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Decade of hydrodynamics - what have we learnt?

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The use of hydrodynamics to model the collisions of elementary particles has a long tradition dating back to the works of Landau and Fermi. However, it is during the last decade when hydrodynamics has became an indispensable tool for describing the heavy-ion collisions at ultra-relativistic energies (RHIC at BNL and LHC at CERN). In this contribution I briefly review the use of hydrodynamics to describe the low momentum particle production (so called bulk) in these collisions, and what we have learnt when applying hydrodynamics to ultrarelativistic heavy-ion collisions.

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The goal of ultrarelativistic heavy-ion collisions is to form strongly interacting matter—matter in a sense that the thermodynamical concepts like temperature and pressure apply. Thus it is natural to try to use fluid dynamics to describe the expansion stage of the collision. In a case of no conserved charges, the equations of motion are the conservation laws for energy and momentum. In the ideal fluid approximation, *i.e.* when there is no dissipation, they can be written as:

$$\partial_\mu T^{\mu\nu} = 0, \qquad \text{where} \qquad T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu},$$

and ε is energy density in the rest frame of the fluid, P pressure, u^μ is the fluid 4-velocity and $g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ is the metric tensor. These four equations contain five unknowns. To close the set of equations we need an equation of state (EoS) connecting pressure to energy density, $P = P(\varepsilon)$. The dynamics is now uniquely defined, but the actual solution depends on the boundary conditions: The initial distribution of matter, and the criterion for the end of evolution. Hydrodynamics does not provide either of these, but they have to be supplied by other models. The end of evolution is usually taken to be a hypersurface of constant temperature or energy density, where the fluid is converted to particles (particlization). In pure hydrodynamical models all interactions are assumed to cease at this point and particle distributions freeze out. In so-called hybrid models fluid dynamics is used to describe only the early dense stage of the evolution, and particles formed at the end of fluid dynamical evolution are fed into a hadron cascade describing the late dilute hadronic stage.

1. There are rescatterings

The particle production in the primary collisions is azimuthally isotropic, but the distribution of observed particles in A+A collisions is not. The anisotropy can be easily explained in terms of rescatterings of the produced particles: In a non-central collision the collision zone has an elongated shape. If a particle is heading to a direction where the collision zone is long, it has a larger probability to scatter and change its direction than a particle heading to a direction where the collision zone is short. Thus more particles end up in direction where the edge of the collision zone is close. Or, in a hydrodynamical language, the pressure gradient between the center of the system and the vacuum is larger in the "short" direction, the flow velocity is thus larger in that direction, and more particles are emitted in that direction than elsewhere.

This anisotropy is quantified in terms of Fourier expansion of the azimuthal distribution. The coefficients of this expansion v_n, and the associated participant angles ψ_n, are defined as

$$v_n = \langle \cos[n(\phi - \psi_n)] \rangle, \qquad \text{and} \qquad \psi_n = \frac{1}{n} \arctan \frac{\langle p_T \sin(n\phi) \rangle}{\langle p_T \cos(n\phi) \rangle}.$$

Of these coefficients v_1 is called directed, v_2 elliptic, and v_3 triangular flow. Elliptic flow of charged hadrons as a function of centrality was one of the first measurements at RHIC [1]. The result is shown in Fig. 1, and compared to early fluid dynamical calculations [2]. As seen, the elliptic flow is quite large and increases with decreasing centrality, as expected if it has the described geometric origin. Thus there must be rescatterings among the particles formed in the collision, and an A+A collision is not just a sum of independent pp collisions. The observed elliptic flow is also very close to the hydrodynamically calculated one, which is a very strong indication of hydrodynamical behaviour of the matter. If the produced matter is not close to kinetic equilibrium, at least it behaves as if it was.

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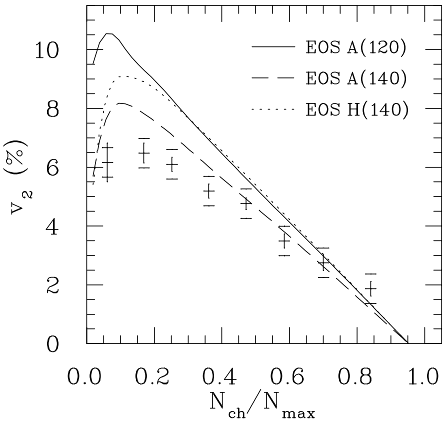

Figure 1: The elliptic flow parameter v_2 of charged hadrons as function of centrality ($\mathrm{N_{ch}/N_{max}} = 1$ is the most central collision) in Au+Au collisions at $\sqrt{s_{\mathrm{NN}}} = 130$ GeV. The data are from Ref. [1] and the calculation using different EoSs and freeze-out temperatures from Ref. [2].

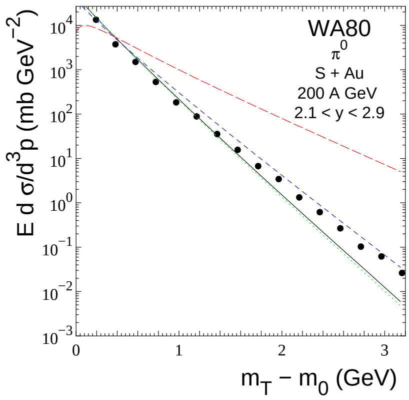

Figure 2: Transverse momentum distribution of neutral pions in S+Au collision at $E_{\mathrm{lab}} = 200A$ GeV using four different EoSs. The red curve corresponds to ideal pion gas EoS, whereas the other curves correspond to hadron resonance gas with or without phase transition to ideal parton gas. The figure is from Ref. [3], and the data are from Ref. [4].

2. Equation of State has many degrees of freedom

The equation of state (EoS) of strongly interacting matter is an explicit input to hydrodynamical models. Thus one might expect hydrodynamical modelling of heavy-ion collisions to tell us a lot about the equation of state, but unfortunately that is not the case. The collective motion of the system is directly affected by the pressure gradients in the system, and thus by the EoS, but the effects of the EoS on the final particle p_T distributions can to very large extent be compensated by changes in the initial state of the evolution and the final decoupling temperature. This makes constraining the properties of the EoS very difficult. However, what we do know is that the number of degrees of freedom has to be large.

It was already seen when modelling S+Au collisions at the CERN SPS at $E_{\mathrm{lab}} = 200A$ GeV energy, that if we use ideal pion gas EoS, the transverse momentum distribution of pions becomes too flat [3], see Fig. 2. If one changes the freeze-out temperature to reduce the transverse flow velocity, the increasing temperature compensates the lower velocity, and the spectrum stays too hard. As well, if we use an EoS containing several hadrons and resonances, the distributions can be fitted. In the present calculations the “large number” usually means all hadrons and resonances in the Particle Data Book up to ~ 2 GeV mass in the low temperature region and a parton gas in the high temperature region.

One might want to use the elliptic flow to constrain the EoS after the initial state and freeze-out temperature are fixed to reproduce the p_T distributions. Unfortunately elliptic flow is only very weakly sensitive to the details of the EoS [5]: The only observable affected by the EoS seems to be the p_T-differential anisotropy of heavy particles, *e.g.* protons. As shown in Fig. 3, the $v_2(p_t)$ of pions is unchanged within the experimental errors no matter whether one uses an EoS with (EoS A) or without phase transition (EoS H), or an EoS with a first order phase transition (EoS A)

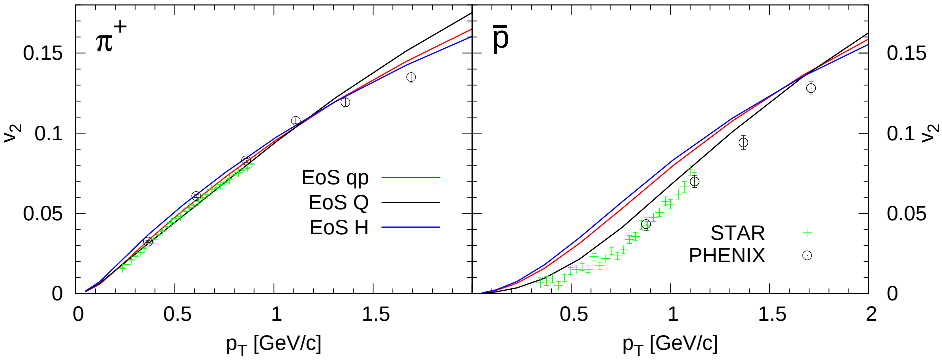

Figure 3: Elliptic flow of pions and antiprotons vs. transverse momentum in minimum bias Au+Au collisions at $\sqrt{s_{\rm NN}} = 200$ GeV calculated using three different EoSs [5] and compared with the data by the STAR and PHENIX collaborations [6]. The labels stand for a lattice QCD inspired quasiparticle model (qp), EoS with a first order phase transition (Q), and pure hadron resonance gas with no phase transition (H).

or a smooth crossover (EoS qp). On the other hand, the proton $v_2(p_t)$ is sensitive to the EoS, but surprisingly the EoS with the first order phase transition is closest to the data.

Consequently, distinguishing between different parametrizations of the lattice QCD EoSs is very difficult, see Ref. [7]. In the present calculations the lattice QCD EoS is taken as given, but in the long run a systematic study of collisions at different energies may reveal some sensitivity to EoS and thus help to test the applicability of the lattice QCD EoS to describe the mesoscopic amount of matter created in ultrarelativistic heavy ion collisions.

3. Shear viscosity over entropy density ratio has very low minimum

Once it became clear that the ideal fluid dynamics can describe the particle spectra and their anisotropies fairly well, it was reasonable to assume that the matter formed in the collision has very low shear viscosity coefficient to entropy density ratio η/s. But how low in particular? To answer that question required the development of relativistic dissipative fluid dynamical models. Unfortunately the relativistic generalization of Navier-Stokes hydrodynamics allows acausal and unstable solutions, and is thus not suitable to describe heavy-ion collisions. The stability and causality can be restored by assuming that the dissipative currents (shear stress tensor $\pi^{\mu\nu}$, heat flow q^{μ} and bulk pressure Π) are not directly related to the gradients in the system, but are dynamical variables which relax to their Navier-Stokes values on time scales given by the corresponding relaxation times τ_π, τ_q, and τ_Π (for a more detailed discussion see *e.g.* Ref [8] and references therein). The present studies of heavy-ion collisions are concentrated on the midrapidity region where the net-baryon density is tiny and thus the heat flow is negligible. The bulk viscosity coefficient is expected to be large around the phase transition, but small below and above it. The effect of bulk viscosity has been evaluated to be smaller than the effect of shear viscosity [9], and since there is no reliable method to distinguish the effects of bulk from the effects of shear, the former is largerly ignored, and the calculations concentrate on studying the effects of shear viscosity and on extracting the η/s ratio from the experimental data.

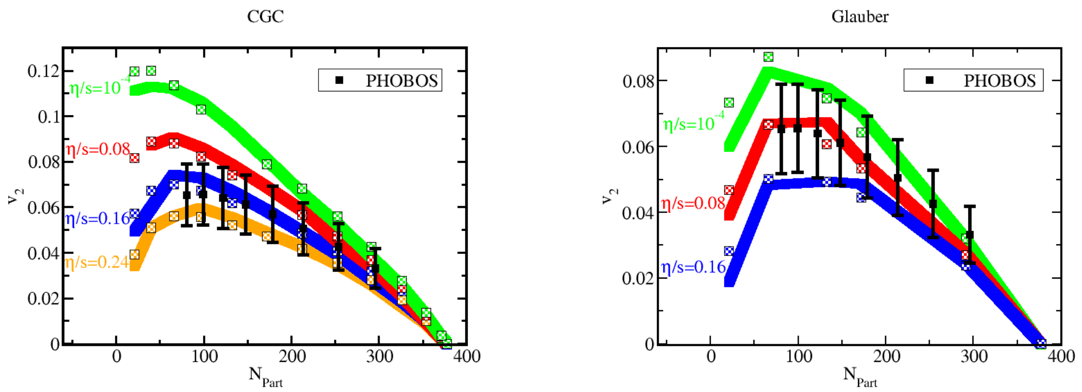

Figure 4: Charged hadron v_2 as function of centrality in Au+Au collisions at $\sqrt{s_{\rm NN}} = 200$ GeV using different values of η/s and CGC (left) and Glauber (right) initial conditions. The data are from Ref. [11] and the Figures from Ref. [12].

It has been shown that the shear viscosity strongly reduces v_2 [10]. Thus in principle extracting the η/s ratio from the data is easy: One needs to calculate the p_T-averaged v_2 of charged hadrons using various values of η/s and choose the value of η/s which reproduces the data, see Fig. 4. Unfortunately this approach is hampered by our ignorance of the initial state of the evolution. In Fig. 4 a viscous fluid calculation of v_2 of charged hadrons is shown [12]. As seen, a curve corresponding to a finite value of η/s fits the data best, but the preferred value depends on how the initial state of hydrodynamic evolution is chosen: Whether one uses so-called MC-Glauber [13] or MC-KLN [14] model causes a factor two difference in the preferred value ($\eta/s = 0.08$–0.16).

The calculations have been improved since Ref. [12] by a better treatment of the hadronic phase (see *e.g.* Ref. [15]), but the same uncertainty remains. This uncertainty can be reduced by studying the higher flow coefficients ($v_n, n > 2$) event-by-event. Because of the fluctuations of the positions of nucleons in the nuclei, the initial collision region has an irregular shape which fluctuates event-by-event, see Fig. 5, and thus all the coefficients v_n are finite [16]. As illustrated in Fig. 6, the larger the n, the more sensitive the coefficient v_n is to viscosity [18]. This provides a possibility to distinguish between different initializations, and preliminary results for the p_T-dependence of v_2 and v_3 seem to favour the MC-Glauber initialization [19].

On the other hand, in event-by-event studies it is not sufficient to reproduce only the average values of v_n, but the fluctuations of the flow coefficients should be reproduced as well. Neither MC-Glauber nor MC-KLN model seems to be able to reproduce the measured fluctuations [20], whereas the recent calculation using so-called IP-Glasma [21] initialization reproduces both the fluctuations and the average values of v_2, v_3 and v_4 [22], making this approach very promising.

However, in the calculations discussed above the η/s-ratio is assumed to be constant. We know no fluid where the η/s-ratio would be temperature independent, and there are theoretical reasons to expect it to depend on temperature with a minimum around T_c [23]. Thus the temperature independent η/s is only an effective viscosity, and its connection to the physical, temperature dependent, shear viscosity coefficient is unclear. What complicates the determination of the physical shear viscosity coefficient, is that the sensitivity of the anisotropies to dissipation varies during

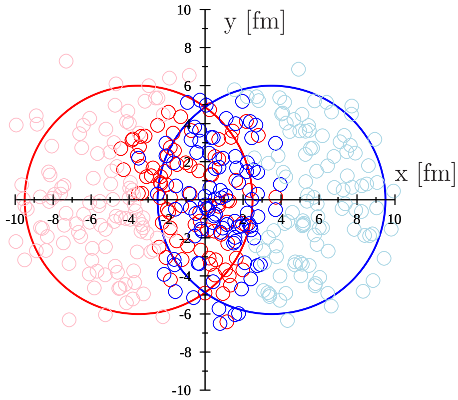

Figure 5: An example of the positions of interacting nuclei in MC-Glauber model. Figure is from Ref. [17].

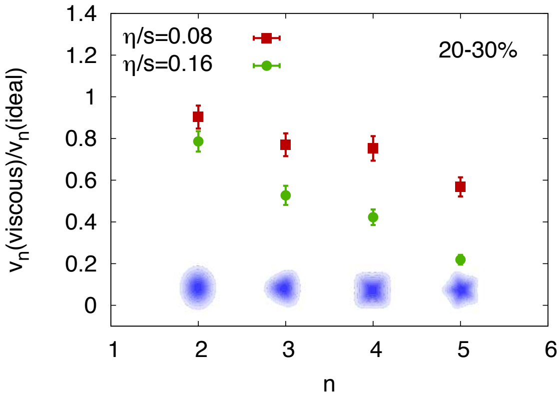

Figure 6: Ratio of the anisotropy coefficients of charged hadrons in viscous calculation to the coefficients in ideal fluid calculation [18]. Figure courtesy to Bjoern Schenke.

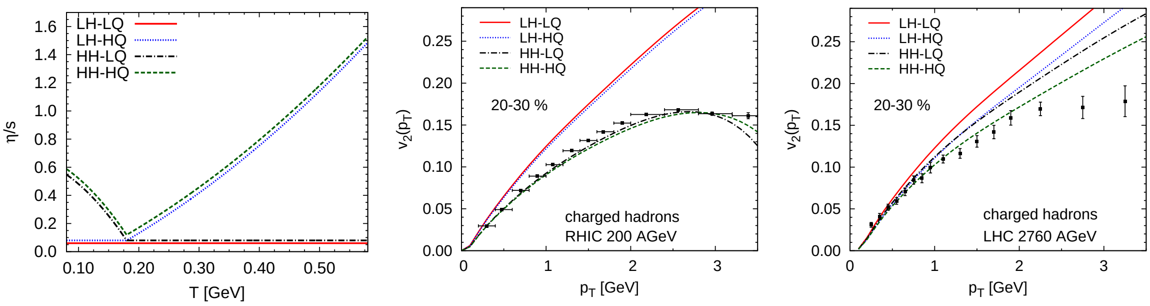

Figure 7: (left) Different parametrizations of η/s as a function of temperature. (center) $v_2(p_T)$ of charged hadrons in the 20-30% Au+Au collisions at $\sqrt{s_{\rm NN}} = 200$ GeV (RHIC). Data are from Ref [25]. (right) $v_2(p_T)$ of charged hadrons in the 20-30% Pb+Pb collisions at $\sqrt{s_{\rm NN}} = 2.76$ TeV (LHC). Data are from Ref [26]. All the figures are from Ref. [27].

the evolution of the system. As studied in Ref. [24], and illustrated in Fig. 7, at RHIC ($\sqrt{s_{\rm NN}} = 200$ GeV) v_2 is insensitive to the value of η/s above T_c, but very sensitive to its minimum value around T_c, and to its value in the hadronic phase below T_c. At the present LHC energy ($\sqrt{s_{\rm NN}} = 2.76$ TeV) the shear viscosity in the plasma phase does affect the final v_2, but not more than the shear viscosity in the hadronic phase. Thus disentangling the effect of viscosity during different stages of the evolution is challenging. Additional factor complicating the determination of the temperature dependence of η/s is that the effect of viscosity on the anisotropies does not depend only on the ratio η/s, but also on the relaxation time τ_π of the shear stress tensor. This is demonstrated in Fig. 8. If the minimum value of η/s is increased by a factor two, v_2 is reduced as expected, but if the relaxation time is also increased by a factor two, the effect of the increase in η/s is almost completely compensated. Disentangling the effects of relaxation time and shear viscosity will be difficult and has not yet been tried. Because of all these complications we can only say that the minimum value of the η/s ratio of strongly interacting matter is small, and in the vicinity of the

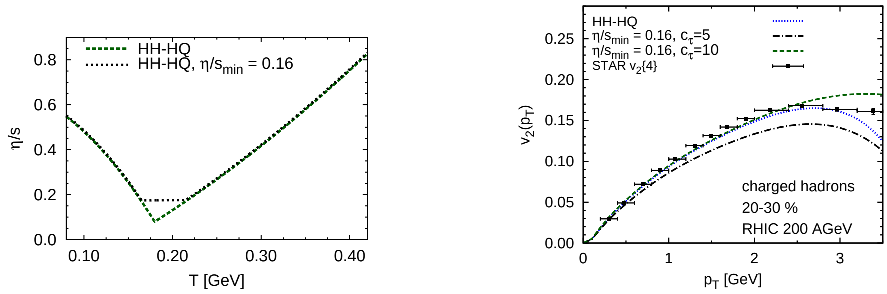

Figure 8: (left) Parametrizations of η/s with as a function of temperature with different minima. The (HH-HQ) line is the same than in Fig. 7. (right) $v_2(p_t)$ of charged hadrons at RHIC using $\eta/s(T)$ with different minima and different relaxation times. Figures are from Ref. [24].

postulated minimum of $\eta/s = 1/4\pi$, but how small, is too early to say.

Finally, in this kind of a short summary it is not possible to discuss all features of flow and hydrodynamical modeling. An interested reader can find an up-to-date review *e.g.* in Ref. [8].

References

[1] K. H. Ackermann *et al.* [STAR Collaboration], *Elliptic flow in Au + Au collisions at (S(NN))**(1/2) = 130 GeV,* Phys. Rev. Lett. **86** (2001) 402 [nucl-ex/0009011].

[2] P. F. Kolb, P. Huovinen, U. W. Heinz and H. Heiselberg, *Elliptic flow at SPS and RHIC: From kinetic transport to hydrodynamics*, Phys. Lett. B **500** (2001) 232 [hep-ph/0012137].

[3] J. Sollfrank, P. Huovinen, M. Kataja, P. V. Ruuskanen, M. Prakash and R. Venugopalan, *Hydrodynamical description of 200-A/GeV/c S + Au collisions: Hadron and electromagnetic spectra*, Phys. Rev. C **55** (1997) 392 [nucl-th/9607029].

[4] R. Santo *et al.* [WA80 Collaboration], *Single photon and neutral meson production from WA80*, Nucl. Phys. A **566** (1994) 61C.

[5] P. Huovinen, *Anisotropy of flow and the order of phase transition in relativistic heavy ion collisions*, Nucl. Phys. A **761** (2005) 296 [nucl-th/0505036].

[6] J. Adams *et al.* [STAR Collaboration], *Azimuthal anisotropy in Au+Au collisions at s(NN)**(1/2) = 200-GeV,* Phys. Rev. C **72** (2005) 014904 [nucl-ex/0409033]; S. S. Adler *et al.* [PHENIX Collaboration], *Elliptic flow of identified hadrons in Au+Au collisions at s(NN)**(1/2) = 200-GeV,* Phys. Rev. Lett. **91** (2003) 182301 [nucl-ex/0305013].

[7] P. Huovinen and P. Petreczky, *QCD Equation of State and Hadron Resonance Gas*, Nucl. Phys. A **837** (2010) 26 [arXiv:0912.2541 [hep-ph]].

[8] U. W. Heinz and R. Snellings, *Collective flow and viscosity in relativistic heavy-ion collisions*, arXiv:1301.2826 [nucl-th].

[9] H. Song and U. W. Heinz, *Interplay of shear and bulk viscosity in generating flow in heavy-ion collisions*, Phys. Rev. C **81** (2010) 024905 [arXiv:0909.1549 [nucl-th]].

[10] D. Teaney, *The Effects of viscosity on spectra, elliptic flow, and HBT radii*, Phys. Rev. C **68** (2003) 034913 [nucl-th/0301099].

[11] B. Alver [PHOBOS Collaboration], *Elliptic flow fluctuations in Au+Au collisions at* $\sqrt{s_{NN}} = 200$ *GeV,* Int. J. Mod. Phys. E **16** (2007) 3331.

[12] M. Luzum and P. Romatschke, *Conformal Relativistic Viscous Hydrodynamics: Applications to RHIC results at s(NN)**(1/2) = 200-GeV,* Phys. Rev. C **78** (2008) 034915 [Erratum-ibid. C **79** (2009) 039903] [arXiv:0804.4015 [nucl-th]].

[13] M. L. Miller, K. Reygers, S. J. Sanders and P. Steinberg, *Glauber modeling in high energy nuclear collisions,* Ann. Rev. Nucl. Part. Sci. **57** (2007) 205 [nucl-ex/0701025].

[14] D. Kharzeev, E. Levin and M. Nardi, *The Onset of classical QCD dynamics in relativistic heavy ion collisions,* Phys. Rev. C **71** (2005) 054903 [hep-ph/0111315]; H. -J. Drescher and Y. Nara, *Effects of fluctuations on the initial eccentricity from the Color Glass Condensate in heavy ion collisions,* Phys. Rev. C **75** (2007) 034905 [nucl-th/0611017]; *Eccentricity fluctuations from the color glass condensate at RHIC and LHC,* **76** (2007) 041903 [arXiv:0707.0249 [nucl-th]].

[15] H. Song, S. A. Bass, U. Heinz, T. Hirano and C. Shen, *200 A GeV Au+Au collisions serve a nearly perfect quark-gluon liquid,* Phys. Rev. Lett. **106** (2011) 192301 [Erratum-ibid. **109** (2012) 139904] [arXiv:1011.2783 [nucl-th]].

[16] B. Alver and G. Roland, *Collision geometry fluctuations and triangular flow in heavy-ion collisions,* Phys. Rev. C **81** (2010) 054905 [Erratum-ibid. C **82** (2010) 039903] [arXiv:1003.0194 [nucl-th]].

[17] H. Holopainen, *Event-by-event hydrodynamics and thermal photon production in ultrarelativistic heavy ion collisions,* Research report 4/2011, Department of Physics, University of Jyväskylä, `http://urn.fi/URN:ISBN:978-951-39-4371-4`

[18] B. Schenke, S. Jeon and C. Gale, *Higher flow harmonics from (3+1)D event-by-event viscous hydrodynamics,* Phys. Rev. C **85** (2012) 024901 [arXiv:1109.6289 [hep-ph]].

[19] Z. Qiu, C. Shen and U. Heinz, *Hydrodynamic elliptic and triangular flow in Pb-Pb collisions at* $\sqrt{s} = 2.76ATeV$*,* Phys. Lett. B **707** (2012) 151 [arXiv:1110.3033 [nucl-th]].

[20] J. Jia [ATLAS Collaboration], *Measurement of the distributions of event-by-event flow harmonics in Pb-Pb Collisions at* $\sqrt{s_{NN}} = 2.76$ *TeV with the ATLAS detector,* arXiv:1209.4232 [nucl-ex].

[21] B. Schenke, P. Tribedy and R. Venugopalan, *Fluctuating Glasma initial conditions and flow in heavy ion collisions,* Phys. Rev. Lett. **108** (2012) 252301 [arXiv:1202.6646 [nucl-th]]; *Event-by-event gluon multiplicity, energy density, and eccentricities in ultrarelativistic heavy-ion collisions,* Phys. Rev. C **86** (2012) 034908 [arXiv:1206.6805 [hep-ph]].

[22] C. Gale, S. Jeon, B. Schenke, P. Tribedy and R. Venugopalan, *Event-by-event anisotropic flow in heavy-ion collisions from combined Yang-Mills and viscous fluid dynamics,* Phys. Rev. Lett. **110** (2013) 012302 [arXiv:1209.6330 [nucl-th]]; *Initial state fluctuations and higher harmonic flow in heavy-ion collisions,* arXiv:1210.5144 [hep-ph].

[23] L. P. Csernai, J. I. Kapusta, L. D. McLerran, *On the Strongly-Interacting Low-Viscosity Matter Created in Relativistic Nuclear Collisions,* Phys. Rev. Lett. **97**, 152303 (2006) [nucl-th/0604032].

[24] H. Niemi, G. S. Denicol, P. Huovinen, E. Molnar and D. H. Rischke, *Influence of a temperature-dependent shear viscosity on the azimuthal asymmetries of transverse momentum spectra in ultrarelativistic heavy-ion collisions,* Phys. Rev. C **86** (2012) 014909 [arXiv:1203.2452 [nucl-th]].

[25] Y. Bai, Ph.D. Thesis, Nikhef and Utrecht University, The Netherlands (2007); A. Tang [STAR Collaboration], *Flow Results and Hints of Incomplete Thermalization,* arXiv:0808.2144 [nucl-ex].

[26] K. Aamodt *et al.* [ALICE Collaboration], *Elliptic flow of charged particles in Pb-Pb collisions at 2.76 TeV,* Phys. Rev. Lett. **105**, 252302 (2011) [arXiv:1011.3914 [nucl-ex]].

[27] H. Niemi, G. S. Denicol, P. Huovinen, E. Molnar and D. H. Rischke, *Effect of temperature-dependent eta/s on flow anisotropies,* Acta Phys. Polon. Supp. **5** (2012) 305 [arXiv:1112.4081 [nucl-th]].