Center Phase Transition from Fundamentally Charged Matter Propagators

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The center phase transition at non-vanishing temperatures is investigated in Landau gauge Quantum Chromodynamics (QCD) and scalar QCD. For each theory novel order parameters for the transition are introduced. The matter-gluon vertex which occurs in the Dyson-Schwinger equations of the propagators has to be modeled in contemporary studies. It is found that the nature of the phase transition depends strongly on the detailed structure of this vertex. Our investigation motivates a precise determination of the matter-gluon vertex at non-vanishing temperatures.

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1. Introduction

Investigations of the phase structure of strongly-interacting matter have received a considerable amount of attention in the last years, both, theoretically as well as experimentally. Among the most prominent features of the QCD phase diagram is the crossover from a confined phase with spontaneously broken chiral symmetry to a chirally symmetric and deconfined phase. A confined phase can be linked to a ground state that respects center symmetry,[1] and the associated order parameter is the Polyakov loop. It vanishes in the center symmetric phase and becomes finite as soon as the center symmetry is broken [1]. At vanishing quark chemical potential both transitions occur roughly at the same temperature which led to the idea and introduction of new dual observables in lattice QCD [3–6]. In particular, the dual chiral condensate pioneered in [3] is constructed from the chiral condensate, the order parameter of chiral symmetry breaking. More recently, dual observable have also become accessible within functional methods [7–9] and have been successfully applied to investigate the center transition. In the present work novel order parameters for the center symmetry and its breaking are introduced and analyzed in QCD as well as fundamentally charged scalar QCD. The order parameters are determined by the corresponding matter propagators without any additional renormalization.

2. (Scalar) Quantum Chromodynamics

We address the deconfinement transition in ordinary QCD as well as in scalar QCD, where the quarks are replaced by fundamentally charged scalars (see e.g. [10]) both in Landau gauge.

The matter propagators are calculated by means of the corresponding Dyson-Schwinger equation (DSE) [11]. As an example, the DSE for the quark propagator is shown diagrammatically in Fig. 1, where thin lines and dots represent bare propagators and one-particle irreducible vertices while thick lines and dots denote the corresponding dressed quantities. Accordingly, the DSE for the scalar propagator, which is shown in Fig. 2, is more involved and contains more diagrams due to the presence of additional bare vertices such as the scalar self-interaction and the quartic scalar-gluon vertices. In one-loop approximation only the momentum-independent tadpole diagrams are left in addition to the gluon exchange diagram. The tadpoles, however, can be treated by adjusting the renormalization constants appropriately. Hence, in a one-loop approximation, the DSE for the scalar propagator is of the same structure as the one for the quark propagator, cf. Fig. 1.

Explicitly, at finite temperature T the DSE for the quark propagator $S(p)$ reads

$$S^{-1}(p) = Z_2 S_0^{-1}(p) - Z_{1F} C_F g^2 T \sum_{\omega_k(\theta)} \int \frac{d^3k}{(2\pi)^3} \gamma^\mu S(k) \Gamma^\nu(k,p;q) D^{\mu\nu}(q) \tag{2.1}$$

and correspondingly for the scalar propagator $D_S(p)$

$$D_S^{-1}(p) = \hat{Z}_2(p^2 + \hat{Z}_m m_0^2) - \hat{Z}_{1F} C_F g^2 T \sum_{\omega_k(\theta)} \int \frac{d^3k}{(2\pi)^3} (p+k)^\mu D_S(k) \Gamma_S^\nu(k,p;q) D^{\mu\nu}(q) \,. \tag{2.2}$$

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[1]In a strict sense center symmetry is realized only in the limit of infinitely heavy quarks while in real QCD the symmetry is always explicitly broken, see e.g. [1, 2].

Figure 1: DSE for the quark propagator.

Figure 2: DSE for the fundamentally charged scalar propagator.

For the four-momenta we use $k = (\vec{k}, \omega_k(\theta))$ and the gluon momentum is constrained by momentum conservation to $q = p - k$. The wave function renormalization of the quark (scalar) fields are denoted by Z_2 ($\hat{Z}_2$), the renormalization constants of the quark-gluon (scalar-gluon) vertex by Z_{1F} ($\hat{Z}_{1F}$) and Z_m labels the scalar mass renormalization constant. The quadratic Casimir invariant in the fundamental representation of the gauge group $SU(3)$ is $C_F = 4/3$ and the coupling constant at the renormalization scale is given by g.

In general, $\exp(i\theta)$-valued boundary conditions in the fourth spacetime direction are realized by introducing generalized Matsubara frequencies $\omega_k(\theta) = (2\pi n_k + \theta)T$, where the sums over $\omega_k(\theta)$ in the DSEs run over the corresponding discrete values $n_k \in \mathbb{Z}$. The usual (anti-)periodic boundary conditions for (fermions) bosons are obtained by setting ($\theta = \pi$) $\theta = 0$, respectively.

Both DSEs depend on the gluon propagator $D^{\mu\nu}$ as well as on the corresponding matter-gluon vertices Γ^ν and Γ_S^ν. Solutions of the DSE for the Landau gauge gluon propagator at non-vanishing temperatures have been obtained in [12]. In addition, data are available from (quenched) lattice simulations at finite temperatures and have already been successfully implemented in functional equations for the quark propagator [8, 13][2]. In this work we will apply the fit functions for the gluon propagator proposed in [13] and hence we omit explicit expressions.

For the matter-gluon vertices the situation is less satisfactory since the temperature behavior of these vertices is not known generally. Some modeling has to be employed such as in [8] where the following expression

$$\Gamma^\nu(k,p;q) = \tilde{Z}_3 \left(\delta^{4\nu}\gamma^4 \frac{C(k)+C(p)}{2} + \delta^{j\nu}\gamma^j \frac{A(k)+A(p)}{2} \right)$$
$$\times \left\{ \frac{d_1}{d_2+q^2} + \frac{q^2}{q^2+\Lambda^2} \left(\frac{\beta_0\alpha(\mu)\ln\left[q^2/\Lambda^2+1\right]}{4\pi} \right)^{2\delta} \right\} \tag{2.3}$$

for the quark-gluon vertex can be found and will be used also in this work. The Ansatz is motivated by Slavnov-Taylor identities of the Abelian gauge theory (see e.g. [15]) and by the running of the non-perturbative coupling of the Yang-Mills theory. The purely phenomenological parameters d_1

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[2]Recently, also unquenched lattice data for the Landau gauge gluon propagator are available [14], which will be used in future studies of the system.
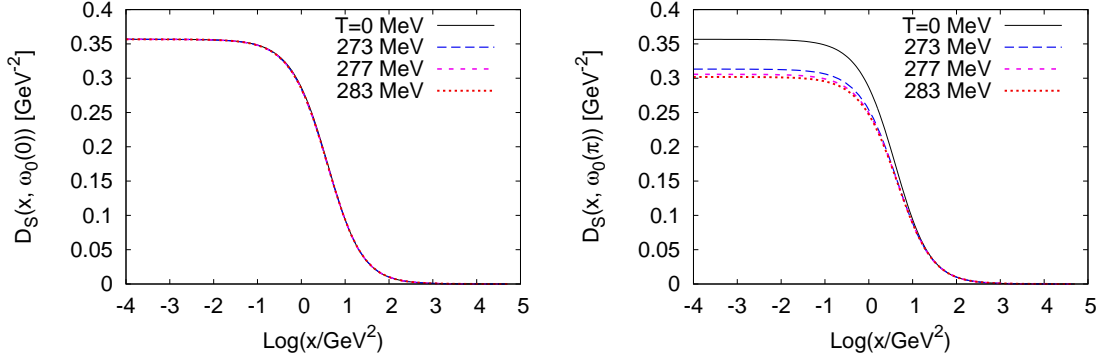
Figure 3: Solution of the truncated DSE Eq. (2.2) for the scalar propagator as function of $\vec{p}^2 = x$ at the lowest Matsubara frequency for periodic $\theta = 0$ (left panel) and antiperiodic $\theta = \pi$ (right panel) boundary conditions at different temperatures with renormalization scale $\mu = 4$ GeV and mass $m = 1.5$ GeV.

and d_2 are specified in [13], whereas the anomalous dimension $2\delta = -18/44$ and $\beta_0 = 11N_c/3$ ensure a correct perturbative running coupling in the UV regime for $SU(N_c)$ gauge theory. The renormalization scale of the Yang-Mills sector has been fixed by $\alpha(\mu) = 0.3$ and $\Lambda = 1.4$ GeV. The factor $\tilde{Z}_3$ allows to apply the Slavnov-Taylor identity $Z_{1F} = Z_2/\tilde{Z}_3$ in Landau gauge [16] which yields finally only a dependence on the quark wave function renormalization Z_2. The gluon momentum is denoted by q and k and p are the in- and outgoing quark momenta, respectively.

In a similar context we employ

$$\Gamma_S^\nu(k,p;q) = \tilde{Z}_3 \frac{D_S^{-1}(p^2) - D_S^{-1}(k^2)}{p^2 - k^2}(p+k)^\nu$$
$$\times d_1 \left\{ \frac{\Lambda^2}{\Lambda^2 + q^2} + \frac{q^2}{q^2 + \Lambda^2} \left(\frac{\beta_0 \alpha(\mu) \ln\left[q^2/\Lambda^2 + 1\right]}{4\pi} \right)^{2\delta} \right\} \quad (2.4)$$

for the scalar-gluon vertex, where one additional parameter $d_1 = 0.53$ has been introduced and all remaining parameters are the same as in the quark-gluon vertex. In contrast to the quark-gluon vertex, the scalar-gluon vertex is dressed only with the vacuum propagators $D_S^{-1}(p^2)$. For the numerical solution of the corresponding DSEs we rewrite the propagators $S^{-1}(p) = i\gamma_4 \omega_p(\theta)C(p) + i\vec{p}A(p) + B(p)$ and $D_S(p) = Z_S(\vec{p}^2, \omega_p(\theta))/(\vec{p}^2 + \omega_p(\theta)^2)$ in terms of dressing functions in a standard way. Details on the numerical implementation as well as on the renormalization scheme will be published elsewhere [17], cf. also [18, 19].

Numerical results for the scalar propagator around temperatures of the center transition in the quenched theory with $T_c \approx 277$ MeV are shown in Fig. 3 for periodic (left panel) as well as antiperiodic boundary conditions (right panel). The mass of the scalars has been fixed to $m = 1.5$ GeV which results in a quite inert behavior around T_c. These results demonstrate that there are no direct modifications of the scalar propagator in the vicinity of the transition. This motivates the construction of more sensitive order parameters for the center phase transition.

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3. Center Phase Transition and Dual Order Parameters

Order parameters for the center phase transition can be composed with functional methods via dual observables like e.g. the dual chiral condensate [7, 8] or the dual density [9]. As has been discussed in the latter reference, dual quantities can be evaluated in two different ways. In general, dual order parameters are constructed from some boundary-condition dependent operator $\hat{O}_\theta$, where θ denotes the phase of the $U(1)$-valued boundary conditions. Originally, such operators were introduced in lattice calculations [3–6] and evaluated in QCD with the original boundary conditions, i.e., with (anti-)periodic boundary conditions for (fermions) bosons.

On the other hand, these operators can also be evaluated with functional methods in various theories with general boundary conditions, referred to as QCD_θ in [9]. However, dual observables evaluated in this way can serve as order parameters only if the deconfinement transition temperature at physical boundary conditions is a lower bound for the transition temperatures in the different theories QCD_θ [9].

In previous studies of the center phase transition the dual chiral condensate has been calculated with functional methods based on the quark propagator [7, 8, 13]. It is given by an expansion in complex Fourier modes

$$\Sigma_n = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{-in\theta} \langle \bar{\psi}\psi \rangle_\theta \tag{3.1}$$

with a θ-dependent quark condensate

$$\langle \bar{\psi}\psi \rangle_\theta = Z_2 N_c T \sum_{\omega_p(\theta)} \int \frac{d^3 p}{(2\pi)^3} tr_D S(\vec{p}, \omega_p(\theta)) \,. \tag{3.2}$$

For arbitrary n that is not a multiple of N_c, Σ_n can then serve as an order parameter for center symmetry. Usually, the dual chiral condensate Σ_1 is used, also called dressed Polyakov loop. In a lattice formulation it contains contributions from all time-like loops around the torus with winding number $n = 1$ [3, 4, 6] and transforms similar to the ordinary Polyakov loop [1] under center transformations.

Hence, the calculation of the dual chiral condensate requires the chiral condensate with general boundary conditions, which has to be regularized for non-vanishing quark masses. As an already finite alternative we propose

$$\Sigma_Q = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{-i\theta} \Sigma_{Q,\theta} \,, \quad \Sigma_{Q,\theta} = T \sum_{\omega_p(\theta)} \left[\frac{1}{4} tr_D S(\vec{0}, \omega_p(\theta)) \right]^2 \tag{3.3}$$

as an order parameter for the center phase transition. $\Sigma_{Q,\theta}$ is finite, because the sum scales like $1/\omega_p^4$ for large Matsubara modes. Similarly, for scalar QCD we propose the order parameter

$$\Sigma_S = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{-i\theta} \Sigma_{S,\theta} \,, \qquad \Sigma_{S,\theta} = T \sum_{\omega_p(\theta)} D_S^2(\vec{0}, \omega_p(\theta)) \,. \tag{3.4}$$

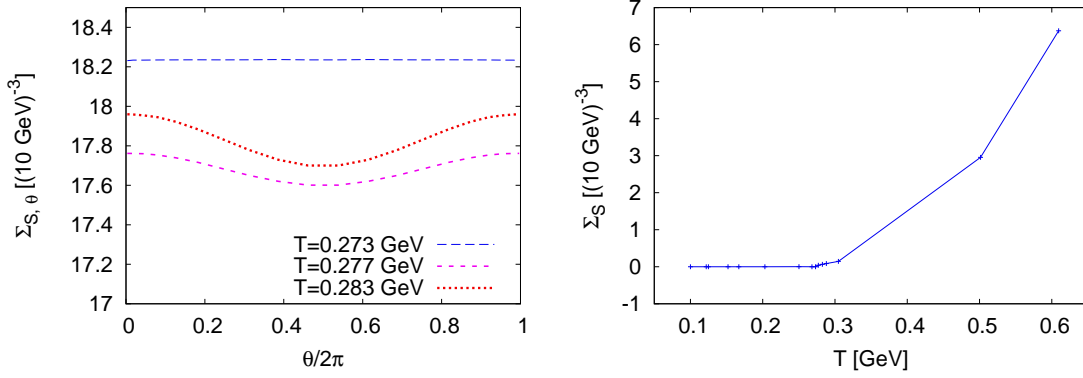More details on these order parameters will be presented in an upcoming publication [17].

Figure 4: Left: $\Sigma_{S,\theta}$ of Eq. (3.4) as a function of the boundary conditions for temperatures around the transition. Right: The dual condensate Σ_S as a function of the temperature ($\mu = 4$ GeV and $m = 1.5$ GeV).
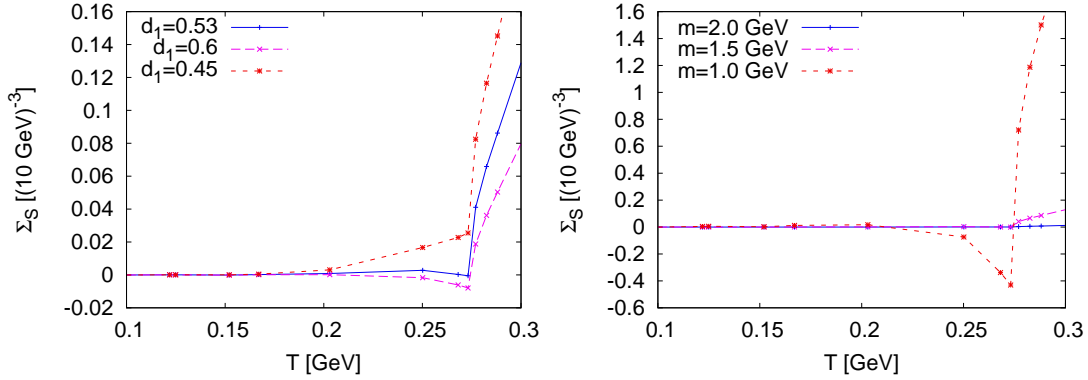

Figure 5: Parameter dependency of the dual condensate Σ_S as a function of temperature for three different values of the vertex parameter d_1 (left) and for three different values of the mass of the scalar field (right); ($\mu = 4$ GeV and $m = 1.5$ GeV).

4. Results

In order to confirm that both quantities, Σ_Q in Eq. (3.3) and Σ_S in Eq. (3.4), are well-defined order parameters we investigate both theories, QCD and scalar QCD, at finite temperatures.

In the left panel of Fig. 4 the θ-dependence of $\Sigma_{S,\theta}$ for the quenched scalar theory is shown for temperatures around the transition. From the definition Eq. (3.4) it is clear that Σ_S vanishes as long as the $\Sigma_{S,\theta}$ is constant while a θ-dependency is necessary for a non-vanishing order parameter. However, these findings for $\Sigma_{S,\theta}$ are qualitatively similar to QCD with finite quark masses [3]. In the right panel of Fig. 4 the dual condensate Σ_S is shown as a function of the temperature which nicely demonstrates its property as an order parameter for the center symmetry. Below the transition temperature of the quenched theory around $T_c \approx 277$ MeV it vanishes and is finite at higher temperatures. In the vicinity of the critical temperature the temperature behavior of the order parameter depends crucially on the model parameters used for the scalar-gluon vertex. This is demonstrated in Fig. 5, where the temperature behavior of Σ_S is shown for different values of the d_1 parameter in the scalar-gluon vertex Eq. (2.4) (left panel) and for various values of the mass of the scalar field (right panel). Stronger deviations from a vanishing order parameter slightly below
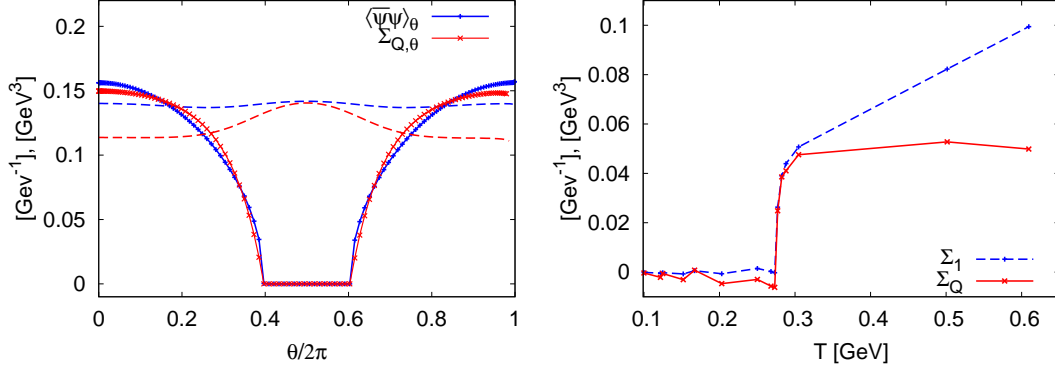
Figure 6: Left panel: The quark condensates $\langle\bar{\psi}\psi\rangle_\theta$ and $\Sigma_{Q,\theta}$, as defined in Eq. (3.3), as a function of the boundary angle for different temperatures in the chiral limit (dashed lines: $T = 273$ MeV, solid lines: $T = 283$ MeV). Right panel: The order parameters Σ_Q and Σ_1 as defined in Eq. (3.1) and (3.3) as a function of the temperature.

the transition temperature are observed for smaller mass. This indicates that the vertex sensitivity increases towards smaller masses.

Finally, the proposed new order parameter for ordinary QCD shows a similar behavior which is presented in the left panel of Fig. 6. In the figure the θ-dependency of $\Sigma_{Q,\theta}$ in comparison to the chiral condensate is plotted as a function of the generalized boundary conditions for two different temperatures. The results have been obtained in the chiral limit of the quenched theory. Above the transition temperature, $\Sigma_{Q,\theta}$ shows the same characteristic plateau as the chiral condensate for θ's close to the physical antiperiodic boundary conditions $\theta = \pi$. Due to the restoration of chiral symmetry the condensate vanishes which also affects $\Sigma_{Q,\theta}$ through the scalar dressing function $B(p)$ in the quark propagator. The slight variations of Σ_Q in the center symmetric phase below T_c results from a non-constant $\Sigma_{Q,\theta}$ and can be attributed to lattice artifacts as well as the choice of the quark-gluon vertex model. However, this effect is more pronounced in $\Sigma_{Q,\theta}$ than in the chiral condensate. The right panel of Fig. 6 shows the corresponding order parameter Σ_Q in comparison to the dual chiral condensate. Both vanish below T_c and jump immediately to non-vanishing values above T_c. Their deviation at larger temperatures can be assigned to different dimensionalities.

5. Conclusions

We investigated the center phase transition of QCD as well as fundamentally charged scalar QCD in a quenched formulation. Novel order parameters are proposed along the lines of previously constructed dual observables accessible by functional methods. Solving the Dyson-Schwinger equations for the corresponding matter propagators numerical results for these order parameters are presented. A parameter dependency of the employed matter-gluon vertices on the presented results is found and motivates a more detailed investigation of its temperature dependence (see also [20] for corresponding investigations at vanishing temperature).

Acknowledgements

References

[1] A. M. Polyakov, Phys. Lett. B **72** (1978) 477; L. Susskind, Phys. Rev. D **20** (1979) 2610.

[2] F. Karsch and E. Laermann, In *Hwa, R.C. (ed.) et al.: Quark gluon plasma* 1-59 [hep-lat/0305025]; Y. Aoki, Z. Fodor, S. D. Katz and K. K. Szabo, Phys. Lett. B **643** (2006) 46 [hep-lat/0609068].

[3] C. Gattringer, Phys. Rev. Lett. **97** (2006) 032003 [hep-lat/0605018]; E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, Phys. Rev. D **77** (2008) 094007 [arXiv:0801.4051 [hep-lat]].

[4] F. Bruckmann, C. Gattringer and C. Hagen, Phys. Lett. B **647** (2007) 56 [hep-lat/0612020].

[5] E. Bilgici, F. Bruckmann, J. Danzer, C. Gattringer, C. Hagen, E. M. Ilgenfritz and A. Maas, Few Body Syst. **47** (2010) 125 [arXiv:0906.3957 [hep-lat]]; B. Zhang, F. Bruckmann, C. Gattringer, Z. Fodor and K. K. Szabo, AIP Conf. Proc. **1343** (2011) 170 [arXiv:1012.2314 [hep-lat]].

[6] F. Synatschke, A. Wipf and C. Wozar, Phys. Rev. D **75** (2007) 114003 [hep-lat/0703018 [HEP-LAT]]; F. Synatschke, A. Wipf and K. Langfeld, Phys. Rev. D **77** (2008) 114018 [arXiv:0803.0271 [hep-lat]].

[7] C. S. Fischer, Phys. Rev. Lett. **103** (2009) 052003 [arXiv:0904.2700 [hep-ph]].

[8] C. S. Fischer and J. A. Mueller, Phys. Rev. D **80** (2009) 074029 [arXiv:0908.0007 [hep-ph]].

[9] J. Braun, L. M. Haas, F. Marhauser and J. M. Pawlowski, Phys. Rev. Lett. **106** (2011) 022002 [arXiv:0908.0008 [hep-ph]].

[10] L. Fister, R. Alkofer and K. Schwenzer, Phys. Lett. B **688** (2010) 237 [arXiv:1003.1668 [hep-th]].

[11] R. Alkofer and L. von Smekal, Phys. Rept. **353** (2001) 281 [arXiv:hep-ph/0007355]; P. Maris and C.D. Roberts, Int. J.Mod. Phys. E**12** (2003) 297 [arXiv:nucl-th/0301049]; C. S. Fischer, J. Phys. G **32** (2006) R253 [hep-ph/0605173].

[12] A. Maas, J. Wambach, B. Gruter and R. Alkofer, Eur. Phys. J. C **37** (2004) 335 [hep-ph/0408074]. Eur. Phys. J. C **42** (2005) 93 [hep-ph/0504019].

[13] C. S. Fischer, A. Maas and J. A. Mueller, Eur. Phys. J. C **68** (2010) 165 [arXiv:1003.1960 [hep-ph]].

[14] R. Aouane, F. Burger, E. -M. Ilgenfritz, M. Muller-Preussker and A. Sternbeck, arXiv:1212.1102 [hep-lat].

[15] J. S. Ball and T. -W. Chiu, Phys. Rev. D **22** (1980) 2542.

[16] J. C. Taylor, Nucl. Phys. B **33** (1971) 436.

[17] M. Mitter, M. Hopfer, B. -J. Schaefer and R. Alkofer, in preparation (2013).

[18] M. Hopfer, M. Mitter, B. -J. Schaefer and R. Alkofer, arXiv:1211.0166 [hep-ph].

[19] M. Q. Huber and M. Mitter, Comput. Phys. Commun. **183** (2012) 2441 [arXiv:1112.5622 [hep-th]].

[20] A. Windisch, M. Hopfer and R. Alkofer, arXiv:1210.8428 [hep-ph]; M. Hopfer, A. Windisch and R. Alkofer, PoS CONFINEMENTX (2013) 073.