E. Amaro, E. Hernandez and J. Nieves, Nucl. Phys. A **740**, 333 (2004).
- [22] C. Albertus, E. Hernandez, J. Nieves and J. M. Verde-Velasco, Eur. Phys. J. A **31**, 691 (2007); Erratum-ibid. A **36**, 119 (2008).
- [23] K. Nakamura et al. (Particle Data Group), J. Phys. G **37**, 075021 (2010).
- [24] W. Roberts and M. Pervin, Int. J. Mod. Phys. A **23**, 2817 (2008).
- [25] C. Albertus, E. Hernandez, J. Nieves, Phys. Lett. **B690**, 265 (2010).
- [26] T. Branz et al., Phys. Rev. D **81**, 114036 (2010).