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In-nedium mass reduction of η'-meson

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In this talk, we present the qualitative reduction of the mass of η'-meson which leads to the enhancement of η' contributions to the dilepton spectra from relativistic heavy ion collisions. QCD low energy theorem, together with the Witten-Veneziano formula, provides the relation of the η' mass with temperature-dependent gluon condensates.

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1. Introduction

Classical QCD Lagrangian in the limit of massless quarks is chirally symmetric. It is known, however, that the quantum-mechanical vacuum fluctuation causes the breakdown of the symmetry. As a consequence, SU(3) flavor singlet axial-vector current is no longer conserved, a phenomenal called the U$_A(1)$ anomaly. Then the corresponding Goldstone mode acquires unexpectedly large mass, which is η'-meson of mass near 1 GeV in vacuum. The breaking of the U$_A(1)$ symmetry is an operator relation which remains valid even when the spontaneously broken chiral symmetry is restored. However, it has been a phenomenological question [1, 2, 3, 4, 5] if its effect on the η' mass survives even at the chirally restored phase of QCD. The question has recently revived as the RHIC data on the two pion Bose-Einstein correlation at $\sqrt{s} = 200\,\text{GeV}$ Au+Au colision seems to suggest the quenching of the η' mass in medium [6, 7, 8]. Its partial quenching in nuclear medium is also of great interest as such effects could be probed in finer detail in nuclear target experiments [9, 10].

It has been clarified [3, 4, 5] how the chiral symmetry and topological configurations contribute to the restoration of U$_A(1)$ symmetry. It was shown that in the chiral limit, with N_f flavors, the symmetry will effectively restored in correlation functions composed of up to $N_f - 1$ points [11]. However, the argument is still based on correlation functions and does not explicitly relate the η' mass to the other pseudoscalar masses. To establish this relation, we revisit the Witten-Veneziano (WV) mass relation [12, 13] for the η' mass in vacuum and generalize it to finite temperature. All contents are based on Ref. [14] in which more detailed discussions can be found.

2. Witten-Veneziano formula

We start with reviewing the derivation of the WV mass formula [12, 13].

$$U(k) = i\int \mathrm{d}^4x\, e^{ik\cdot x}\langle \mathscr{T} G\tilde{G}(x)\, G\tilde{G}(0)\rangle. \tag{2.1}$$

One should note that, in the large N_c limit, Eq. (2.1) is of order N_c^2, as can be seen by the two loops representing two gluon lines in Fig. 1-(a). There is also a well known low energy theorem for the correlation function at zero external momentum $U(k=0) \neq 0$. However, when massless quarks are added to the theory, the low energy theorem leads to the vanishing correlation function $U(k=0) = 0$ through the anomaly relation that relates the pseudo-scalar gluon current to axial current. The light quark effects U_q, however, are suppressed in $1/N_c$ compared to the pure gluonic contributions U_g. The only quark effect that survives here comes from η'-meson of which mass squared is assumed to be of order $1/N_c$. Then, to make sense of $U(0) = U_g(0) + U_q(0) = 0$, the gluonic part $U_g(0)$ of Eq. (2.1) can be represented in terms of the η' mass, otherwise suppressed in $1/N_c$:

$$U_g(0) = -\frac{|\langle 0|G\tilde{G}|\eta'\rangle|^2}{m_{\eta'}^2} = \frac{N_c^2}{4N_f} m_{\eta'}^2 f_\pi^2 \left(\frac{4\pi^2}{\alpha_s}\right)^2 \tag{2.2}$$

where we made use of $f_{\eta'} = f_\pi$ to the lowest order in N_c. Eq. (2.2) is the celebrated WV formula.

At finite temperature, the thermal correction to Eq. (2.2) comes from the interactions with thermal gluons or quarks. The contribution from thermal gluons scales as N_c^2 as can be seen in

Fig. 1-(b). On the other hand, for external quark effects, it is of N_c order in Fig. 1-(c). And corresponding mesonic parts (Fig. 1-(d) and (e)) with thermal interactions are suppressed in comparison with the gluonic parts in large N_c limit. Therefore the same argument holds as in the vacuum. Namely, the additions of quark somehow have to cancel the leading N_c behavior at $k = 0$. This cancelation cannot be done by collective states, as quark collective states are also suppressed in $1/N_c$, and hence it has to come from a modified η'-contribution. The the WV formula still holds at finite temperature as in the vacuum. If we are in the confined phase, same arguments suggestes that, at finite density, the effects of nucleons are suppressed at the large N_c limit as the effects of the quarks.

3. Low energy theorem

Now, $U_0(0)$ can be obtained from the low energy theorem. Here we use the derivation using the heavy quark expansion [15].

$$U_g(0) = -\frac{16\pi^2}{9g_s^2}\frac{\mathrm{d}}{\mathrm{d}(-1/4g_s^2)}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle. \tag{3.1}$$

And also by using the renormalizaion group argument, any matrix element with canonical dimension d should be proportional to the d^{th} power of the scale:

$$\langle\mathscr{O}\rangle_{T,\mu} = \text{const}\left[M_0\exp\left(-\frac{8\pi^2}{bg_s^2}\right)\right]^d f(T,\mu), \tag{3.2}$$

where $b = 11 - \frac{2}{3}N_f$ and M_0 is the ultraviolet cutoff [16]. Here an arbitrary function, $f(T,\mu)$, of extra scales has been introduced to generalize this argument to the finite temperature and density case.

By inserting Eq. (3.2) into Eq. (3.1), it becomes as follows:

$$U_g(0) = \frac{2}{9b}\left(\frac{16\pi^2}{g_s^2}\right)^2\left(4 - T\frac{\partial}{\partial T} - \mu\frac{\partial}{\partial\mu}\right)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{T,\mu}. \tag{3.3}$$

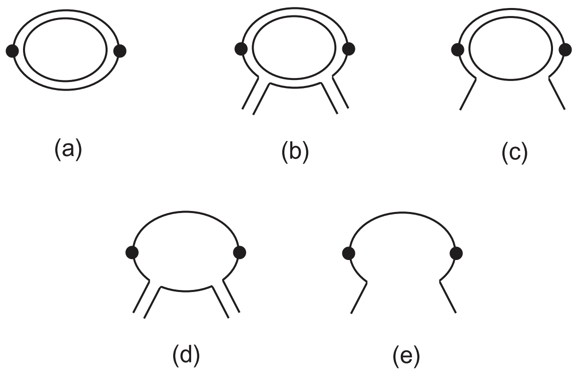

Figure 1: The diagrams of the correlation function in double-line notation: (a) representing two gluon lines in a vacuum, (b) gluonic and (d) mesonic parts of the correlation function interacting with thermal gluons, (c) gluonic and (e) mesonic parts interacting with external quarks.

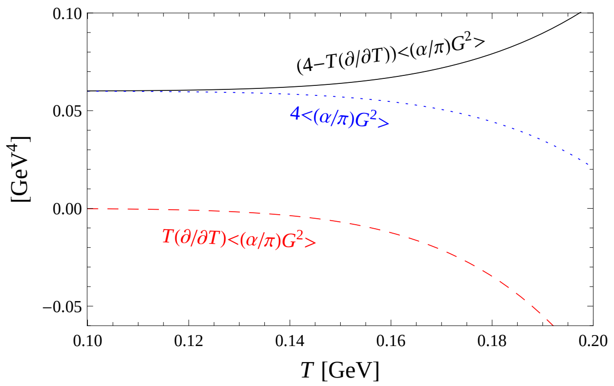

Figure 2: T-dependence of gluon condensate and its derivative fitted to Wuppertal-Budapest lattice data in full QCD.

Finally combining Eq. (2.2) and Eq. (3.3) with $\mu = 0$, one finds

$$\left(\frac{3\alpha_s}{4\pi}\right)^2 \frac{|\langle 0|G\tilde{G}|\eta'\rangle|^2}{m_{\eta'}^2} m_{\eta'}^2 f_\pi^2 = \frac{2}{b}\left(4 - T\frac{\partial}{\partial T}\right)\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_T . \tag{3.4}$$

The effect of subtracting out the second term in Eq. (3.4) is to get rid of the perturbative correction. The leading perturbative correction to the gluon condensate is proportional to $g^4(T)T^4$ [17, 18]. Therefore, assuming that the temperature dependence is of the following form,

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_T = G_0(T) + a g_s^4 T^4, \tag{3.5}$$

we find

$$\left(4 - T\frac{\partial}{\partial T}\right)\left\langle \frac{\alpha}{\pi} G^2 \right\rangle_T = \left(4 - T\frac{\partial}{\partial T}\right) G_0(T), \tag{3.6}$$

if the temperature dependence of g_s is neglected. The only temperature dependence that survives is $G_0(T)$, whose scale dependence is coming from dimensional transmutation and not from the external temperature only. It is the nonperturbative part which dominates the behavior of the right-hand side of Eq. (3.4).

In the dotted line of Fig. 2, we show a resonance gas model result for the gluon condensate [19], which has been fit to reproduce the Wuppertal-Budapest's full QCD data of the trace anomaly [20]: the gluon condensate part is obtained by taking the chiral limit in the resonance gas model, which corresponds to subtracting out the fermion part in the trace anomaly. The dashed and solid lines are obtained by operating on the T-dependent operators as need in Eq. (3.4).

4. η' mass at finite temperature

The final step in obtaining the mass of η', when chiral symmetry is restored, is estimating the change fo the coupling $\langle 0|G\tilde{G}|\eta'\rangle$. For that purpose, let us consider $U(k)$ in Eq. (2.1) in the full theory, but rewrite it in terms of the quark axial current using the anomaly relation.

$$\begin{aligned} U(k) &= i\int \mathrm{d}^4x\, e^{ik\cdot x}\langle \mathscr{T} G\tilde{G}(x)\, G\tilde{G}(0)\rangle \\ &= k^\mu k^\nu i\int \mathrm{d}^4x\, e^{ik\cdot x}\left(\frac{4\pi}{\alpha_s N_f}\right)\Big[\langle \mathscr{T}\bar{q}i\gamma_\mu\gamma_5 q(x)\,\bar{q}i\gamma_\nu\gamma_5 q(0)\rangle - \langle \mathscr{T}\bar{q}\gamma_\mu q(x)\,\bar{q}\gamma_\nu q(0)\rangle\Big], \end{aligned} \tag{4.1}$$

where we have subtracted out the contribution from the conserved vector current. Using the previous terminology, when chiral symmetry is restored, the connected piece will cancel, as they are the same as the difference between flavored chiral partners, and only the disconnected pieces will remain. Assuming that the spectral sum starts from the η', we find Eq. (4.1) can be written as follows:

$$\begin{aligned} U(k) &= -\frac{|\langle 0|G\tilde{G}|\eta'\rangle|^2}{k^2 - m_{\eta'}^2} - \cdots \\ &\propto \int \Big[\mathrm{Tr}[S_A(x,x) i\gamma_\mu \gamma_5] \mathrm{Tr}[S_A(0,0) i\gamma_\nu \gamma_5] - \mathrm{Tr}[S_A(x,x)\gamma_\mu] \mathrm{Tr}[S_A(0,0)\gamma_\nu] \Big]. \end{aligned} \tag{4.2}$$

However, the disconnected pieces are all of the same order in m_q when chiral symmetry is restored.

$$\mathrm{Tr}[S_A(x,x)] \sim \mathrm{Tr}[S_A(x,x)\Gamma] \sim O(m_q), \tag{4.3}$$

where Γ is a Hermitian gamma matrix. This is so because the chiral order parameter can be written as $\langle \bar{q}q \rangle = -\frac{1}{Z} \int d\mu \ \mathrm{Tr}[S_A(0,0] = -\pi\rho(\lambda = 0)$, the density of zero eigenvalues of the Dirac operator in the presence of the gauge field [21]. Therefore, using the spectral representation, we find that

$$\langle 0|G\tilde{G}(x)|\eta'\rangle \sim O(m_q), \tag{4.4}$$

when chiral symmetry is restored. Therefore, going back to Eq. (3.4), we find that, when chiral symmetry is restored,

$$m_{\eta'}^2 = \left(\frac{3\alpha_s}{4\pi}\right)^2 \frac{|\langle 0|G\tilde{G}|\eta'\rangle^2|}{\frac{2}{b}\left(d - T\frac{\partial}{\partial T}\right)\langle \frac{\alpha}{\pi} G^2 \rangle_T} \xrightarrow{\langle \bar{q}q \rangle \to 0} 0. \tag{4.5}$$

Therefore, one can conclude that in QCD large N_c limit, η' mass will become degenerate with the other goldstone bosons.

5. Conclusions

It should be noted that the η' mass that is being quenched is the part of that comes from the breaking of the $\mathrm{U}_A(1)$ part. Going back to Eq. (2.2) and substituting the vacuum value of Eq. (3.3) one find,

$$m_{\eta'} = \sqrt{\frac{8}{33} \frac{1}{f_\pi} \langle \frac{\alpha}{\pi} G^2 \rangle} \approx 464\,\mathrm{MeV}, \tag{5.1}$$

where we have used $f_\pi = 130\,\mathrm{MeV}$ and $\langle \frac{\alpha}{\pi} G^2 \rangle = (0.35\,\mathrm{GeV})^4$. This is smaller than the vacuum value of the η' mass as expected. Assuming that the pseudo scalar mesons do not change their mass towards the phase transition point, it is this extra $\mathrm{U}_A(1)$ mass of η' that is going to quenched in the chiral symmetry restored phase. At this stage, it is hard to make a quantitative estimate on how this mass is partially restored in the nuclear medium. However, from Eq. (4.2) we can to first approximation assume that the $\langle 0|G\tilde{G}|\eta'\rangle \propto \mathrm{Tr}[S_A(x,x)] \propto \langle \bar{q}q \rangle$ and then using Eq. (3.4) deduce $m_{\eta'} \propto \langle \bar{q}q \rangle$. Therefore, if the chiral order parameter reduces by 20% in nuclear medium the $\mathrm{U}_A(1)$ breaking part of the η' mass will also reduce by the same fraction.

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