PROCEEDINGS OF SCIENCE

Effective Strange Quark/Antiquark Masses from Chiral Soliton Models for Exotic Baryons

Vladimir Kopeliovich[*†]
Institute for Nuclear Research of Russian Academy of Sciences
E-mail: kopelio@inr.ru

The effective strange quark and antiquark masses are estimated from the chiral soliton model (CSM) results for the spectrum of exotic and nonexotic baryons. There are problems when one tries to project results of the CSM on the quark models (QM): expansion parameter in $1/N_c$ is large for the case of spectrum, extrapolation to real world is not possible in this way. Rigid (soft as well) rotator and bound state models coincide in the first order of $1/N_c$, but differ in the next orders.

There is correspondence of the CSM and simple QM predictions for pentaquarks spectra in negative S sector of $\{27\}$ and $\{35\}$ plets: the effective mass of strange quark is about $135-130\,MeV$, slightly smaller for $\{35\}$. For positive strangeness components the link between CSM and QM requires strong dependence of effective $\bar{s}$ mass on particular $SU(3)$ multiplet. $SU(3)$ configuration mixing is important and pushes spectra towards simplistic model (with equal masses of the strange quark and antiquark), but reasons for this are not clear.

The success of the CSM in many respects means that predictions of pentaquark (PQ) states should be considered seriously. Existence of PQ by itself is without any doubt, although very narrow PQ may not exist.

Xth Quark Confinement and the Hadron Spectrum
8-12 October 2012
TUM Campus Garching, Munich, Germany

[*]Speaker.
[†]Based on the papers JETP 108, 770 (2009) [Zh.Eksp.Teor.Fiz. 135, 885 (2009)], Phys.Rev. D73, 094018 (2006)

1. Studies of baryon spectrum - nonstrange, strange, charmed, etc. - remain to be one of main subjects of accelerator physics. Discovery of baryon states besides well established octet and decuplet, in particular, exotic baryons, could help to the progress in understanding the hadron structure.

In the absence of the complete theory of strong interactions there are different approaches and models; each has some advantages and certain drawbacks. Interpretation of hadrons spectra in terms of the quark models (QM) is widely accepted, QM are very successful tool for the classification and interpretation of hadron spectrum. QM are to large extent phenomenological; simplicity of QM becomes a fiction when we try to go behind e.g. 3-quark picture for baryons, since there is no regular methods of solving relativistic many-body problem. The number of constituents (e.g. additional $q\bar{q}$-pairs) is not fixed in a true relativistic theory.

Alternative approaches, in particular, the chiral soliton approach (CSA) [1] has certain advantages. It is based on few principles and ingrediemts incorporated in the model lagrangian. Baryons and baryonic systems are considered on equal footing (the look "from outside", at fixed external quantum numbers B,S,I,J). CSA looks like a theory, but still it is a model, and some elements of phenomenology are present necessarily in the CSA. It has been noted first in [2] and, for any baryon numbers, in [3] that so called exotic (i.e. with additionalquark-antiquark pairs) states appear naturally within the CSA. More definite numerical predictions were made somewhat later in[4] and (quite definite!) in [5].

Results obtained within the CSA mimic some features of baryons spectrum within the QM due to the Gell-Mann - Okubo relations. Masses, binding energies of classical configurations, moments of inertia $\Theta_K,\ \Theta_\pi$ (so called kaonic and pionic inertia) and some other characteristics of chiral solitons depend on three profile functions $\{f,\ \alpha,\ \beta\}(x,y,z)$ and contain implicitly the information about interaction between baryons. Minimization of the mass functional M_{class} provides 3 profiles and allows to calculate moments of inertia, etc.

2. The observed spectrum of states is obtained by means of quantization procedure and depends on quantum numbers and moments of inertia, Σ-term (Γ), etc. In $SU(2)$ case, the rigid rotator model (RRM) is most effective and successfull in describing the properties of nucleons, Δ [1], of light nuclei [6] and also "symmetry energy" of nuclei with $A <\sim 30$ [7].

In the $SU(3)$ case the mass formula takes place, also for RRM

$$M(p,q,Y,I,J) = M_{cl} + \frac{K(p,q,J)}{2\Theta_K} + \frac{J(J+1)}{2\Theta_\pi} + \delta M(Y,I),$$

where these terms scale as functions of N_c as $\sim N_c,\ \sim 1,\quad \sim N_c^{-1}$ and ~ 1,correspondingly; it is in fact expansion in powers of $1/N_c$. $K(p,q,J) = C_2(SU_3) - I_r(I_r+1) - N_c^2B^2/12$ - difference of the well known Casimir operators, "Right" isospin $I_r = J$ for $theB = 1$ states. For all minimal multiplets $K = N_cB/3$, K is greater by few units for nonminimal (exotic) multiplets, see table 2. Some paradox is in the fact that total splitting of the whole multiplet is $\sim N_c$.

The δM which contribute to the mass splittings inside of $SU(3)$ multiplets, is due to the term in the lagrangian $\mathscr{L}_M \simeq -\tilde{m}_K^2\Gamma s_\nu^2/2,\ \nu$ is the angle of rotation into "strange" direction, $\tilde{m}_K^2 = F_K^2m_K^2/F_\pi^2 - m_\pi^2$ includes $SU(3)$-symmetry violation in flavor decay constants, $\Gamma \sim 5\,Gev^{-1} \sim \Sigma$, moments of inertia $\Theta_\pi \sim (5-6)Gev^{-1}$, $\Theta_K \sim (2-3)Gev^{-1}$. $\Theta \sim N_c$.

"Strangeness contents" of the quantized baryon state $C_S =< s_V^2/2 >_B$ can be calculated exactly with the help of wave functions in $SU(3)$ configuration space, for arbitrary number of colors N_c [9]. Approximately, at large N_c $C_S \simeq \frac{2+|S|}{N_c}$. The Gell-Mann - Okubo formula takes place in the form

$$C_S = -A(p,q)Y - B(p,q)\left[Y^2/4 - \vec{I}^2\right] + C(p,q),$$

$A_(p,q), B_(p,q), C_(p,q)$ depend on particular $SU(3)$ multiplet. For the "octet"

$$A(''8'') = \frac{N_c+2}{(N_c+3)(N_c+7)}, \quad B(''8'') = \frac{2}{(N_c+3)(N_c+7)}, \quad C(''8'') = \frac{3}{(N_c+7)}.$$

If we try to make expansion in $1/N_c$, then parameter is $\sim 7/N_c$. For "decuplet" and "antidecuplet" expansion parameter is $\sim 9/N_c$ and becomes worse for greater multiplets, "27"-plet, "35"-plet, etc. Apparently, for realistic world with $N_C = 3$ the $1/N_c$ expansion *does not work*.

Any chain of states connected by relation $I = C' \pm Y/2$ reveals linear dependence on hypercharge (strangeness), so, the CSA mimics the quark model with the effective strange quark mass

$$m_S^{eff} \sim \tilde{m}_K^2 \Gamma[A(p,q) \mp 3B(p,q)/2],$$

for decuplet (antidecuplet). This is valid if the FSB is included in the lowest order of perturbation theory. At large N_c $m_S^{eff} \sim \tilde{m}_K^2 \Gamma/N_c$, too much, $\sim 0.6\,GeV$ if extrapolated to $N_c = 3$.

If we make expansion in RRM, we obtain for the "octet" of baryons

$$\delta M_N = 2\tilde{m}_K^2 \frac{\Gamma}{N_c}\left(1 - \frac{6}{N_c}\right), \;\ldots\; \delta M_\Xi = \tilde{m}_K^2 \frac{\Gamma}{N_c}\left(4 - \frac{28}{N_c}\right),$$

3. Within the bound state model (BSM) [8] anti-kaon is bound by $SU(2)$ skyrmion. The mass formula takes place

$$M = M_{cl} + \omega_S + \omega_{\bar{S}} + |S|\omega_S + \Delta M_{HFS}$$

where flavor and antiflavor excitation energies

$$\omega_S = N_c(\mu - 1)/8\Theta_K, \;\; \omega_{\bar{F}} = N_c(\mu + 1)/8\Theta_K,$$

$$\mu = \sqrt{1 + \bar{m}_K^2/M_0^2} \simeq 1 + \frac{\bar{m}_K^2}{2M_0^2}, \quad M_0^2 = N_c^2/(16\Gamma\Theta_K) \sim N_c^0, \quad \mu \sim N_c^0.$$

The hyperfine splitting correction depending on hyperfine splitting constants c and $\bar{c}$, and "strange isospin" $I_S = |S|/2$ equals

$$\Delta M_{HFS} = \frac{J(J+1)}{2\Theta_\pi} + \frac{(c_S - 1)[J(J+1) - I(I+1)] + (\bar{c}_S - c_S)I_S(I_S+1)}{2\Theta_\pi}$$

and is small at large N_c, $\sim 1/N_c$, and for heavy flavors. For anti-flavor (positive strangeness) certain changes should be done: $\omega_S \to \omega_{\bar{S}}$ and $c_S \to c_{\bar{S}}$ in the last term.

We obtain for the total splitting of the "octet" in BSM and in RRM [9, 10]:

$$\Delta_{tot}(''8'', BSM) = \tilde{m}_K^2 \frac{\Gamma}{N_c}\left(2 - \frac{4}{N_c}\right), \quad \Delta_{tot}(''8'', RRM) = \tilde{m}_K^2 \frac{\Gamma}{N_c}\left(2 - \frac{16}{N_c}\right).$$

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For the "decuplet"

$$\Delta_{tot}(\text{"10"}, BSM) = \tilde{m}_K^2 \frac{\Gamma}{N_c}\left(3 - \frac{15}{N_c}\right), \quad \Delta_{tot}(\text{"10"}, RRM) = \tilde{m}_K^2 \frac{\Gamma}{N_c}\left(3 - \frac{33}{N_c}\right).$$

In BSM the mass splittings are bigger than in RRM. The conclusion is that the RRM used for prediction of pentaquarks [2, 4] is different from the BSM model, used in [11] to disavow the pentaquark Θ^+. The case of exotic $S = +1$ Θ hyperons is especially interesting. In BSM we obtain

$$\delta M^{BSM}_{\Theta_0, J=1/2} = \bar{m}_K^2 \Gamma\left(\frac{3}{N_c} - \frac{9}{N_c^2}\right), \qquad \delta M^{RRM}_{\Theta_0, J=1/2} = \bar{m}_K^2 \Gamma\left(\frac{3}{N_c} - \frac{27}{N_c^2}\right),$$

and again considerable difference takes place, similar for the $\Theta_1(J = 3/2) \in \{27\}$ and $\Theta_2(J = 5/2) \in \{35\}$, see table 1 and [10].

	$\{8\}$	$\{10\}$	$\{\overline{10}\}$	$\{27\}$	$\{35\}$
m_s^{RRM}	$1-8/N_c$	$1-11/N_c$	–	–	–
m_s^{BSM}	$1-2/N_c$	$1-5/N_c$	–	–	–
$m_{\bar{s}}^{RRM}$	–	–	$1-15/N_c$	$1-13/N_c$	$1-11/N_c$
$m_{\bar{s}}^{BSM}$	–	–	$1-9/N_c$	$1-7/N_c$	$1-5/N_c$

Table 1. First terms of the $1/N_c$ expansion for the effective strange quark and antiquark masses within different $SU(3)$ multiplets, in units $\bar{m}_K^2 \Gamma/N_c$. Empty spaces are left in the cases of theoretical uncertainty. The assumption concerning strange quarks/antiquarks sea should be kept in mind.

The addition of the term to the BSM result, possible due to normal ordering ambiguity present in BSM (I.Klebanov, VBK, 2005), $\Delta M_{BSM} = -6\bar{m}_K^2 \frac{\Gamma}{N_c^2}(2 + |S|)$ brings results of RRM and BSM in agreement - for nonexotic and exotic states. This procedure looks not quite satisfactorily: if we believe to RRM, why we need BSM at all? Anyway, RRM and BSM in its accepted form are different models.

For anti-decuplet the $SU(3)$ configuration mixing decreases slightly the total splitting, and pushes N^* and Σ^* toward higher energy. Mixing with components of the octet is important. Apparent contradiction takes place with simplest assumption of equality of masses of strange quarks and antiquarks $m(s) = m(\bar{s})$ (so called simplistic model). For decuplet mixing increases total splitting considerably, but approximate equidistancy still remains! Mixing with components of the $\{27\}$-plet is important. The states with different numbers of $q\bar{q}$ pairs can mix, and such mixing should be taken into account within the QM.

4. It is possible to make comparison of CSA results with expectations from simple quark model in *pentaquark* approximation (projection of CSM on QM). The masses m_s, $m_{\bar{s}}$ and $m(s\bar{s})$ come into play.

$\lvert\overline{10},2,0>$	$\lvert\overline{10},1,\frac{1}{2}>$	$\lvert\overline{10},0,1>$	$\lvert\overline{10},-1,\frac{3}{2}>$		
$m_{\bar{s}}+3/\Theta_K+..$	$2m_{s\bar{s}}/3+3/\Theta_K+..$	$m_s+m_{s\bar{s}}/3+3/\Theta_K$	m_s+3/Θ_K		
564	655	745	836		
600	722	825	847		
$\lvert 27,2,1>$	$\lvert 27,1,\frac{3}{2}>$	$\lvert 27,0,2>$	$\lvert 27,-1,\frac{3}{2}>$	$\lvert 27,-2,1>$	
$m_{\bar{s}}+2/\Theta_K+..$	$m_{s\bar{s}}/2+2/\Theta_K+..$	$m_s+2/\Theta_K+..$	$2m_s+2/\Theta_K$	$3m_s+2/\Theta_K$	
733	753	772	889	1005	
749	887	779	911	1048	
$\lvert 35,2,2>$	$\lvert 35,1,\frac{5}{2}>$	$\lvert 35,0,2>$	$\lvert 35,-1,\frac{3}{2}>$	$\lvert 35,-2,1>$	$\lvert 35,-3,\frac{1}{2}>$
$m_{\bar{s}}+1/\Theta_K+..$	$1/\Theta_K+..$	$m_s+1/\Theta_K+..$	$2m_s+1/\Theta_K$	$3m_s+1/\Theta_K$	$4m_s+1/\Theta_K$
1152	857	971	1084	1197	1311
1122	853	979	1107	1236	1367

Table 2. Strange quark (antiquark) masses contributions and calculation results for the baryon mass (nucleon mass is subtracted) within the RRM without and with configuration mixing. Contributions depending on Θ_π are not shown in the first line.

Simple relations can be obtained from this Table for effective $s-$quark masses:
from the total splitting of antidecuplet $[2m_s - m_{\bar{s}}]_{\overline{10}} = 247\,MeV$ $(272\,Mev$ without mixing).
from splittings within 27-plet $[m_s - m_{\bar{s}}]_{27} = 30\,Mev$ $(39\,Mev)$, $[m_s]_{27} \simeq 135\,Mev$ $(117\,Mev)$,
and from 35-plet $[m_s]_{35} = 130\,Mev$ $(114\,Mev)$, $[m_{\bar{s}}]_{35} \simeq 270\,Mev$ $(295\,Mev)$. Strong dependence of s-antiquark mass on the multiplet takes place, which certainly needs further understanding and should be considered as a challenge.

In view of theoretical uncertainties and problems, further experimental investigations could play a very important role.

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