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Thermal production rate of non-relativistic Majorana neutrinos in an effective field theory framework

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Heavy Majorana neutrinos represent an attempt to explain the smallness of the ν_e, ν_μ and ν_τ masses via the See-Saw mechanism. Moreover heavy Majorana neutrinos have been considered to play a relevant role in the dark matter problem and the leptogenesis mechanism as well. In the latter a heavy Majorana neutrino decays within a hot and dense plasma of particles producing an imbalance of leptons and anti-leptons. Hence a thermal treatment is needed and we focus here on the temperature regime $M \gg T$, which may be relevant for the leptogenesis scenario. Building on QCD thermal field theory methods, we develop an effective theory for non-relativistic Majorana neutrinos which simplifies computations in a thermal medium of standard model particles. As a proof of concept, we perform the computation of the decay rate for the Majorana neutrinos to test our effective Lagrangian, reproducing a result already known in the literature. Moreover the present effective field theory approach may be considered within different models involving non-relativistic Majorana fermions.

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1. Motivation and Introduction

The standard model (SM) of particle physics, together with general relativity, is believed to correctly describe almost all phenomena in nature. Nevertheless some experimental evidences and observations demand for physics beyond the standard model and the connection between particle physics and cosmology plays an important role. In the past decade neutrino experiments provided convincing evidence for neutrino masses and mixing [1], [2]. Therefore neutrino mass terms have to be implemented in a quantum field theory Lagrangian. This issue may be connected with the dark matter problem and the baryon asymmetry in the Universe which arise as the main open problems of the standard cosmology. Even if the dark matter existence is widely accepted, we still do not know which is the right dark matter particle. Many models suggest many different and interesting candidates each of them with its own advantages and drawbacks: neutralino and gravitino from supersymmetry, extra dimensions [3], [4] and heavy Majorana neutrinos [5], to cite few of them. Finally the baryon asymmetry in the Universe poses a puzzle in fundamental physics. In fact the SM of particle physics contains all the requirements necessary to dynamically generate such an asymmetry but it fails to explain an asymmetry as large as the one observed [6], which is by now accurately determined by Cosmic Microwave Background (CMB) Anisotropy measurements [7].

The dynamical generation of the baryon asymmetry in the context of quantum field theories of particle physics is called *baryogenesis*. Baryogenesis through leptogenesis [8] is a simple mechanism to explain this baryon asymmetry. In fact a lepton asymmetry may be converted in a baryon one by the sphaleron transitions which exist in the SM. Leptogenesis may be implemented in a conservative extension of the SM adding $\mathcal{N}$ species of right-handed Majorana neutrinos. A see-saw mechanism (Type I) [5], [9] has to be considered to complete the basic scheme. Alternative scenarios are possible, such as supersymmetric leptogenesis [10] and soft leptogenesis [11] involving Majorana super fields. Therefore almost all the models describing leptogenesis involve Majorana fields because the inclusion of a Majorana mass is a simple way to obtain the lepton number violation needed for leptogenesis. Moreover Majorana fermions are considered to be a basic ingredient in models describing dark matter particles, such as the neutralino, gravitino and heavy neutrinos.

In models that try to explain the origin of dark matter, leptogenesis or both, it is quite common to find a heavy Majorana particle whose decay has a central role. In order to obtain the desired result for a lepton asymmetry or production of dark matter, it is required a departure from thermal equilibrium, and this naturally happens in the regime

$$M \gg T \,, \tag{1.1}$$

where M is the mass of the heavy Majorana fermion and T the temperature of the thermal plasma. The relation (1.1) represents an important regime for both dark matter generation and leptogenesis mechanism. We want to stress the following aspects with relation to (1.1): first, the heavy Majorana fermion is barely affected by the temperature and it may be considered as a non-relativistic particle in the plasma; second, we may perform computations of both scattering process and decays involving the hard scale M putting $T \to 0$ and introduce the thermal effects as small corrections afterwards. The possibility to build up an effective field theory arises since a hierarchy of energy scales exists and we may identify heavy degrees of freedom (heavy Majorana fermions) and light degrees of freedom (SM particles). This is important because there are a lot of simplifications,

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arising from the fact that $e^{-M/T}$ is small, that are not obvious in the full theory Lagrangian but are implemented automatically by using the effective field theory techniques.

As a proof of concept of the effective field theory (EFT), we compute in [12] the decay rate for non-relativistic neutrinos within a hot plasma, that has already been found in [13, 14] but with a different computation procedure. In [14] an effective field theory is not set up and the imaginary time formalism (ITF) [15] is used to deal with thermal corrections. In the present work the real time formalism (RTF) [15] is exploited, which is suitable to the extension to non-equilibrium reactions. In [13] the real time formalism is also used, nevertheless the use of an EFT simplifies the problem of the doubling of degrees of freedom that arises in this formalism, the details of this will be explained later. In the future the EFT developed in [12] and presented here can be used to simplify computations of the decay rates taking into account *CP* violation and a medium far away from thermal equilibrium, as well as studies of thermal effects in other models in which a heavy Majorana particle appears.

2. Thermal leptogenesis and Majorana neutrinos

In Sec. 4 and Sec. 5 we show the effective formalism at work in a specific model as a proof of concept. Therefore we briefly discuss its main features. A conservative extension of the SM Lagrangian may be obtained adding a set of $\mathscr{N}$ right-handed neutrinos. They are also called *sterile neutrinos* because they are singlet under all the gauge group of the SM Lagrangian. Different models consider different number of sterile neutrinos and the Lagrangian may be written as follows

$$\mathscr{L} = \mathscr{L}_{SM} + i\bar{N}_I \partial_\mu \gamma^\mu N_I - \left(F_{\alpha i} \bar{L}_\alpha a_R \tilde{\Phi} N_I - \frac{M_I}{2} \bar{N}_I^c N_I + h.c. \right), \tag{2.1}$$

where the Higgs doublet is embedded in the $\tilde{\phi}$, defined as $\tilde{\phi} = i\sigma^2 \phi^*$, N stands for the Majorana neutrino, L_α is the SM lepton doublet and a_R is the right-handed projector. The index I labels the number of species of heavy neutrinos. In this simple extension of the SM the usual scenario that is considered, called *thermal leptogenesis* [8], consists of a hierarchical spectrum for right handed neutrinos. In this scenario the lightest heavy neutrino, N_1, is produced by thermal scattering after inflation, and subsequently decays out of thermal equilibrium in lepton number and CP-violating processes still in a hot plasma environment. Hence a deep comprehension of the interaction among heavy Majorana neutrinos and light SM particle in a thermal environment may be crucial for both the production and decay processes relevant for the leptogenesis mechanism.

The temperature of the system has to be such that the thermal leptogenesis is efficiently active, i.e. $T = \mathscr{O}(10^8 \div 10^9)$ GeV [16], [17]. This is the first relevant energy scale of the system under consideration. In this framework all the SM particles are thermalized. Hence their four-momenta are of the order of T and we can consider them as massless particles. Because of we consider only the lightest neutrino relevant for the leptogenesis mechanism we put $M_{I=1} = M$. The leptogenesis mechanism starts when the heavy neutrino decouples from the plasma reaching the out of equilibrium condition. When the temperature drops to $T \simeq M$, the decays of the heavy neutrinos into leptons and anti-leptons become effective, being the inverse decays suppressed. During the Universe expansion the decay process continues later on in the regime $T < M$ fulfilling the relation in (1.1) and the inverse decays are almost not present in such a temperature regime. With the exception of the model in [17] in general at least one heavy neutrino has a mass bigger than the EW scale, therefore it is interesting to compute the decay at finite T in the range

$$M \gg T \gg M_W \,. \tag{2.2}$$

3. Non-relativistic Majorana fermions

In order to construct an EFT, one of the most important steps is to identify the correct degrees of freedom in the given region of validity. In the present case the correct degrees of freedom at low energies are non-relativistic Majorana neutrinos. In this section we derive the appropriate propagator and the canonical anti-commutation relations for the non-relativistic Majorana fields. The main feature of a Majorana spinor ψ_M is the following

$$\psi_M = (\psi_M)^C \,, \tag{3.1}$$

where ψ^C denotes the conjugate spinor and C is the conjugation matrix which in the Weyl basis reads:

$$C = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix} .$$

A Majorana spinor is thus self-conjugate, hence we have a spinor with four real components at the very beginning which are connected two by two. As a result we have four degrees of freedom taking into account the real and imaginary parts. On the contrary a Dirac spinor has eight degrees of freedom. Note that the combinations $\langle \psi\psi \rangle$ and $\langle \bar{\psi}\bar{\psi} \rangle$ are non vanishing due to the Majorana nature of the fermion. Let us discuss the suitable propagator for the effective theory. In order to do this it is convenient to remind what is the situation in the heavy quark effective theory (HQET) [19]. In the HQET Lagrangian [19] the original QCD Dirac spinor that describes the heavy quark is separated in a Pauli spinor field that describes only heavy quark particles and another Pauli spinor field than describes only heavy quark anti-particles. The heavy quark particle is described by the two component field, let us call it h. It obeys, in the rest frame reference frame, to

$$\frac{1+\gamma^0}{2} h = h. \tag{3.2}$$

Moreover the h annihilates a heavy quark, and does not create an antiquark. Hence the h field contains only annihilation operators and satisfy the following equal-time anti-commutation relations:

$$\left\{ h^{\alpha}(\vec{x},t), h^{\beta}(\vec{y},t) \right\} = \left\{ h^{\alpha\dagger}(\vec{x},t), h^{\beta\dagger}(\vec{y},t) \right\} = 0, \tag{3.3}$$

$$\left\{ h^{\alpha}(\vec{x},t), h^{\beta\dagger}(\vec{y},t) \right\} = \delta^3(x-y)\, \delta^{\alpha\beta}. \tag{3.4}$$

We want to build an effective field theory for Majorana fermions in a way similar to HQET. Hence we consider a decomposition like in the HQET, involving non relativistic projectors. We decompose the four component Majorana spinor as follows

$$N = \left(\frac{1+\gamma^0}{2} \right) N + \left(\frac{1-\gamma^0}{2} \right) N = N_< + N_>. \tag{3.5}$$

According to the projectors properties and considering appropriate hermitian conjugate quantities one may get the following conditions:

$$N_< = i\gamma^2 N_>^\dagger\,, \quad N_> = i\gamma^2 N_<^\dagger \tag{3.6}$$

Referring to the relations in (3.6) one may argue that there is no particle-antiparticle disentanglement unlike in the HQET case. This is in agreement with the Majorana nature of the fermion: we can not distinguish a particle from its own antiparticle. Nevertheless by solving the Dirac equation for a free Majorana field, one can check that in the non-relativistic limit the operator content of $N_<$ is like the h field: $N_<$ contains only annihilation operators. Starting from the expression of the full relativistic Majorana spinors given in [20], we have obtained the anti-commutation relations for the non-relativistic spinor ending with the following result

$$\left\{N_<^a(\vec{x},t), N_<^b(\vec{y},t)\right\} = \left\{N_<^{a\dagger}(\vec{x},t), N_<^{b\dagger}(\vec{y},t)\right\} = 0, \tag{3.7}$$

$$\left\{N_<^a(\vec{x},t), N_<^{b\dagger}(\vec{y},t)\right\} = \delta^3(x-y)\,\delta^{ab}. \tag{3.8}$$

The conclusion is that in the non-relativistic EFT the heavy Majorana particle can be described by only one Pauli spinor field, as opposite to what happens with heavy Dirac particles that need two of them. This is basically due to the relation (3.1). Finally we provide the expression of the non-relativistic propagator, describing the heavy degrees of freedom, and obtained in a way similar to non-relativistic QCD (NRQCD) [21] (the difference between HQET and NRQCD is just the power counting). We obtain the following expression

$$\langle 0|T\left\{N_<^a(x)N_<^{b\dagger}(y)\right\}|0\rangle = \left(\frac{1+\gamma^0}{2}\right)\int \frac{d^4k}{(2\pi)^4}\frac{ie^{-ik(x-y)}}{k^0 - \frac{\vec{k}^2}{2M} + i\varepsilon}\,\delta^{ab}, \tag{3.9}$$

whereas the other possible time ordered combinations are vanishing because they contain only creation or only annihilation operators.

4. Effective Lagrangian and matching

We have defined both the hierarchy of energy scales and the relevant degrees of freedom of the low energy Lagrangian. The effective Lagrangian should reproduce the propagation of the heavy Majorana neutrinos and their interaction with the SM particles. Another important step in building the effective Lagrangian is to define the symmetries in order to write down all the possible operators which they allow for. The symmetries are the Poincaré invariance and the gauge invariance. The hard scale at which the low energy Lagrangian loses its validity is M. It is also the scale entering in the interaction terms to control the operator expansion. The effective Lagrangian involves a string of local operators describing the interaction among Majorana neutrinos and the Higgs field, heavy quarks (t,b), leptons and gauge bosons (see diagrams in Fig. 1). According to the temperature relevant for the leptogenesis mechanism, well above 100 GeV, we consider the Higgs field and gauge bosons in the phase in which the electroweak symmetry is not broken. In the following we do not use the subscripts for the non-relativistic Majorana fields as we did in Sec. 3 for illustrative purposes. Therefore we identify $N_< \equiv N$. The effective Lagrangian can be written as

$$\mathscr{L}_{EFT} = \mathscr{L}_N + \mathscr{L}_{SM} + \mathscr{L}_{N-SM}. \tag{4.1}$$

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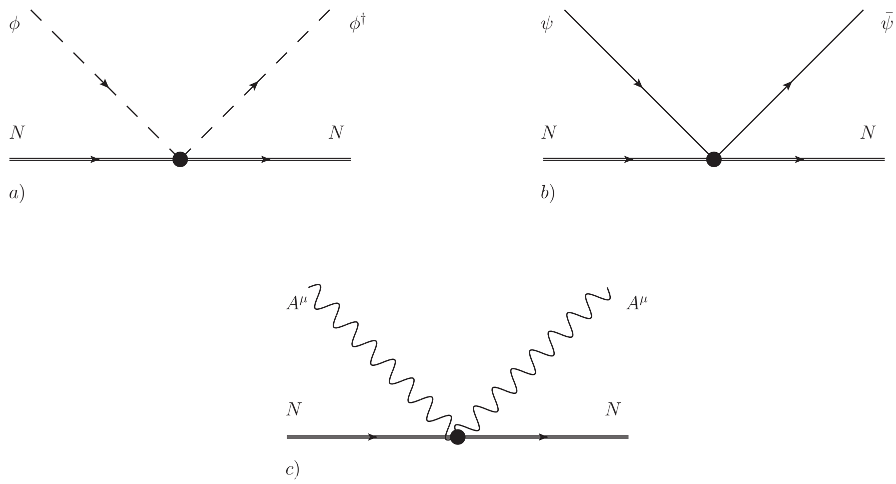

Figure 1: The diagrams represent the effective vertices derived by the effective Lagrangian. They represent the interactions among heavy Majorana neutrinos and Higgs field in a), fermions in b) and gauge bosons in c).

As in any EFT, the Lagrangian contains operators of any possible dimension. These dimensions are compensated by powers of M so that the action is dimensionless. According to this, the number of operators in the Lagrangian is infinite but we only need to consider a finite number of operators to achieve a given precision. In our case we consider only operators of dimension seven or smaller. $\mathscr{L}_N$ is the part of the Lagrangian that contains the kinetic term of heavy Majorana particles, $\mathscr{L}_{SM}$ is the usual standard model Lagrangian and the last term explicitly reads

$$\begin{aligned}\mathscr{L}_{N-SM} = \frac{a}{M}N^\dagger N\phi^\dagger\phi + \frac{b}{M^3}N^\dagger N D_0\phi^\dagger D_0\phi + \frac{c_1}{M^3}\left(Na_R\bar{L}_\alpha\right)\left(iD_0L_\beta a_L N^\dagger\right) + \\ \frac{c_2}{M^3}\left(Na_R\gamma_\mu\gamma_\nu\bar{L}_\alpha\right)\left(iD_0L_\beta\gamma^\nu\gamma^\mu a_L N^\dagger\right) + \frac{c_3}{M^3}N^\dagger N\left(\bar{t}a_L\gamma^0\, iD_0 t\right) + \\ \frac{c_4}{M^3}N^\dagger N\left(\bar{q}a_L\gamma^0\, iD_0 q\right) + \frac{d_1}{M^3}tr\left\{T^aT^b\right\}N^\dagger N F^a_{0i}F^b_{0i} + \frac{d_2}{M^3}N^\dagger N W_{0i}W_{0i}\,. \end{aligned} \tag{4.2}$$

In order to get a contribution for the decay rate process one needs only the imaginary part arising from the Wilson coefficients in (4.2) because tadpoles only contribute with a real part. Therefore we focus on the derivation of their imaginary part in the matching procedure through the Green functions, getting the following expressions:

$$\begin{aligned} Im(a) = -i\frac{3}{4\pi}|F|^2\lambda \;\;,\quad Im(b) = -i\frac{5}{32\pi}(3g^2+g'^2)|F|^2 \\ Im(c_1) = i\frac{3}{8\pi}|\lambda_{tb}|^2|F|^2 - i\frac{3}{16\pi}(3g^2+g'^2)|F|^2 \;\;,\quad Im(c_2) = i\frac{1}{384\pi}(3g^2+g'^2)|F|^2 \\ Im(c_3) = -i\frac{1}{24\pi}|\lambda_{tb}|^2|F|^2 \;\;,\quad Im(c_4) = -i\frac{1}{48\pi}|\lambda_{tb}|^2|F|^2 \\ Im(d_1) = -i\frac{1}{48\pi}g^2|F|^2 \;\;,\quad Im(d_2) = -i\frac{1}{96\pi}g'^2|F|^2\,. \end{aligned} \tag{4.3}$$

We use the notation $|F|^2 = \sum_{\alpha=1}^{3}F^*_\alpha F_\alpha$. The power of dimensional analysis together with the explicit expression of the effective Lagrangian allows us to predict the thermal corrections to the

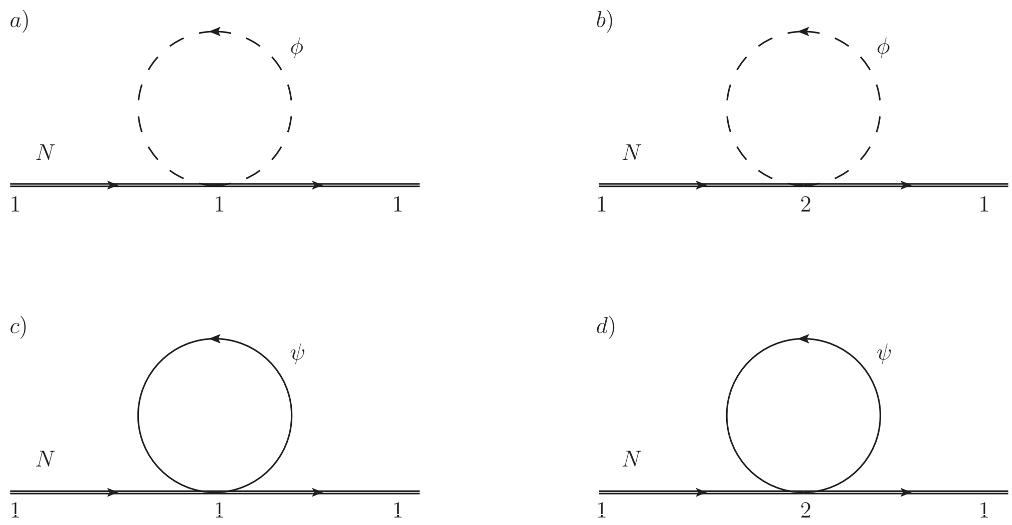

Figure 2: The diagrams show a thermal loop affecting the propagation of the heavy Majorana neutrino (thick double line). A scalar particle (Higgs field) is circulating in the loop diagrams in a) and b) whereas a fermion propagates in the loop diagrams in c) and d). The doubling of degrees of freedom is taken into account by the labelling 1 and 2 in each diagram. We may extract the thermal correction to decay width by the computation of such diagrams in the effective field theory.

decay rate due to each type of particle in the thermal plasma:

$$\delta\Gamma_{\phi} \propto \frac{T^2}{M}\,, \quad \delta\Gamma_{\partial\phi} \propto \frac{T^4}{M^3}\,, \quad \delta\Gamma_{\psi} \propto \frac{T^4}{M^3}\,, \quad \Gamma_{F,W} \propto \frac{T^4}{M^3}\,. \tag{4.4}$$

In the next section we explain how to calculate the thermal corrections.

5. Thermal corrections and results

At this point we may get the thermal corrections as simple tadpole diagrams build up using the effective vertices in Fig. 1. Examples of such diagrams are shown in Fig. 2, for a scalar and a fermion particle, where the vertices are the ones obtained in the effective Lagrangian in (4.1). We use the RTF (Real Time Formalism) to express thermal propagators in the loop. Dealing with RTF one has to consider the doubling of degrees of freedom (for more details see [15]). In principle one has to take into account the configuration of the fields as shown in Fig. 2, considering both the fields of type 1 and 2. However the doubling of the degrees of freedom actually does not affect our computation. The reason is that the inclusion of any vertex of type 2 fields involving a heavy Majorana fermion in time-ordered Green functions requires the inclusion of a heavy Majorana propagator of type 21, but in complete analogy with the heavy quark in HQET [22], this is zero. As a consequence all the vertices involving heavy Majorana fermions can be considered to be of type 1. Therefore in a one loop computation in the EFT only the fields of type 1 are taken into account and hence the 11 component of the thermal propagators, which is the only one we need for this computation. The 11 component of the thermal propagators have the following expressions for bosons and fermions respectively:

$$i\Delta(x-y) = \int \frac{d^4p}{(2\pi)^4} \left[\frac{i}{p^2 - m^2 + i\varepsilon} + (2\pi) n_B(|p_0|) \delta(p^2 - m^2) \right] e^{-k(x-y)} \tag{5.1}$$

$$iS(x-y) = \int \frac{d^4p}{(2\pi)^4} (\not{p} + m) \left[\frac{i}{p^2 - m^2 + i\varepsilon} - (2\pi) n_F(|p_0|) \delta(p^2 - m^2) \right] e^{-p(x-y)} \tag{5.2}$$

The procedure to extract the decay width is the following. We compute

$$\langle \Omega | N^i(x) N^{\dagger j}(y) | \Omega \rangle \tag{5.3}$$

where Ω is the minimal energy state. In momentum space this Green function has a pole around $k_0 \sim \frac{\mathbf{k}^2}{2M}$ from which we can identify the decay width

$$\frac{iZ}{k^0 - \frac{\vec{k}^2}{2M} - \delta E + i\frac{\Gamma}{2}} = \frac{iZ}{k^0 - \frac{\vec{k}^2}{2M} + i\varepsilon} - Z(i\delta E + \frac{\Gamma}{2}) \left(\frac{i}{k^0 - \frac{\vec{k}^2}{2M} + i\varepsilon} \right)^2 + \cdots \tag{5.4}$$

The thermal effects are going to come from the loops corrections to this Green function. As an example let us consider the contribution from the term proportional to a in the EFT Lagrangian in eq. (4.2) represented by the diagram *a)* in Fig. 1, which reads

$$i\frac{a}{M} \langle 0 | \int d^4t N^i(x) N^{\dagger j}(y) N^{\dagger k}(t) N^k(t) \phi_m^\dagger(t) \phi_m(t) | 0 \rangle. \tag{5.5}$$

The first term of the scalar propagator gives a vanishing quantity since a scale-less integral in dimensional regularization. The second term, which contains the thermal Bose-Einstein distribution, gives

$$\frac{ia}{M} \int \frac{d^4k}{(2\pi)^4} \frac{d^4\ell}{(2\pi)^4} \left(\frac{i}{k^0 - \frac{\vec{k}^2}{2M} + i\varepsilon} \right)^2 e^{-ik(x-y)} \left(\frac{1+\gamma^0}{2} \right)^{ij} (2\pi) n_B(|\ell_0|) \delta(\ell^2) \ell_0^2, \tag{5.6}$$

where ℓ^μ is the momentum circulating in the thermal loop. One gets from eq. (5.6) the leading thermal correction to the heavy Majorana decay width due to the Higgs field which reads

$$\delta\Gamma_\phi = -\frac{|F|^2 \lambda}{8\pi} \frac{T^2}{M}, \tag{5.7}$$

where we used the $Im(a)$ in eq. (4.3). We stress that the power counting already predicted such result in (4.4), revealing the power of the effective Lagrangian approach. Performing the thermal loop for all the particle species in the plasma we finally get the decay width at finite temperature

$$\Gamma(M,T) = \frac{|F|^2 M}{8\pi} \left\{ 1 - \lambda \left(\frac{T}{M} \right)^2 - \frac{\pi^2}{80} \left(\frac{T}{M} \right)^4 (3g^2 + g'^2) - \frac{7\pi^2}{60} \left(\frac{T}{M} \right)^4 |\lambda_{tb}|^2 + \mathcal{O}\left(\frac{T}{M} \right)^6 \right\}. \tag{5.8}$$

The expression in eq. (5.8) agrees with the result calculated in [14] and it has been computed in [12] within an effective field theory framework.

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