

# Non-perturbative QCD effects in $\eta_c$ and $\eta_b$ decays into baryons and WIMP scattering off nuclei

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In this work we estimate the helicity suppressed decay rates of  $\eta_b$  resonances into baryon pairs due to instanton-induced effects by rescaling the corresponding partial widths of the experimentally measured branching ratios for the  $\eta_c(1S) \rightarrow p\bar{p}$  and  $\eta_c(1S) \rightarrow \Lambda\bar{\Lambda}$  decay modes. Thus we point out that both  $\eta_b(1S) \rightarrow p\bar{p}$  and  $\eta_b(1S) \rightarrow \Lambda\bar{\Lambda}$  channels could be detected at a Super B factory and LHC experiments. Furthermore, we examine related instanton-induced effects on WIMP scattering off nuclei concluding, albeit with large uncertainties, that they might enhance the spin-dependent cross section for a light pseudoscalar Higgs mediator, thereby inducing a dependence on the momentum transfer to the recoiling nucleus.

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## 1. Introduction

As is well-known, non-abelian gauge theories show non-trivial properties associated to a tunneling process between different classical field configurations. In particular instanton effects can be related to the  $U(1)$ -problem,  $\theta$ -problem and chiral symmetry breaking in QCD, leading to many phenomenological implications in hadron physics. For general reviews see, e.g., [1], [2], [3].

In this work we first examine the decays of  $\eta_c$  pseudoscalar resonances (later extending our analysis to  $\eta_b$  states) into baryon pairs which pose a long-standing puzzle to perturbative QCD, as commented below.

Let us get started by remarking that instantons can be interpreted in either way:

- a) Instantons are large fluctuations of the gluon field corresponding to a tunneling process occurring in time whose amplitude is proportional to  $\exp(-2\pi/\alpha_s)$ , where  $\alpha_s$  denotes the strong coupling constant. It is apparent that such effects are non perturbative and instantons are missed in all orders of perturbation theory.
- b) Instantons are localized pseudoparticles in Euclidean space-time that can induce interactions between quarks and gluons.

The physical QCD vacuum should resemble a kind of “liquid”, provided that the typical instanton size  $\bar{\rho} \simeq 1/600 \text{ MeV}^{-1} = 1/3 \text{ fm}$  is smaller than the instanton separation  $R \approx 1 \text{ fm}$ .

### 1.1 Motivating ideas

It has been found experimentally that the decay rate of the  $\eta_c$  resonance into baryon pairs is much larger than expected according to perturbative QCD, where such processes should be largely suppressed by helicity conservation.

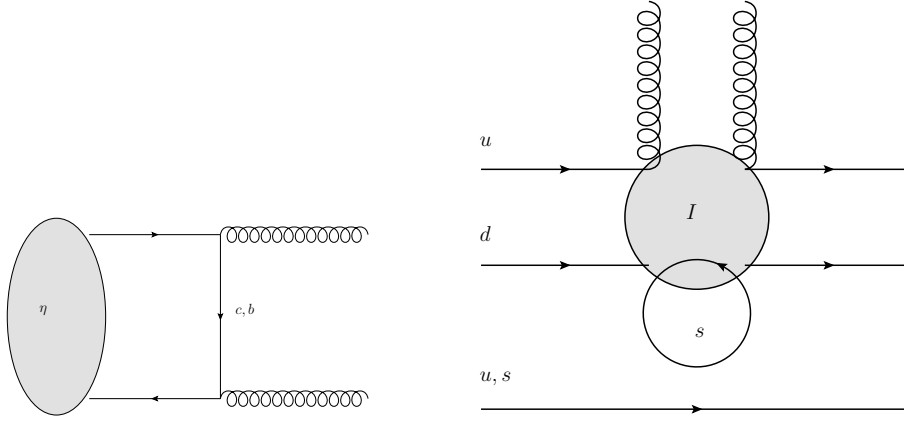
In particular, an explanation of the large observed decay rates of  $\eta_c(1S)$  resonance to baryon pairs [4]:

- $BF[\eta_c(1S) \rightarrow p\bar{p}] = (1.43 \pm 0.17) \times 10^{-3}$
- $BF[\eta_c(1S) \rightarrow \Lambda\bar{\Lambda}] = (0.94 \pm 0.32) \times 10^{-3}$

seems to require a fundamental modification of the perturbative approach.

Different proposals have been put forward in terms of a non-leading or non-perturbative mechanism: mixing of the resonance and gluonium states [6], instanton effects [7], intermediate meson loop contribution [9] or higher Fock components of the hadronic resonance [10]. Despite many uncertainties, it is conceivable that long-distance contributions may also affect  $\eta_b(nS)$  resonances. In Fig. 1 we show the possible instanton interaction via a couple of gluons perturbatively coupled to the  $\eta_{c,b}$  meson (left) and non-perturbatively coupled to a baryon pair (right) via instanton interactions.

On the other hand, one can wonder whether a non-perturbative contribution may have an influence in the scattering of dark matter (e.g. WIMP) off nuclei, when the mediator couples to nucleons via a triangle-loop diagram with two gluons as depicted in Fig.2.



**Figure 1:** Left: Perturbative coupling of two gluons to a pseudoscalar resonance  $\eta_{c,b}$ . Right: Non-perturbative coupling of a couple of gluons to a  $p\bar{\rho}$  or to  $\Lambda\bar{\Lambda}$  pairs through an instanton-induced interaction.

## 2. Instanton size distribution

The analogy with a liquid pointed out in the Introduction allows one to introduce an instanton density with a size distribution which can be calculated from QCD under some approximations and regularization scheme.

At the one-loop approximation the instanton size distribution is given by [11]:

$$n(\rho) \sim \frac{1}{\rho^5} [\alpha_s(\rho^{-1})]^{-6} \exp \left[ -\frac{2\pi}{\alpha_s(\rho^{-1})} \right] \quad (2.1)$$

where

$$\alpha_s(\rho^{-1}) = \frac{g_s^2(\rho^{-1})}{4\pi} \simeq \frac{2\pi}{b \log(1/\Lambda\rho)} \quad (2.2)$$

with  $b = 11 - 2N_f/3$ , where  $N_f$  denotes the the number of active flavors;  $\Lambda$  is of the order of 250 MeV. Let us remark that  $\alpha_s(\mu)$  in the pre-exponent factor of Eq.(2.1) starts to run only at the two-loop approximation; hence its argument is set at the ultraviolet cutoff  $\mu$ .

Note that at small  $\rho$  and  $N_f = 3$ , the size distribution is given approximately by:

$$n(\rho) \sim \rho^{b-5} = \rho^4 \quad (2.3)$$

i.e. it increases with increasing instanton size. Thus the integral of  $n(\rho)$  over  $\rho$  diverges, representing an infrared problem. Actually, such infrared divergence can be seen as an artifact of using the one-loop formula for  $\alpha_s(1/\rho)$  in Eq.(2.2). Moreover, if the semiclassical approximation is meaningful at all, a solution to this problem has to be found in the context of the full instanton ensemble. The simplest way is to cut the integrations off at some given value  $\rho_0$ . A dynamical cut-off could originate from configurations where instantons start to overlap, undergoing a repulsive interaction.

On the other hand, there are arguments in favor of a suppression of the type  $n(\rho) \sim \exp(-c\rho^2)$ , which is physically equivalent to set a cutoff  $\rho_0$  for large instanton sizes. We will turn to this point in section 4.

## 2.1 The case of heavy quarkonium decays into baryons

In the following we make the hypothesis that heavy quarkonium (charmonium and bottomonium) mainly couples to instantons of size  $\rho_{c,b} \approx (m_{c,b} v_{cb})^{-1}$ , where  $v_{c,b}$  denote the heavy quark velocities, respectively. In particular we consider  $\eta_{c,b}$  states with the simplifying assumption

$$n(\rho) = n_{c,b} \delta(\rho - \rho_{c,b}) \quad (2.4)$$

where  $n_{c,b}$  is assumed to follow the same scaling as Eq.(2.3). (As we shall see, heavy quarks in the quarkonium system do not play a role in the instanton-gluon interaction, although they couple perturbatively to the gluon pair for pseudoscalar states.). Therefore

$$r_c = \frac{n_b}{n_c} \simeq \left[ \frac{\rho_b}{\rho_c} \right]^4 \simeq 0.1 \quad (2.5)$$

## 2.2 Effective instanton induced gluon-quark vertex

The relevant Lagrangian can be written as

$$\mathcal{L} = \int d\rho n(\rho) \left[ \text{t'Hooft}(\gamma_5) + \text{two-gluon interaction} \right] \quad (2.6)$$

with the following rules in our case:

- i) Each quark in zero mode in instanton field gives a factor  $\rho^3$
- ii) Each closed quark loop gives a factor  $m_{current} \rho$
- iii) Each gluon coupling to the instanton field gives a factor  $\rho^2/g_s(\rho^{-1})$

According to the previous rules, the lowest order diagram contributing to the  $\eta_b$  decay into a baryon pair is shown in Fig.1. A caveat is in order however: Although diagrams with extra light quark loops are suppressed because of small masses of *up* and *down* quarks, such diagrams would contribute with lower  $\rho$  powers; hence the suppression might be somewhat balanced at the end. Nonetheless, in the following we focus on diagram of Fig.1 as the leading contribution to the effective coupling of gluons to baryons.

The amplitude of the diagram depicted in Fig. 1 (right) in the case of the pseudoscalar quantum number for two gluon state should scale with the instanton size  $\rho$  according to (see also [8]):

$$(\rho^3)^2 \times (m_s \rho) \times (\bar{u}u \bar{d}d \gamma_5 + \bar{d}d \bar{u}u \gamma_5) \times \left[ \frac{\rho^2}{g_s(\rho^{-1})} \right]^2 G_{\mu\nu} \tilde{G}_{\mu\nu} \quad (2.7)$$

which represents that the interaction amplitude should be  $\sim \rho^{11}/\alpha_s(\rho^{-1})$ , to be convoluted with the instanton density  $n(\rho)$ , leading to

$$\int d\rho n(\rho) \rho^{11}/\alpha_s(\rho^{-1}) \sim \rho_{c,b}^{15}/\alpha_s(\rho_{c,b}^{-1}) \quad (2.8)$$

in the approximation given in Eq.(2.4).

### 3. Numerical estimates for $\eta_b \rightarrow p\bar{p}$ and $\eta_c \rightarrow \Lambda\bar{\Lambda}$ partial widths

We shall rescale the decay rate of the  $\eta_c(1S)$  into a baryon pair  $B\bar{B}$  taking into account both kinematical and dynamical factors according to the ansatz:

$$\frac{\Gamma[\eta_b \rightarrow B\bar{B}]}{\Gamma[\eta_c \rightarrow B\bar{B}]} \sim \frac{K[\eta_b]}{K[\eta_c]} \times r_c \times r_\rho \quad (3.1)$$

where

$$\frac{K[\eta_b]}{K[\eta_c]} = \frac{m_{\eta_b}^4}{m_{\eta_c}^4} \times \frac{\sqrt{1 - 4m_p^2/m_{\eta_b}^2}}{\sqrt{1 - 4m_p^2/m_{\eta_c}^2}} \times \left[ \frac{R_{\eta_b(1S)}(0)}{R_{\eta_c(1S)}(0)} \right]^2 \sim 10^3 \quad (3.2)$$

$$r_c = \left[ \frac{n_b}{n_c} \right]^2 \simeq \left[ \frac{\rho_b}{\rho_c} \right]^8 \sim 10^{-2}; \quad r_\rho = \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^2 \times \left[ \frac{\alpha_s(\rho_c^{-1})}{\alpha_s(\rho_b^{-1})} \right]^2 \times \left[ \frac{\rho_b}{\rho_c} \right]^{22} \sim 10^{-6} - 10^{-5} \quad (3.3)$$

Setting numerical values:  $|R_{\eta_b(1S)}(0)/R_{\eta_c(1S)}(0)|^2 \simeq 6.477/0.81 \simeq 8$  [12],  $BF[\eta_c \rightarrow p\bar{p}] = (1.41 \pm 0.17) \times 10^{-3}$ , and  $BF[\eta_c \rightarrow \Lambda\bar{\Lambda}] = (9.4 \pm 3.2) \times 10^{-4}$  [4], we are led to:

$$\frac{\Gamma[\eta_b \rightarrow B\bar{B}]}{\Gamma[\eta_c \rightarrow B\bar{B}]} \sim 10^{-5} - 10^{-4} \rightarrow \Gamma[\eta_b(1S) \rightarrow B\bar{B}] \approx \mathcal{O}(1 - 10) \text{ eV} \quad (3.4)$$

Next, making use of the total width of the  $\eta_b(1S)$  resonance, found to be  $\Gamma[\eta_b(1S)] \simeq 10$  MeV by Belle [13], we are able to provide the following order-of-magnitude predictions:

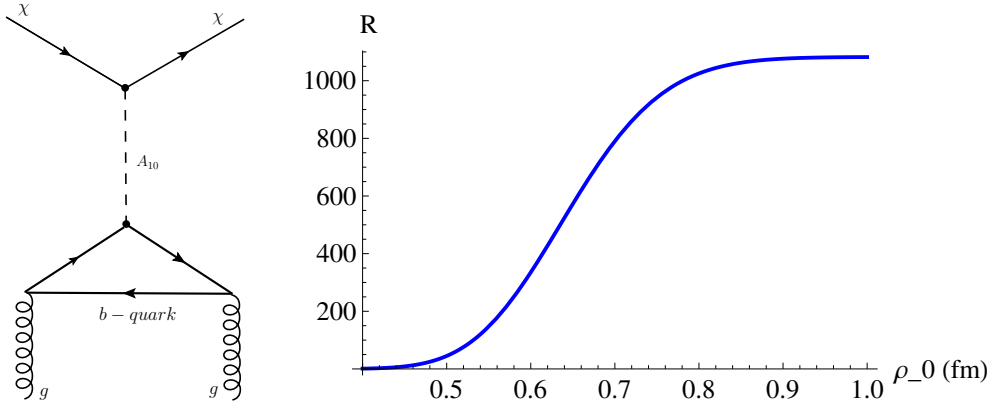
$$BF[\eta_b(1S) \rightarrow p\bar{p}] \simeq BF[\eta_b(1S) \rightarrow \Lambda\bar{\Lambda}] \sim 10^{-7} - 10^{-6} \quad (3.5)$$

Such values of the branching fractions imply that  $\eta_b(1S)$  decays to baryon pairs should be within reach of LHC experiments [14] and a Super B Factory.

### 4. Extrapolation to small transfer momentum in WIMP scattering off nuclei

Direct detection of dark matter through WIMP scattering off nuclei has become one of the hottest (and controversial) points in physics today. Indeed, some experiments claim to have already found evidence of it whereas others fail to detect any signal at all. On the one hand, DAMA/LIBRA, CoGeNT, and more recently CRESST experiments have reported the observation of events in excess of the expected background, hinting at the existence of a light WIMP [15, 16, 17]. On the other hand, exclusion limits set by other direct searches, such as Xenon10 [18] and Xenon100 [19], are in tension with the above claims.

The total WIMP-nucleus cross section has contributions from both spin-independent and spin-dependent interactions, though one contribution is expected to dominate the other depending on the target nucleus (e.g. according to the even/odd number of protons and neutrons) and the detection technique employed in the experiment. The contributions to the spin-independent cross section arise in the interaction Lagrangian of the WIMP with quarks and gluons of the nucleon from scalar and vector couplings whereas the spin-dependent part is attributed to the axial-vector couplings. Pseudoscalar interaction is usually neglected because of a strong velocity and/or momentum transfer suppression.



**Figure 2:** Left: Contribution of a light CP-odd Higgs boson to the spin-dependent cross section of WIMPs (e.g. a light neutralino  $\chi$ ) off nuclei via two gluons whose coupling to nucleons would be enhanced by instanton-induced effects as explained in the main text. Right: The ratio  $R$  defined in Eq.(4.3) as a function of the upper limit of the instanton size  $\rho_0$  (in fm). The curve is only indicative of a possible large enhancement and numerical values from the curve should be taken with great care.

In fact, direct detection through WIMP scattering off nuclei typically implies momentum transfer  $q$  of the order of 100 MeV, i.e. nuclear target recoil energies of the order of tens of keV.

Nevertheless, momentum-dependent interactions have been put forward [20, 21, 22] in order to alleviate the tension between the DAMA signal and the null results from other experiments. Specifically we focus on a scenario beyond the Standard Model, with a light pseudoscalar Higgs boson  $A_1$  acting as a mediator, as shown in Fig.2 (left). A detailed discussion of the motivations for such a scenario (in particular, a light neutralino  $\chi$  as a dark matter candidate, and a light CP-odd Higgs boson according to the NMSSM) can be found in [23, 24] and references therein.

At large  $\tan\beta$ , the bottom quark running in the triangle-loop of the diagram of Fig.2 should give the dominant contribution, leading to an effective coupling of a (highly) virtual pseudoscalar  $b\bar{b}$  state to gluons which, in turn, would couple to the target nucleon.

In view of our previous analysis of the  $\eta_b \rightarrow p\bar{p}$  decay, we next examine the possibility that the QCD vacuum might also play a non-trivial role in WIMP scattering off nuclei, eventually causing a (huge) enhancement of the spin-dependent cross section. The underlying reason of such an enhancement comes from a larger number of instantons involved in the gluon-quark interaction, whose size satisfies  $\rho \lesssim 1/q \simeq 1$  fm. Hence large values of  $\rho_0$  correspond to low values of the momentum transfer to nucleons.

As already mentioned in section 2, instanton self-interactions should lead to the modification of the (otherwise divergent) size distribution (2.1). We parametrize the new distribution as

$$n(\rho) \rightarrow n(\rho) e^{-c\rho^2} \quad (4.1)$$

where  $c = (b-4)/2\bar{\rho}^2$  [25]. Setting numerical values, one finds  $c \simeq 0.9 \text{ GeV}^2$ , also in agreement with Eq.(4.20) of Ref.[26]. A similar suppression factor, namely  $\exp(-2\pi\sigma\rho^2)$ , where  $\sigma = (0.44\text{GeV})^2$ , was obtained in [27].

In order to take into account all the  $\rho$  powers in the instanton-induced interaction from Eq.(2.8), let us define

$$f(\rho_0) = \int_0^{\rho_0} d\rho \rho^{15} e^{-c\rho^2} \quad (4.2)$$

where  $\rho_0$  denotes the instanton size upper limit set by the typical energy scale of the process.

Notice that the instanton-induced  $\chi$ -nucleon cross section should be proportional to  $f(\rho_0)^2$ , i.e. largely depending on the instanton size range defined by  $\rho_0$ . In view of our previous assumption on the enhancement of the  $\eta_c \rightarrow p\bar{p}$  decay rate due to instanton effects, let us introduce the ratio

$$R(\rho_0) = \left[ \frac{f(\rho_0)}{f(\rho_c)} \right]^2 \quad (4.3)$$

In Fig. 2 (right) we plot  $R(\rho_0)$  as a function of the upper limit  $\rho_0$  of integration in Eq.(4.2). In spite of many (and large) uncertainties (mainly due to the strong dependence on a power of  $\rho$ ) and the risky extrapolation to low momentum transfer, we tentatively infer from the curve that a very important enhancement of the cross section for spin-dependent  $\chi$ -nucleon cross section can occur.

## 5. Summary

Instantons, fluctuations of (non-abelian) gauge fields representing topological changing tunneling transitions, yield interactions between quarks and gluons which are absent in perturbation theory. In this work we have first examined possible instanton-induced effects in the decay of  $\eta_b$  resonances into baryon pairs. By rescaling the results experimentally found for  $BF[\eta_c(1S) \rightarrow p\bar{p}]$  and  $BF[\eta_c(1S) \rightarrow \Lambda\bar{\Lambda}]$  we have been able to make the order-of-magnitude estimate  $BF[\eta_b(1S) \rightarrow p\bar{p}] \simeq BF[\eta_c(1S) \rightarrow p\bar{p}] \sim 10^{-7} - 10^{-6}$ , although with large uncertainties mainly coming from the strong  $\rho$  power dependence of the decay width as can be seen from Eq.(2.8).

On the other hand, we have examined possible instanton-induced effects in WIMP scattering off nuclei, by extrapolation from the charmonium mass down to very small momentum transfer ( $q \simeq \Lambda$ ) implying large size instantons whose contribution is cut off by an exponential factor. This possibility covers different scenarios involving a light CP-odd Higgs boson [24] or a light  $Z'$  mediator (see e.g. [28]). We tentatively conclude that an important enhancement of the spin-dependent cross section can be caused by an instanton-induced interaction of gluons and nucleons. Phrased in other way, WIMP scattering off nuclei would somewhat resemble (diffractive) proton-proton collisions at high energy (see e.g. [29]).

Whether or not this kind of instanton-induced interaction associated to helicity flip of the target nucleons is called to play a relevant role in the description of dark matter scattering off nuclei, has to be investigated further.

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