

Neutron rich hypernuclei in chiral soliton model

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The binding energies of neutron rich strangeness $S = -1$ hypernuclei are estimated in the chiral soliton approach using the bound state rigid oscillator version of the $SU(3)$ quantization model. Additional binding of strange hypernuclei in comparison with nonstrange neutron rich nuclei takes place at not large values of atomic (baryon) numbers, $A = B \leq \sim 10$. This effect becomes stronger with increasing isospin of nuclides, and for the "nuclear variant" of the model with rescaled Skyrme constant e . Binding energies of ${}^8_{\Lambda}He$ and recently discovered ${}^6_{\Lambda}H$ satisfactorily agree with data. Hypernuclei ${}^7_{\Lambda}H$, ${}^9_{\Lambda}He$ are predicted to be bound stronger in comparison with their nonstrange analogues 7H , 9He ; hypernuclei ${}^{10}_{\Lambda}Li$, ${}^{11}_{\Lambda}Li$, ${}^{12}_{\Lambda}Be$, ${}^{13}_{\Lambda}Be$ etc. are bound stronger in the nuclear variant of the model.

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1. Introduction

Studies of nuclear states with unusual properties — nontrivial values of flavor quantum numbers (strangeness, charm or beauty), or large isospin (so called neutron rich nuclides) are important not only for the nuclear physics itself, but for astrophysics and cosmology. Studies of neutron rich hypernuclei recently got new impact due to discovery of the hypernucleus ${}^6_{\Lambda}H$ (heavy hyperhydrogen) by FINUDA Collaboration (2012) which followed its search during several years [1].

Theoretical discussion began with the work by R.H. Dalitz and R. Levi-Setti [3], see [4, 5, 6] in parallel with experimental searches [7, 8, 9]. Dalitz and Levi-Setti noted: Lambda particle may act as additional glue for the nuclear matter, increasing the binding energy in comparison with nuclei having zero strangeness. This observation is confirmed within the chiral soliton approach (CSA). Moreover, this effect becomes stronger for the neutron rich nuclei, with increasing excess of neutrons inside the nucleus.

The advantage of the CSA [10], [11] is its universality, i.e. the possibility to consider different nuclei on equal footing, and considerable predictive power. The drawback of the CSA is its relatively low accuracy in describing the properties of each particular nucleus. In this respect the CSA cannot compete with traditional approaches and models like shell model, Hartree-Fock method, etc. [3], [4, 5, 6].

The quantization of the model performed first in the $SU(2)$ configuration space for the baryon number one states [12], somewhat later for configurations with axial symmetry [13] and for multiskyrmions [14], allowed, in particular, to describe the properties of nucleons and Δ -isobar and, more recently, some properties of light nuclei [15, 16], including so called "symmetry energy" [15]. Recently the neutron rich isotope ${}^{18}B$ has been found to be unstable relative to the decay ${}^{18}B \rightarrow {}^{17}B + n$ [17], in agreement with [15].

The $SU(3)$ quantization of the model has been performed first within the rigid rotator approach [18], and also in the bound state model [19]. The binding energies of the ground states of light hypernuclei have been described in [20] within a version of the bound state chiral soliton model [19], in qualitative agreement with data [21].

The collective motion contributions, only, have been taken into account (single particles excitations should be added), and special subtraction scheme has been used to remove uncertainties in absolute values of masses intrinsic to the CSA [22, 23]. This investigation has been extended to the higher in energy (excited) states, with baryon number $B = 2$ and 3, some of them may be interpreted as antikaon-nuclei bound states [24]. Some of these states are bound stronger than predicted originally by Akaishi and Yamazaki [25, 26]. These states could overlap and appear in experiment as a broad enhancement, in qualitative agreement with data obtained by FINUDA [27] and more recently by DISTO [28].

2. Features of the Chiral Soliton Approach (CSA)

Principles and ingredients of the CSA incorporated in the *truncated* effective chiral lagrangian [10]:

$$L^{eff} = -\frac{F_{\pi}^2}{16} Tr l_{\mu} l_{\mu} + \frac{1}{32e^2} Tr [l_{\mu} l_{\nu}]^2 + \frac{F_{\pi}^2 m_{\pi}^2}{8} Tr (U + U^{\dagger} - 2), \quad (1)$$

the chiral derivative $l_\mu = \partial_\mu U U^\dagger$, $U \in SU(2)$ or $U \in SU(3)$ - unitary matrix depending on chiral fields, m_π is the pion mass, F_π - the pion decay constant known experimentally, e - the only parameter of the model in its minimal variant proposed first by Skyrme.

The chiral and flavor symmetry breaking term in the lagrangian density depends on kaon mass and decay constant m_K and F_K ($F_K/F_\pi \simeq 1.23$ from experimental data):

$$L^{FSB} = \frac{F_K^2 m_K^2 - F_\pi^2 m_\pi^2}{24} \text{Tr}(U + U^\dagger - 2)(1 - \sqrt{3}\lambda_8) - \frac{F_K^2 - F_\pi^2}{48} \text{Tr}(U l_\mu l_\mu + l_\mu l_\mu U^\dagger)(1 - \sqrt{3}\lambda_8). \quad (2)$$

λ_8 is the $SU(3)$ Gell-Mann matrix. This term defines the mass splittings between strange and nonstrange baryons (multibaryons), modifies some properties of skyrmions and is crucial. The whole lagrangian given by (1),(2) is proportional to the number of colors of underlying QCD, $L^{eff} \sim N_c$. The mass term in (1) $\sim F_\pi^2 m_\pi^2$, changes asymptotics of the profile f and the structure of multiskyrmions at large B , in comparison with the massless case. For the $SU(2)$ case

$$U = \cos f + i(\vec{n}\vec{\tau})\sin f, \quad (3)$$

the unit vector \vec{n} depends on 2 functions, α, β, τ_k are the Pauli matrices. Three profiles $\{f, \alpha, \beta\}(x, y, z)$ parametrize the 4-component unit vector on the 3-sphere S^3 . The topological soliton (the skyrmion) is configuration of chiral fields, possessing topological charge identified with the baryon number B (for the nucleus it is the atomic number A : $B = A$).

3. Properties of multiskyrmions

Minimization of the mass functional M_{cl} provides 3 profiles $\{f, \alpha, \beta\}(x, y, z)$ and allows to calculate moments of inertia Θ_I, Θ_F , the Σ -term (we call it Γ) and some other characteristics of chiral solitons shown in tables 1 and 2.

B	Θ_I	Θ_J	Θ_F^0	Θ_S	Γ	$\tilde{\Gamma}$	μ_S	ω_S
1	5.55	5.55	2.05	2.636	4.80	14.9	3.155	307
6	25.4	178	13.1	16.64	29.0	38.0	3.125	287
7	28.9	221	14.7	18.64	32.3	44.0	3.009	283
8	33.4	298	17.4	22.15	38.9	47.0	3.125	288
9	37.8	376	20.6	26.25	46.3	47.5	3.269	292
10	41.4	455	23.0	29.35	52.0	50.0	3.289	293
11	45.2	547	25.6	32.74	58.5	52.4	3.340	295
13	52.1	737	30.5	39.07	70.2	56.8	3.372	296
14	56.1	865	33.7	43.15	78.2	58.9	3.460	299
16	63.2	1110	38.9	50.07	91.5	62.8	3.517	302

Table 1. Characteristics of classical skyrmion configurations which enter the nuclei — hypernuclei binding energies differences: moments of inertia Θ , Σ -term Γ and $\tilde{\Gamma}$ - in units GeV^{-1} , ω_S - in MeV , μ_S is dimensionless Parameters of the model $F_\pi = 186 MeV$; $e = 4.12$. The numbers are taken from [29, 30].

B	Θ_I	Θ_F^0	Θ_S	Γ	$\tilde{\Gamma}$	μ_S	ω_S
1	12.8	4.66	5.893	10.1	19.6	6.407	344
6	62.6	30.7	38.60	64.7	50.6	6.728	334
7	69.6	34.9	43.75	72.5	54.4	6.500	330
8	79.9	41.3	51.97	87.4	58.2	6.785	334
9	88.9	47.1	59.43	101	61.7	6.927	337
10	97.4	52.6	66.40	113	64.9	6.957	336
11	106	58.5	73.88	126	67.9	7.038	337
12	114	63.8	80.65	138	70.8	7.049	337
13	122	69.5	87.94	151	73.6	7.102	338
14	132	76.3	96.81	168	76.3	7.289	341
15	140	82.3	104.5	182	78.8	7.353	342
16	148	88.1	112.0	196	81.2	7.402	343

Table 2. Same as in Table 1 for rescaled (nuclear) variant of the model with constant $e = 3.0$ [15, 31].

These characteristics of classical configurations contain implicitly information about interaction between baryons. Θ_S given in Tables 1 and 2 is certain combination of Θ_F^0 and sigma term Γ :

$$\Theta_S = \Theta_F^0 + \frac{1}{4} \left(\frac{F_K^2}{F_\pi^2} - 1 \right) \Gamma. \quad (4)$$

The strangeness excitation energies ω_S given in Tables 1, 2 are somewhat overestimated, especially for nuclear variant of the model — this is an artefact of the CSA. However, this overestimation is cancelled in the nuclear binding energies differences considered below. The rational map approximation [32] simplifies considerably calculations of various characteristics of multiskyrmions presented in Tables 1, 2.

4. Mass formula for multibaryons in the $SU(3)$ bound state model

The observed spectrum of strange multibaryon states (hypernuclei) is obtained by means of the $SU(3)$ quantization procedure and depends on the quantum numbers of multibaryons and characteristics of skyrmions presented in Tables 1, 2. Within the bound state model the antikaon field is bound by the $SU(2)$ skyrmion. The mass formula takes place

$$M(A, S, I, J) = M_{cl} + \omega_S + \omega_{\bar{S}} + |S| \omega_S + \Delta M_{HFS} \quad (5)$$

where strangeness and antistrangeness excitation energies

$$\omega_S = N_c(\mu_S - 1)/8\Theta_S, \quad \omega_{\bar{S}} = N_c(\mu_S + 1)/8\Theta_S, \quad (6)$$

$$\Theta_S = \Theta_F^0 + \frac{1}{4} \left(\frac{F_K^2}{F_\pi^2} - 1 \right) \Gamma, \quad \mu_S = \sqrt{1 + \bar{m}_K^2/M_0^2},$$

$$M_0^2 = N_c^2/(16\Gamma\Theta_S) \sim N_c^0, \quad \bar{m}_K^2 = m_K^2 F_K^2/F_\pi^2. \quad (7)$$

The hyperfine splitting correction to the energy of the baryon state, depending on hyperfine splitting constants c_S , \bar{c}_S , observed isospin I , "strange isospin" I_S , the isospin of skyrmion without added antikaons \vec{I}_r and the angular momentum J , equals in the case when interference between usual space and isospace rotations is negligible or not important (Westerberg, Klebanov, 1996):

$$\Delta M_{HFS} = \frac{c_S I_r(I_r + 1) - (c_S - 1)I(I + 1) + (\bar{c}_S - c_S)I_S(I_S + 1)}{2\Theta_I} + \frac{J(J + 1)}{2\Theta_J}. \quad (8)$$

The hyperfine splitting constants are equal

$$c_S = 1 - \frac{\Theta_I}{2\Theta_S \mu_S}(\mu_S - 1), \quad \bar{c}_S = 1 - \frac{\Theta_I}{\Theta_S \mu_S^2}(\mu_S - 1), \quad (9)$$

Strange isospin equals $I_S = 1/2$ for $S = \mp 1$. We recall that body-fixed isospin $\vec{I}^{bf} = \vec{I}_r + \vec{I}_S$, [?, 30]. \vec{I}_r is quite analogous to the so called "right" isospin within the rotator quantization scheme. When $I_S = 0$, i.e. for nonstrange states, $I = I_r$ and this formula goes over into $SU(2)$ formula for multiskyrmions. Correction $\Delta M_{HFS} \sim 1/N_c$ is small at large N_c , and also for heavy flavors.

5. Total binding energies of neutron rich hypernuclei

The mass splitting within $SU(3)$ multiplets of multibaryons contains the smallest uncertainty: the unknown for the $B > 1$ solitons Casimir energy [22, 23] cancels in the mass splittings. For the difference of energies of states with strangeness S and with $S = 0$ which belong to multiplets with equal values of (p, q) -numbers ($p = 2I_r$), we obtain, using the above expressions for the constants c_S and \bar{c}_S (first subtraction):

$$\Delta E(p, q; I, S; I_r, 0) = |S| \omega_S + \frac{\mu_S - 1}{4\mu_S \Theta_S} [I(I + 1) - I_r(I_r + 1)] + \frac{(\mu_S - 1)(\mu_S - 2)}{4\mu_S^2 \Theta_S} I_S(I_S + 1). \quad (10)$$

The term $\sim (I_r + 1/4)$ in Eq. (11) is responsible for the additional binding of neutron rich hypernuclei in comparison with the $S = 0$ neutron rich nuclei.

$A - {}_\Lambda A$	ϵ_2^{exp}	$\epsilon_{3/2}^{exp}$	$\epsilon_{3/2}^{th}$	$\epsilon_{3/2}^{th,*}$
${}^6H - {}_\Lambda^6H$	5.8	10.8	14.8	17
${}^8He - {}_\Lambda^8He$	31.4	36.0	34.8	40
${}^{10}Li - {}_\Lambda^{10}Li$	45.3		40.6	50
${}^{12}Be - {}_\Lambda^{12}Be$	68.6		59.3	71
${}^{14}B - {}_\Lambda^{14}B$	85.4		70.4	84
${}^{16}C - {}_\Lambda^{16}C$	111		92.3	107

Table 3. The total binding energies (in MeV) of nonstrange isotopes with $I = 2$, $N - Z = 4$ (in MeV) [33] and hypernuclei with isospin $I = 3/2$ for the original variant, $e = 4.12$, and for the variant with rescaled constant, $e = 3$ (numbers with the *). Experimental values of binding energy are available only for ${}^8_\Lambda He$ [7] and ${}^6_\Lambda H$ FINUDA [1].

For the difference of binding energies of the hypernucleus with strangeness $S = -1$, isospin $I = I_r - 1/2$ and the nonstrange nucleus with isospin $I = I_r$ (the neutron excess $N - Z = 2I_r$) we obtain (second subtraction):

$$\Delta\varepsilon = \omega_{S,1} - \omega_{S,B} - \frac{3}{8} \frac{\mu_{S,1} - 1}{\mu_{S,1}^2 \Theta_{S,1}} + \left(I_r + \frac{1}{4} \right) \frac{\mu_{S,B} - 1}{4\mu_{S,B} \Theta_{S,B}} - \frac{3}{16} \frac{(\mu_{S,B} - 1)(\mu_{S,B} - 2)}{\mu_{S,B}^2 \Theta_{S,B}}. \quad (11)$$

The value of binding energy of hyperhydrogen shown here, $\varepsilon({}_\Lambda^6H) = 10.8 \text{ MeV}$ is the sum of the binding energy of ${}_\Lambda^6H$ relative to ${}^5H + \Lambda$, measured by FINUDA [1], $\varepsilon({}_\Lambda^6H) = (4.0 \pm 1.1) \text{ MeV}$ and the binding energy of 5H , $\varepsilon({}^5H) \simeq 6.78 \text{ MeV}$ [34].

The value of the binding energy of ${}_\Lambda^8He$ shown in Table 3 is the sum of the Λ separation energy $7.16 \pm 0.70 \text{ MeV}$ measured in [7], and the total binding energy of the 7He nucleus, $\varepsilon({}^7He) \simeq 28.82 \text{ MeV}$ [33].

$A - {}_\Lambda A$	$\varepsilon_{5/2}^{exp}$	ε_2^{th}	$\varepsilon_2^{th,*}$
${}^7H - {}_\Lambda^7H$	8	23	24
${}^9He - {}_\Lambda^9He$	30.3	30	37
${}^{11}Li - {}_\Lambda^{11}Li$	45.64	41	51
${}^{13}Be - {}_\Lambda^{13}Be$	68.1	59	71
${}^{15}B - {}_\Lambda^{15}B$	88.2	72	86
${}^{17}C - {}_\Lambda^{17}C$	111.5	94	108

Table 4. The case of the odd atomic numbers A , nonstrange isotopes with $I = 5/2$, $N - Z = 5$ (total binding energies are taken from [33]), and hypernuclei with $I = 2$. Experimental data on hypernuclei binding energies are not available, still.

The value 8 MeV for the binding energy of 7H is preliminary result published in [35]. The correction to the binding energies depending on the spin of the nucleus J is not included. This correction is small in any case because the moment of inertia Θ_J is large, generally $\Theta_J \sim B^2$ and $\Theta_J > B\Theta_I$.

The decrease of values of $\Delta\varepsilon_{5/2,2}^{th}$ with increasing atomic number may be connected with limited applicability of the rational map approximation for describing multiskyrmions at larger baryon (atomic) numbers.

6. Conclusions and prospects

The difference of total binding energies of neutron rich hypernucleus with atomic number A , strangeness $S = -1$, charge Z (i.e. containing Z protons), isospin $I = (N - Z - 1)/2$, and the zero strangeness nucleus with same atomic number A , Z protons and $N = A - Z$ neutrons, which has isospin $I = (N - Z)/2$ is calculated. Within the CSA this quantity contains the smallest uncertainty.

Calculations are performed for two values of the Skyrme constant, $e = 4.12$, and $e = 3.0$ (the rescaled, or nuclear variant) which allowed to describe the mass splittings of nuclear isotopes with atomic numbers up to ~ 30 [15]. Both variants of the model provide close results for ${}_\Lambda^6H$ and ${}_\Lambda^7H$, but for greater atomic numbers the difference becomes considerable.

Results of the rescaled nuclear variant are more reliable for greater atomic numbers, $A \geq \sim 10$. Further study of the dependence of our results on the only parameter of the model, the Skyrme constant e , is desirable.

Extention of these calculation to hypernuclei with arbitrary excess of neutrons in nuclei is possible without difficulties, as well as to charm and beauty flavours.

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