

## Mass dependence of the deconfinement and chiral restoration critical temperatures in nonlocal SU(2) PNJL models

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In the framework of nonlocal SU(2) chiral quark models with Polyakov loop, we analyze the dependence of the deconfinement and chiral restoration critical temperatures on the explicit chiral symmetry breaking driven by the current quark mass. Our results are compared with those obtained within the standard local Polyakov-Nambu-Jona-Lasinio (PNJL) model and with lattice QCD calculations. For a wide range of pion masses, it is found that both deconfinement and chiral restoration critical temperatures turn out to be strongly entangled, in contrast with the corresponding results within the PNJL model. In addition, it is seen that the growth of the critical temperatures with the pion mass above the physical point is basically linear, with a slope parameter which is close to the existing lattice QCD estimates. On the other hand, at the mean field level one finds an early onset of the first order transition expected in the large quark mass limit.

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## 1. Formalism

Our starting point is the SU(2) quark model described in Ref. [1], a nonlocal extension of the NJL model that includes wave function renormalization (WFR) of the quark fields, and in which quarks are coupled to a background color field. We work in the mean field approximation (MFA), and use the Matsubara formalism to consider finite temperature. Within this framework the MFA thermodynamical potential reads

$$\Omega^{\text{MFA}} = -4T \sum_{c,n} \int \frac{d^3 \vec{p}}{(2\pi)^3} \ln \left[ \frac{(\rho_{n,\vec{p}}^c)^2 + M^2(\rho_{n,\vec{p}}^c)}{Z^2(\rho_{n,\vec{p}}^c)} \right] + \frac{\bar{\sigma}_1^2 + \kappa_p^2 \bar{\sigma}_2^2}{2G_S} + \mathcal{U}(\Phi, \Phi^*, T). \quad (1.1)$$

Here  $M(p) = Z(p)[m_c + \bar{\sigma}_1 g(p)]$  and  $Z(p) = [1 - \bar{\sigma}_2 f(p)]^{-1}$ , where  $g(p)$  and  $f(p)$  are nonlocal form factors, and  $\bar{\sigma}_1$  and  $\bar{\sigma}_2$  are the mean field values of scalar fields introduced after a bosonization of the fermionic theory. We have also defined  $(\rho_{n,\vec{p}}^c)^2 = [(2n+1)\pi T + \phi_c]^2 + \vec{p}^2$ , with  $\phi_{r,g} = \pm\phi_3 + \phi_8/\sqrt{3}$ ,  $\phi_b = -2\phi_8/\sqrt{3}$ , where  $\phi_3$  and  $\phi_8$  parameterize the traced PL according to

$$\Phi = \frac{1}{3} \left[ \exp\left(-\frac{2i}{\sqrt{3}} \frac{\phi_8}{T}\right) + 2 \exp\left(\frac{i}{\sqrt{3}} \frac{\phi_8}{T}\right) \cos(\phi_3/T) \right]. \quad (1.2)$$

Regarding the PL effective potential  $\mathcal{U}(\Phi, \Phi^*, T)$  we consider two alternative functional forms: A polynomial one based on a Ginzburg-Landau ansatz; and a logarithmic expression of the Haar measure associated with the SU(3) color group integration [2].

In addition, we use the same prescription as e.g. in Ref. [3] to regularize  $\Omega^{\text{MFA}}$ .

The mean field values  $\bar{\sigma}_{1,2}$  and  $\phi_{3,8}$  can be obtained from a set of four coupled ‘‘gap’’ equations that follow from the minimization of the real part of the regularized thermodynamical potential,

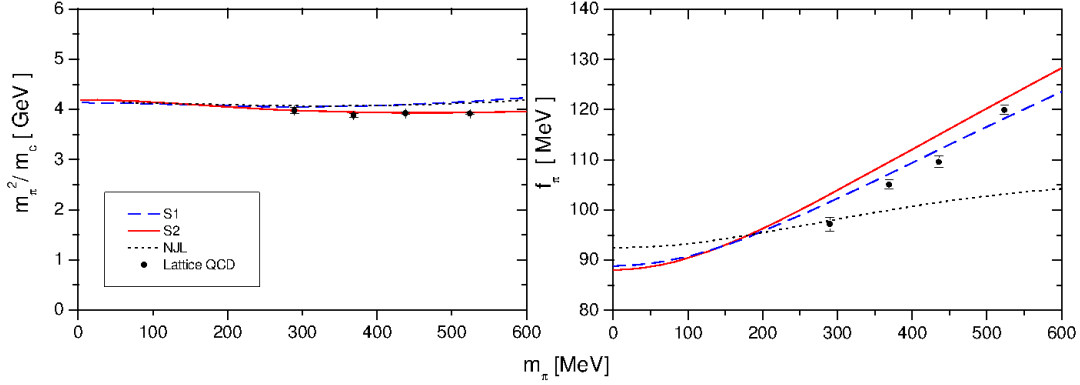
$$\frac{\partial \text{Re} [\Omega_{\text{reg}}^{\text{MFA}}]}{\partial (\bar{\sigma}_1, \bar{\sigma}_2, \phi_3, \phi_8)} = 0. \quad (1.3)$$

Once the mean field values are obtained, the behavior of other relevant quantities as functions of  $T$  can be determined. We concentrate in particular in the chiral quark condensate  $\langle \bar{q}q \rangle = \partial \Omega_{\text{reg}}^{\text{MFA}} / \partial m_c$ , which together with the modulus of the Polyakov loop  $|\Phi|$  will be taken as order parameters of the chiral restoration and deconfinement transitions, respectively. The associated susceptibilities will be defined as  $\chi_{\text{cond}} = d\langle \bar{q}q \rangle / dT$  and  $\chi_{\text{PL}} = d|\Phi| / dT$ .

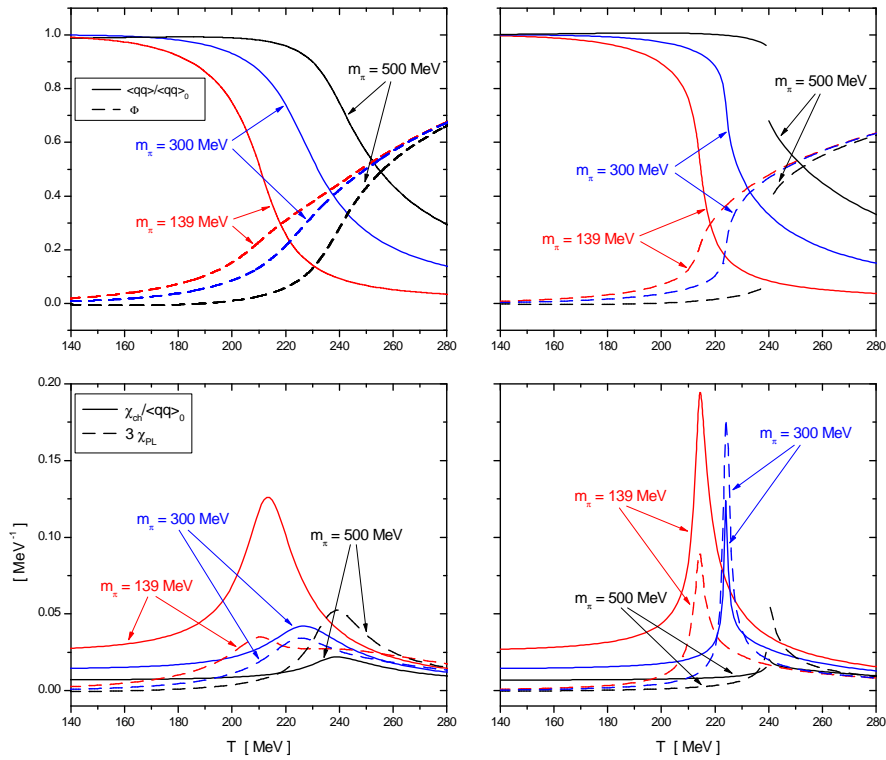
In order to fully specify the model under consideration we have to fix the model parameters,  $G_S$ ,  $\kappa_p$  and  $m_c$ , as well as the form factors  $g(q)$  and  $f(q)$  that characterize the nonlocal interactions. Here we consider the parameter sets 1 and 2, which correspond to sets B and C from Ref. [4].

## 2. Results

We want to study the dependence of nPNJL model predictions on the amount of explicit chiral symmetry breaking. This can be addressed by varying the current quark mass  $m_c$ , while keeping the rest of the model parameters fixed at their values at the physical point. As a first step we analyze the corresponding behavior of the pion mass and decay constant at vanishing temperature, in comparison with that obtained in the local NJL model and in lattice QCD (Fig. 1). While lattice



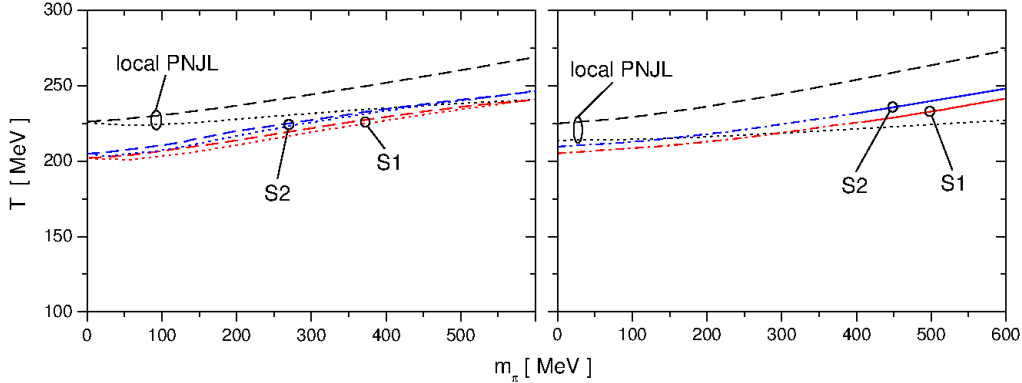
**Figure 1:**  $m_\pi^2/m_c$  (left) and  $f_\pi$  (right) at  $T = 0$  in local and nonlocal models. Lattice results from Ref. [5].



**Figure 2:** Order parameters (top) and susceptibilities (bottom) as functions of  $T$  for some values of the pion mass (Set 2). Left (right) panels correspond to the polynomial (logarithmic) Polyakov potential.

results for the ratio  $m_\pi^2/m_c$  are in agreement with both local and nonlocal models, those for  $f_\pi$  show a significant increase with  $m_\pi$  that can be reproduced only by the predictions of nonlocal models.

We turn to analyze the mass dependence of the critical temperatures at  $\mu = 0$ . We have found that, contrary to the case of the local PNJL model, in nPNJL models both  $T_c$  turn out to be strongly



**Figure 3:** Critical temperatures as functions of  $m_\pi$  for PNJL and nPNJL models, considering polynomial (left) and logarithmic (right) PL potentials. Dashed (dotted) lines correspond to chiral restoration (deconfinement)  $T_c$ . For the nPNJL models with a logarithmic potential, both transitions occur at the same  $T$ , and they can be of first order (solid lines) or proceed as a smooth crossover (dashed-dotted lines).

entangled for the considered range of  $m_\pi$  (Fig. 2). In addition, the growth of  $T_c$  with  $m_\pi$  above the physical point is basically linear (Fig. 3), with a slope parameter in the range of  $0.06 - 0.07$ , which is close to existing lattice QCD estimates,  $\sim 0.05$  [6]. This can be contrasted with the results obtained within pure chiral models, where one finds a strong increase of the chiral restoration temperature with  $m_\pi$  [7, 8]. For example, within the model of Ref.[8] one gets a value of  $0.243$ .

### 3. Conclusions

We found that nPNJL predictions as functions of the amount of chiral symmetry breaking show good agreement with LQCD results for pion properties at  $T = 0$  as well as for the growth of  $T_c$  with  $m_\pi$ . On the other hand, particularly in the case of the logarithmic PL potential, this calculation leads to a too early onset of the first order transition known to exist in the large quark mass limit. We expect that the development of a fully nonperturbative scheme to account for meson fluctuations in nonlocal models might help to solve this problem.

### References

- [1] G. A. Contrera, M. Orsaria and N. N. Scoccola, Phys. Rev. D **82**, 054026 (2010)
- [2] C. Ratti *et al.* Phys. Rev. D **73**, 014019 (2006) ; S. Roessner *et al.* Phys. Rev. D **75**, 034007 (2007)
- [3] D. Gomez Dumm and N. N. Scoccola, Phys. Rev. C **72**, 014909 (2005)
- [4] S. Noguera, N. N. Scoccola, Phys. Rev. D **78**, 114002 (2008)
- [5] J. Noaki *et al.* [JLQCD and TWQCD Collab.], Phys. Rev. Lett. **101**, 202004 (2008)
- [6] F. Karsch *et al.* Nucl. Phys. B **605**, 579 (2001); V. G. Bornyakov *et al.* Phys. Rev. D **82**, 014504 (2010)
- [7] J. Berges *et al.* Phys. Rev. D **59**, 034010 (1999) ; J. Braun *et al.* Phys. Rev. D **73**, 074010 (2006)
- [8] A. Dumitru, D. Roder and J. Ruppert, Phys. Rev. D **70**, 074001 (2004)