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Chiral expansion of the three-nucleon forces

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In this contribution I briefly review a current status of high-order calculations of chiral three-nucleon forces. I present our recent results on the longest-range three-nucleon force which has been calculated up to next-to-next-to-next-to-next-to-leading order in the chiral expansion. Several low-energy constants from the single-nucleon sector are fitted to pion-nucleon scattering data which we had to reanalyze utilizing the same power counting in single-nucleon and few-nucleon sectors. We observe a good convergence of chiral expansion for this topology. Complete next-to-next-to-next-to-next-to-leading order results for the three-nucleon force and their numerical implementation are under way.

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1. Introduction

A quantitative understanding of nuclear spectra and reactions requires, first of all, a quantitative understanding of how nucleons interact with each other. Quantum chromodynamics (QCD) is believed to be a fundamental theory of strong interactions which controls the dynamics between nucleons. In this picture nucleons are composite particles built out of quarks and gluons. So one way to clarify the question about few-nucleon interactions is to put quarks and gluons, the fundamental degrees of freedom in QCD, on the lattice and perform nuclear Monte Carlo simulations on supercomputers. There is indeed a lot of activity along this line (see e.g. Refs. [1, 2]). Few-nucleon lattice QCD simulations are in general very time consuming and have sofar been performed with unphysical quark masses. The field is strongly progressing, so we may hope to see in the future simulations of e.g. nucleon-nucleon (NN) scattering lengths or deuteron binding energy at physical quark masses.

An alternative approach to few-nucleon interactions is to use chiral perturbation theory which is valid in the low-energy sector where all momenta are well below the chiral scale $\Lambda_\chi \sim 1$ GeV. Due to spontaneous (and small explicit) breaking of chiral symmetry, one of the prominent symmetries of QCD, long-range part of nuclear forces is dominated by Goldstone boson dynamics. Since the interaction between Goldstone bosons vanishes when their four momenta vanish, one can make a perturbative expansion in small momenta and masses[1] of Goldstone bosons (identified with pions in the SU(2) sector) divided by the hard chiral-symmetry-breaking scale Λ_χ. Already two decades ago Weinberg [3] suggested in his seminal papers to calculate an effective few-nucleon potential by using chiral perturbation theory. He showed that it is possible to define an effective potential (scheme dependent quantity) which can be systematically calculated order by order in chiral effective field theory. The chiral potential serves as an input in numerical few- and many-body simulations of low-energy observables. This path has beed intensively followed by several groups, so that chiral NN potential was calculated up to next-to-next-to-next-to-leading order (N^{3}LO) in the chiral expansion (see Refs. [4, 5, 6, 7, 8, 9] for various review articles). At this order 24 [2] unknown NN low-energy constants (LECs) were fitted to Nijmegen data. At N^{3}LO NN phase shifts are accurately described up to $E_{\rm lab} \simeq 200$ MeV [10, 11]. In the three-nucleon sector the situation is less clear. The chiral three-nucleon force (3NF) starts to contribute at N^{2}LO. In all numerical implementations only the leading N^{2}LO 3NF has so far been considered. Already at this order many observables in the three-nucleon sector are very well described. There are, however, some observables (like e.g. the nucleon vector analyzing power A_y in elastic neutron-deuteron scattering below 30 MeV laboratory energy for the incident nucleon) which is not well described at this order. The most natural way to deal with this issue is to increase the order of chiral expansion to N^{3}LO for 3NF. This will make a force as precise (from chiral effective field theory point of view) as in the two-nucleon case and will hopefully resolve some long standing problems. The purpose of this manuscript is to review a current status of the construction of N^{3}LO and partly (only the longest-range part) of the N^{4}LO 3NF. In the next section I will discuss the N^{3}LO 3NF and motivate the need to go to at least N^{4}LO in the framework without explicit Δ-isobar degrees of freedom. In Sec. 2 I will discuss the longest-range part of the 3NF at N^{4}LO. Since at this order, pion-nucleon scattering

[1] Goldstone bosons become massive due to small explicit breaking of chiral symmetry.

[2] The number corresponds to the isospin limit.

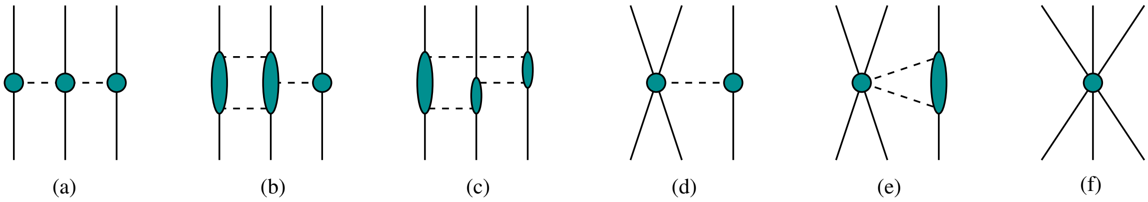

Figure 1: Various topologies contributing to the 3NF up to and including N^{4}LO: two-pion (2π) exchange (a), two-pion-one-pion (2π-1π) exchange (b), ring (c), one-pion-exchange-contact (d), two-pion-exchange-contact (e) and purely contact (f) diagrams. Solid and dashed lines represent nucleons and pions, respectively. Shaded blobs represent the corresponding amplitudes.

up to q^4-order in the chiral expansion appears as a subprocess, we need to reconsider pion-nucleon scattering within the same power-counting which has been used in the calculations of few-nucleon forces (Q/m-corrections are counted as two chiral powers.[3] Here, m denotes the nucleon-mass and Q a small three-momentum of the nucleon). This requires a refit of some LECs in the one-nucleon sector which contribute to the 3NF at N^{4}LO. All this is discussed in Sec. 4. Finally, in Sec. 5, I compare the longest-range contributions at N^{2}LO , N^{3}LO and N^{4}LO and try to make a qualitative statement about convergence of their chiral expansion. More quantitative statements are only possible after the full numerical analysis of three-nucleon observables is performed.

2. Chiral three-nucleon forces up to N^{3}LO

The structure of the 3NF up to N^{4}LO is visualized in Fig. 1 and can be written as

$$V_{3\mathrm{N}} = V_{2\pi} + V_{2\pi-1\pi} + V_{\mathrm{ring}} + V_{1\pi-\mathrm{cont}} + V_{2\pi-\mathrm{cont}} + V_{\mathrm{cont}}. \tag{2.1}$$

While the $2\pi - 1\pi$, ring and two-pion-exchange-contact (2π-cont) topologies start to contribute at N^{3}LO, the two-pion-exchange (2π), one-pion-exchange-contact (1π-cont) and contact interaction (cont) graphs already appear at N^{2}LO. It is important to stress that the N^{3}LO contributions do not involve any unknown LECs. The corresponding parameter-free expressions can be found in Refs. [13, 14], see also Ref. [15]. Another interesting feature of the N^{3}LO 3NF corrections is their rather rich isospin-spin-momentum structure emerging, especially, from the ring topology (c) in Fig. 1. This is in contrast with the quite restricted operator structure of the two-pion exchange 3NF whose effects in the three-nucleon continuum have already been extensively explored. It is, therefore, very interesting to study the impact of the novel structures in the 3NF on nucleon-deuteron scattering and the properties of light nuclei, especially in connection with the already mentioned puzzles. On the other hand, one may ask whether the resulting (leading) contributions to the structure functions accompanying the novel operator structures in the 3NF already allow for their decent description. Stated differently, the question is whether the lowest-nonvanishing-order contributions from the 2π-1π and ring-topologies are already converged or at least provide

[3]Note that usually in the one nucleon sector Q/m is treated as one chiral power [12].

a reasonable approximation to the converged result. There is a strong reason to believe that this is not going to be the case since the contributions due to intermediate $\Delta(1232)$ excitations are not yet taken into account for these topologies at N^{3}LO. In the standard chiral EFT formulation based on pions and nucleons as the only explicit degrees of freedom, effects of the Δ (and heavier resonances as well as heavy mesons) are hidden in the (renormalized) values of certain LECs starting from the subleading effective Lagrangian. The major part of the Δ contributions to the nuclear forces is taken into account in the Δ-less theory through resonance saturation of the LECs $c_{3,4}$ accompanying the subleading $\pi\pi NN$ vertices [16, 17, 18, 19, 20] (see, however, the last two references for some examples of the Δ-contributions that go beyond the saturation of $c_{3,4}$). These LECs turn out to be numerically large and are known to be driven by the Δ isobar [21, 18]. As a consequence, one observes a rather unnatural convergence pattern in the chiral expansion of the two-pion exchange NN potential $V_{\rm NN}^{2\pi}$ with by far the strongest contribution resulting from the formally subleading triangle diagram proportional to c_3 [22]. The (formally) leading contribution to $V_{\rm NN}^{2\pi}$ does not provide a good approximation to the potential so that one needs to go to (at least) the next-higher order in the chiral expansion and/or to include the Δ isobar as an explicit degree of freedom [18]. The situation with the 2π-1π and ring topologies in the 3NF is similar. Based on the experience with the NN potential, one expects significant contributions due to intermediate Δ excitations, see also the discussion in Ref. [23]. For the ring topology, this expectation is confirmed by the phenomenological study of Ref. [24]. In order to include effects of the Δ-isobar one needs

- either to go to (at least) next-to-next-to-next-to-next-to-leading order (N^{4}LO) in the standard Δ-less EFT approach,
- or to include the Δ-isobar as an explicit degree of freedom.

It should be understood that both strategies outlined above are, to some extent, complementary to each other. In particular, N^{3}LO contributions in the Δ-less theory only take into account effects due to single Δ-excitation but not due to the double and triple Δ-excitations (whose inclusion in the Δ-less approach would require the calculation at even higher orders). These effects are taken into account already at N^{3}LO in the Δ-full approach. On the other hand, there are also contributions not related to Δ-excitations which are included/absent in the Δ-less approach at N^{4}LO/Δ-full theory at N^{3}LO. It remains to be seen which strategy will turn out to be most efficient.

Here we concentrate on the Δ-less approach and discuss the longest-range contribution to the 3NF [25] given by the two-pion exchange potential $V_{2\pi}$. It has a very restrictive general spin-isospin-momentum structure in the static limit

$$V_{2\pi} = \frac{\vec{\sigma}_1\cdot\vec{q}_1\,\vec{\sigma}_3\cdot\vec{q}_3}{[q_1^2+M_\pi^2]\,[q_3^2+M_\pi^2]}\Big(\tau_1\cdot\tau_3\,\mathscr{A}(q_2)+\tau_1\times\tau_3\cdot\tau_2\,\vec{q}_1\times\vec{q}_3\cdot\vec{\sigma}_2\,\mathscr{B}(q_2)\Big)\,, \tag{2.2}$$

where $\vec{\sigma}_i$ denote the Pauli spin matrices for the nucleon i and $\vec{q}_i=\vec{p}_i{}'-\vec{p}_i$, with $\vec{p}_i{}'$ and $\vec{p}_i$ being the final and initial momenta of the nucleon i. Here and in what follows, we use the notation: $q_i\equiv|\vec{q}_i|$. The quantities $\mathscr{A}(q_2)$ and $\mathscr{B}(q_2)$ in Eq. (2.2) are scalar functions of the momentum transfer q_2 of the second nucleon whose explicit form is derived within the chiral expansion. Unless stated otherwise, the expressions for the 3NF results are always given for a particular choice of the nucleon labels. The complete result can then be found by taking into account all possible permutations of the nucleons

$$V_{3N}^{\rm full} = V_{3N} + 5 \text{ permutations}\,. \tag{2.3}$$

We now briefly consider the first two terms in the chiral expansion of the functions $\mathscr{A}(q_2)$ and $\mathscr{B}(q_2)$. The leading contributions arise at N^{2}LO which corresponds to the order Q^3 relative to the leading contribution to the nuclear Hamiltonian and have the form

$$\mathscr{A}^{(3)}(q_2) = \frac{g_A^2}{8F_\pi^4}\Big((2c_3 - 4c_1)M_\pi^2 + c_3 q_2^2\Big)\,, \qquad \mathscr{B}^{(3)}(q_2) = \frac{g_A^2 c_4}{8F_\pi^4}\,, \tag{2.4}$$

where g_A, F_π and M_π denote to the physical values of the nucleon axial vector coupling, pion decay constant and pion mass, respectively, and the superscripts of $\mathscr{A}$ and $\mathscr{B}$ refer to the powers of the soft scale Q. The first corrections at N^{3}LO read [13, 15]:

$$\begin{aligned}
\mathscr{A}^{(4)}(q_2) &= \frac{g_A^4}{256\pi F_\pi^6}\Big[A(q_2)\left(2M_\pi^4 + 5M_\pi^2 q_2^2 + 2q_2^4\right) + \left(4g_A^2+1\right)M_\pi^3 + 2\left(g_A^2+1\right)M_\pi q_2^2\Big]\,,\\
\mathscr{B}^{(4)}(q_2) &= -\frac{g_A^4}{256\pi F_\pi^6}\Big[A(q_2)\left(4M_\pi^2 + q_2^2\right) + (2g_A^2+1)M_\pi\Big]\,,
\end{aligned} \tag{2.5}$$

where the loop function $A(q)$ is defined as:

$$A(q) = \frac{1}{2q}\arctan\frac{q}{2M_\pi}\,. \tag{2.6}$$

Notice that the leading-loop contributions to the 2π-exchange topology do not contain logarithmic ultraviolet divergences and, as explained in Ref. [13], turn out to be independent on the LECs d_i entering $\mathscr{L}_{\pi N}^{(3)}$. As usual, we denote by $\mathscr{L}_{\pi N}^{(n)}$ a pion-nucleon Lagrangian at the chiral order n. Explicit expressions in the heavy-baryon formulation for $n \le 4$ can be found in Ref. [27, 28]. At both N^{2}LO and N^{3}LO, all LECs in the effective Lagrangian entering the expressions for the 3NF – including g_A and F_π – can be simply replaced by their physical values.

We emphasize that relativistic corrections to $V_{2\pi}$ have a richer structure than the one given in Eq. (2.2). The explicit form of the $1/m$-corrections to $V_{2\pi}$ at N^{3}LO can be found in Ref. [14], see also [26] for an early work.

3. The two-pion-exchange 3NF at N^{4}LO

We now turn to the sub-subleading contributions to the 2π-exchange 3NF at order Q^5 (N^{4}LO). The final, renormalized N^{4}LO contributions to the functions $\mathscr{A}$ and $\mathscr{B}$ in Eq. (2.2) have the form:

$$\begin{aligned}
\mathscr{A}^{(5)}(q_2) &= \frac{g_A}{4608\pi^2 F_\pi^6}\Big[M_\pi^2 q_2^2\big(F_\pi^2\left(2304\pi^2 g_A(4\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36}) - 2304\pi^2\bar{d}_{18}c_3\right)\\
&\quad + g_A(144c_1 - 53c_2 - 90c_3)\big) + M_\pi^4\left(g_A\left(72\left(64\pi^2\bar{l}_3 + 1\right)c_1 - 24c_2 - 36c_3\right)\right.\\
&\quad + \left.F_\pi^2\left(4608\pi^2\bar{d}_{18}(2c_1 - c_3) + 4608\pi^2 g_A(2\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{36} - 4\bar{e}_{38})\right)\right)\\
&\quad + q_2^4\left(2304\pi^2\bar{e}_{14}F_\pi^2 g_A - 2g_A(5c_2 + 18c_3)\right)\Big]\\
&\quad - \frac{g_A^2}{768\pi^2 F_\pi^6}L(q_2)\left(M_\pi^2 + 2q_2^2\right)\left(4M_\pi^2(6c_1 - c_2 - 3c_3) + q_2^2(-c_2 - 6c_3)\right)\,,
\end{aligned}$$

$$\mathscr{B}^{(5)}(q_2) = -\frac{g_A}{2304\pi^2 F_\pi^6}\Big[M_\pi^2\left(F_\pi^2\left(1152\pi^2\bar{d}_{18}c_4 - 1152\pi^2 g_A(2\bar{e}_{17}+2\bar{e}_{21}-\bar{e}_{37})\right)\right.$$
$$\left. + \; 108 g_A^3 c_4 + 24 g_A c_4\right) + q_2^2\left(5 g_A c_4 - 1152\pi^2\bar{e}_{17}F_\pi^2 g_A\right)\Big]$$
$$+ \; \frac{g_A^2 c_4}{384\pi^2 F_\pi^6} L(q_2)\left(4M_\pi^2 + q_2^2\right), \tag{3.1}$$

where the loop function $L(q)$ is defined according to

$$L(q) = \frac{\sqrt{q^2+4M_\pi^2}}{q}\log\frac{\sqrt{q^2+4M_\pi^2}+q}{2M_\pi}\,. \tag{3.2}$$

The expressions for the 2π-exchange 3NF up to N^{4}LO depend on a number of low-energy constants. The following notation has been used for the LECs: $\bar{d}_i$ are LECs from $\mathscr{L}_{\pi N}^{(3)}$ and $\bar{e}_i$ are LECs from $\mathscr{L}_{\pi N}^{(4)}$. The LECs c_i, $\bar{d}_i$ and $\bar{e}_i$ can be most naturally determined from pion-nucleon scattering (at least) at the subleading-loop order (i.e. Q^4). The heavy-baryon analyses of pion-nucleon scattering at orders Q^3 and Q^4 can be found in Refs. [12, 29, 30], see also Refs. [31, 32, 33] for the calculations within the manifestly covariant framework, Ref. [34] for a related calculation which extends chiral EFT to higher energies by employing constraints set by causality and unitarity and Ref. [35] for a recent review on baryon chiral perturbation theory. Unfortunately, we cannot use the values of the LECs obtained in these studies since we use a different counting scheme for the nucleon mass in the few-nucleon sector, namely $Q/m \sim Q^2/\Lambda_\chi^2$ [3] rather then $m \sim \Lambda_\chi$ as used in the single-nucleon sector, see [4] for an extended discussion. In the next section, I briefly discuss the analysis of pion-nucleon scattering at order Q^4 in the heavy-baryon approach, see Ref. [25] for more details.

4. πN scattering at order Q^4

In the center-of-mass system (cms), the amplitude for the reaction $\pi^a(q_1)+N(p_1)\to\pi^b(q_2)+N(p_2)$ with $p_{1,2}$ and $q_{1,2}$ being the corresponding four-momenta and a,b referring to the pion isospin quantum numbers, takes the form:

$$T_{\pi N}^{ba} = \frac{E+m}{2m}\left(\delta^{ba}\Big[g^+(\omega,t)+i\vec{\sigma}\cdot\vec{q}_2\times\vec{q}_1\,h^+(\omega,t)\Big] + i\varepsilon^{bac}\tau^c\Big[g^-(\omega,t)+i\vec{\sigma}\cdot\vec{q}_2\times\vec{q}_1\,h^-(\omega,t)\Big]\right). \tag{4.1}$$

Here, $\omega = q_1^0 = q_2^0$ is the pion cms energy, $E_1 = E_2 \equiv E = (\vec{q}^{\,2}+m^2)^{1/2}$ the nucleon energy and $\vec{q}_1^{\,2} = \vec{q}_2^{\,2} \equiv \vec{q}^{\,2} = ((s-M_\pi^2-m^2)^2 - 4m^2M_\pi^2)/(4s)$. Further, $t = (q_1-q_2)^2$ is the invariant momentum transfer squared while s denotes the total cms energy squared. The quantities $g^\pm(\omega,t)$ ($h^\pm(\omega,t)$) refer to the isoscalar and isovector non-spin-flip (spin-flip) amplitudes. In Ref. [25], we recalculated these amplitudes in chiral perturbation theory up to the order Q^4 using the power counting which assigns two chiral powers to Q/m. The recalculated amplitudes depend on 13 independent (linear combinations of the) low-energy constants to be fixed from a fit to the data, namely $c_{1,2,3,4}$, $\bar{d}_1+\bar{d}_2$, $\bar{d}_3$, $\bar{d}_5$, $\bar{d}_{14}-\bar{d}_{15}$ and $\bar{e}_{14,15,16,17,18}$.

The fit can be most conveniently performed in the partial wave basis using the available partial wave analyses. In order to estimate a possible uncertainty of the extracted parameters, we considered two different partial wave analyses in our fitting procedure, namely the one of Ref. [36] by the group at the George Washington University, to be referred as GW, and the Karlsruhe-Helsinki analysis of Ref. [37]), to be referred as KH. The energy range of the data fitted corresponds to the πN laboratory momenta $p_{\rm Lab} < 150$ MeV/c. At higher energies the convergence of the chiral expansion becomes doubtful. We follow the strategy which is similar to the one utilized in Ref. [12] and assign the same relative error to all empirical data points.

The partial wave amplitudes $f^{\pm}_{l\pm}(s)$, where l refers to the orbital angular momentum and the subscript '$\pm$' to the total angular momentum ($j = l \pm s$), are given in terms of the invariant amplitudes via

$$f^{\pm}_{l\pm}(s) = \frac{E+m}{16\pi\sqrt{s}} \int_{-1}^{+1} dz \left[g^{\pm} P_l(z) + \vec{q}^{\,2} h^{\pm} \left(P_{l\pm 1}(z) - z P_l(z)\right)\right] ,$$

where $z = \cos(\theta)$ is the angular variable ($t = 2\vec{q}^{\,2}(z-1)$). The amplitude in the isospin basis are related to $f^{\pm}_{l\pm}$ as follows

$$f^{1/2}_{l\pm} = f^{+}_{l\pm} + 2 f^{-}_{l\pm} , \qquad f^{3/2}_{l\pm} = f^{+}_{l\pm} - f^{-}_{l\pm} .$$

The phase shift for a partial wave amplitude with isospin I is obtained using the following unitarization prescription[4] [12]:

$$\delta^{I}_{l\pm}(s) = \arctan\left(|\vec{q}\,| \, \Re\, f^{I}_{l\pm}(s) \right) , \tag{4.2}$$

which reflects the absence of inelasticity below the two-pion production threshold.

We performed a combined fit for all s-, p-, and d-waves since d-waves are the highest partial waves where the order-Q^4 counter terms contribute. The results of the fits using the GW partial wave analysis are visualized in Fig. 2. The KH partial wave analysis leads to a similar fit. In the figure, we show the full, order-Q^4 results (solid curves) as well as the phase shifts calculated up to the order Q^3 (dashed curves) and Q^2 (dashed-dotted curves) using the same values of LECs (from the order-Q^4 fit) in all curves. In the fitted region (from threshold up to $p_{\rm Lab} = 150$ MeV/c), a good description of the data is achieved. As one would expect, the convergence pattern when going from Q^2 to Q^4 is getting worse with increasing the pion momenta. Interestingly, the d-waves are rather well reproduced already at the order Q^3 where there are no counter terms or other contributions depending on free parameters. Both the tree-level and finite loop contributions are important for those four partial waves. Our results for the phase shifts are similar and of a similar quality as the ones reported in Ref. [29].

We finally turn to the discussion of the extracted parameters. The obtained values of the low-energy constants are collected in Table 1. As one can see from the table, the LECs c_i and $\bar{d}_i$ come out rather similar for the two partial wave analyses. The difference does not exceed 30% except for the LECs c_1 and $\bar{d}_5$ which are, however, considerably smaller than the other c_i's and $\bar{d}_i$'s, respectively. Also the LECs $\bar{e}_{14}$ and $\bar{e}_{17}$ are rather stable. These are the only counter terms contributing to d-waves, which is why these two constants are strongly constrained by the threshold behavior of the d-wave phase shifts. In contrast, the other $\bar{e}_i$'s are very sensitive to the energy dependence

[4] It should be understood that this unitarization prescription goes, strictly speaking, beyond the chiral power counting. The resulting model dependence is, however, very small due to the smallness of the phase shifts with the only exception of the P_{33} partial wave, see [38] for a related discussion.

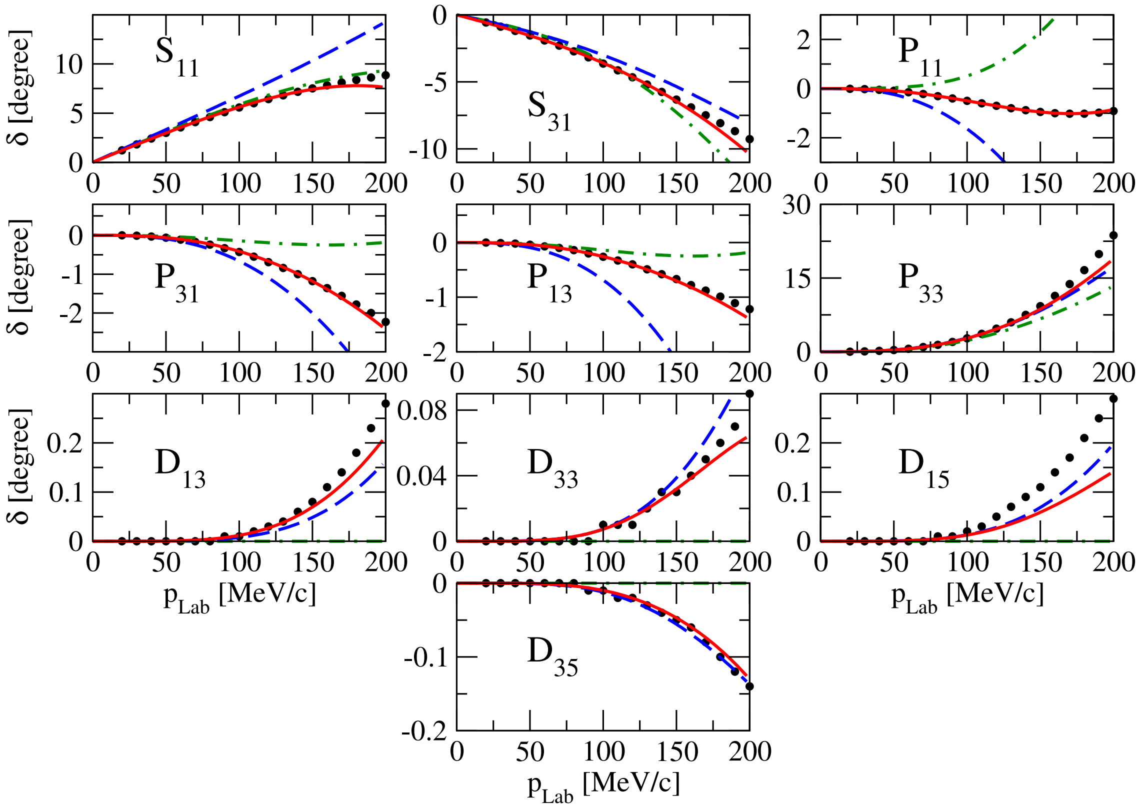

Figure 2: Results of the fit for πN s, p and d-wave phase shifts using the GW partial wave analysis of Ref. [36]. The solid curves correspond to the full Q^4 results, the dashed curves to the order-Q^3 results, and the dashed-dotted curves to the order-Q^2 calculation.

of the s- and p-wave amplitudes and, therefore, vary strongly from one analysis to another. Notice, however, that all extracted constants are of a natural size except for the combination $\bar{d}_{14} - \bar{d}_{15}$ and $\bar{e}_{15}$ which appear to be somewhat large.

We stress that the values for $c_{1,3,4}$ obtained from the fit to the KH partial wave analysis are in an excellent agreement with the ones determined at order Q^3 by using chiral perturbation theory inside the Mandelstam triangle [39]. It is also worth mentioning that the values of $c_{3,4}$ are in a good agreement with the ones determined from the new partial wave analysis of proton-proton and neutron-proton scattering data of Ref. [40].

5. Results for the two-pion exchange 3NF

With all relevant LECs being determined from pion-nucleon scattering, we are now in the position to analyze the convergence of the chiral expansion for the two-pion exchange 3NF. In Fig. 3, we show the results for the functions $\mathscr{A}(q_2)$ and $\mathscr{B}(q_2)$ for small values of the momentum transfer q_2, $q_2 < 300$ MeV at various orders in the chiral expansion. More precisely, we plot $\mathscr{A}^{(3)}$, $\mathscr{A}^{(3)} + \mathscr{A}^{(4)}$ and $\mathscr{A}^{(3)} + \mathscr{A}^{(4)} + \mathscr{A}^{(5)}$ as well as $\mathscr{B}^{(3)}$, $\mathscr{B}^{(3)} + \mathscr{B}^{(4)}$ and $\mathscr{B}^{(3)} + \mathscr{B}^{(4)} + \mathscr{B}^{(5)}$ using

	c_1	c_2	c_3	c_4	$\bar{d}_1+\bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14}-\bar{d}_{15}$
fit to GW, Ref. [36]	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26
fit to KH, Ref. [37]	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02

	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
fit to GW, Ref. [36]	1.75	-5.80	1.76	-0.58	0.96
fit to KH, Ref. [37]	1.52	-10.41	6.08	-0.37	3.26

Table 1: Low-energy constants obtained from a fit to the empirical s, p- and d-wave pion-nucleon phase shifts using partial wave analysis of Ref. [36] and of Ref. [37]. Values of the LECs are given in GeV^{-1}, GeV^{-2} and GeV^{-3} for the c_i, $\bar{d}_i$ and $\bar{e}_i$, respectively.

the values of the LECs c_i, $\bar{d}_i$ and $\bar{e}_i$ determined from the order-Q^4 fit to the KH and GW partial wave analyses as described in the previous section. We use here the same, fixed values for the LECs c_i (and $\bar{d}_i$) listed in Table 1 at all orders and adopt the same conventions regarding the LECs as in the case of pion-nucleon scattering. Notice that $\mathscr{A}^{(5)}$ and $\mathscr{B}^{(5)}$ do not depend on the LECs $\bar{e}_{15,16,18}$ which are very sensitive to a particular choice of the partial wave analysis in pion-nucleon scattering, see Table 1. The relevant LECs $\bar{e}_{14,17}$ are, on the contrary, rather stable as discussed in the previous section.

One observes a very good convergence for the function $\mathscr{A}$ with the subleading-order result (i.e. N^{3}LO) being very close to the one at N^{4}LO. It is also comforting to see that both partial wave analyses lead to similar results for this quantity. The dependence on the input for pion-nucleon phase shifts for $\mathscr{A}$ is bigger than the changes from N^{3}LO to N^{4}LO which can serve as a (conservative) estimation of the theoretical uncertainty at N^{4}LO. The convergence for the function $\mathscr{B}$ is somewhat slower with the shift from N^{3}LO to N^{4}LO being of the order of $\sim 30\%$. Also the difference between the two partial wave analyses of the order of $\sim 20\%$ is larger than for the function $\mathscr{A}$. It should be understood that an accurate description of the low-energy pion-nucleon scattering data at different orders does not automatically guarantee a good convergence of the chiral expansion for $\mathscr{A}$ and $\mathscr{B}$. In particular, these quantities do not depend on the LECs $\bar{d}_i$ (to the order considered) which contribute to πN phase shifts. Thus, the observed reasonable convergence for the 2π-exchange 3NF is a highly non-trivial test of the theoretical approach.

6. Summary

In this proceeding I discussed the convergence of the chiral expansion of the longest-range three-nucleon force given by the two-pion-exchange topology (a) in Fig. 1. This part of the three-nucleon force has been calculated up to next-to-next-to-next-to-next-to-leading order (N^{4}LO) in the chiral expansion as reported in Ref. [25]. At this order, there appear various low-energy constants from the single-nucleon sector which can be fitted to pion-nucleon scattering data. Since the standard power counting in the single-nucleon sector slightly differs from the one adopted in NN sector, a reanalysis of pion-nucleon scattering up to Q^4 was necessary. We used two different partial wave analyses of Ref. [36] and [37]) to fit the relevant low-energy constants. Both fits lead to

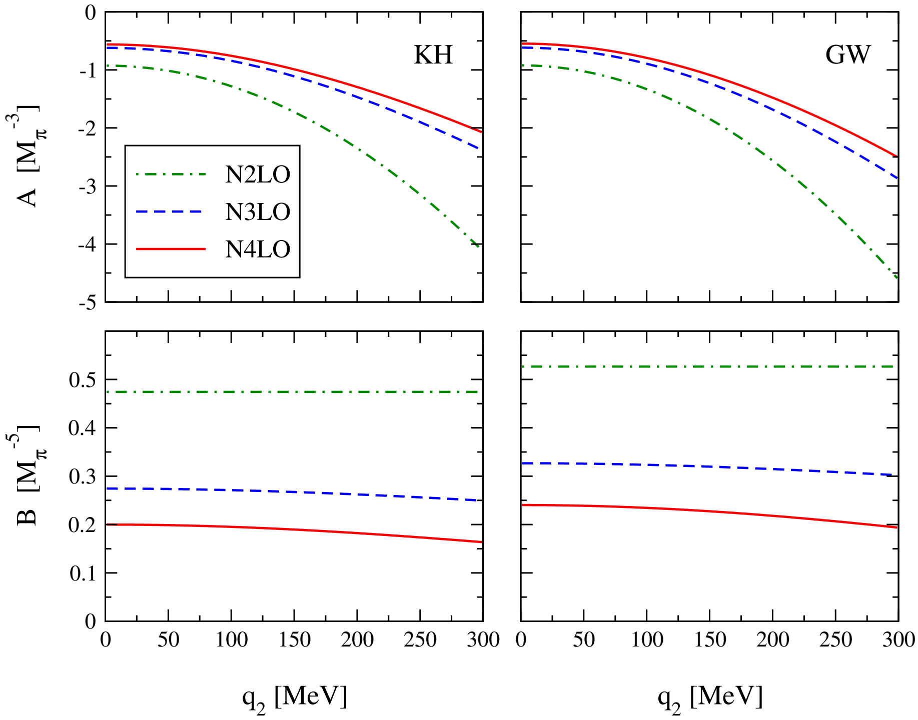

Figure 3: Chiral expansion of the functions $\mathscr{A}(q_2)$ and $\mathscr{B}(q_2)$ entering the two-pion exchange 3NF up to N^{4}LO. Left (right) panel shows the results obtained with the LECs determined from the order-Q^4 fit to the pion-nucleon partial wave analysis of Ref. [37] (Ref. [36]). Dashed, dashed-dotted and solid lines correspond to $\mathscr{A}^{(3)}$, $\mathscr{A}^{(3)}+\mathscr{A}^{(4)}$ and $\mathscr{A}^{(3)}+\mathscr{A}^{(4)}+\mathscr{A}^{(5)}$ in the upper plots while $\mathscr{B}^{(3)}$, $\mathscr{B}^{(3)}+\mathscr{B}^{(4)}$ and $\mathscr{B}^{(3)}+\mathscr{B}^{(4)}+\mathscr{B}^{(5)}$ in the lower plots.

similar results for the three-nucleon force, see Fig. 3. From Fig. 3, we observe a very nice convergence of the chiral expansion of two-pion-exchange three-nucleon force and do not expect further important contributions from higher order corrections to this (longest-range) part of the force.

Obviously, it is important to analyze the chiral expansion of other topologies from Fig. 1 as well. These contributions will be discussed in forthcoming publications. The presented analysis is only qualitative. A quantitative analysis will be possible once we will be able to make predictions for three-nucleon observables. An important milestone in this connection that needs to be achieved is a partial wave decomposition of the derived expressions. While this can be straightforwardly done numerically [41], one needs considerable supercomputing resources to perform several-dimensional integrals for a large number of (angular momentum) channels. First steps in this direction are published in Ref. [42].

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