

Hadronic light-by-light scattering in the muon $g-2$: impact of proposed measurements of the $\pi^0 \rightarrow \gamma\gamma$ decay width and the $\gamma^*\gamma \rightarrow \pi^0$ transition form factor with the KLOE-2 experiment

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The calculation of the hadronic light-by-light scattering contribution to the muon $g-2$ currently relies entirely on models. Measurements of the form factors which describe the interactions of hadrons with photons can help to constrain the models and reduce the uncertainty in $a_\mu^{\text{had. LbyL}} = (116 \pm 40) \times 10^{-11}$. In the numerically dominant pion-exchange contribution, the form factor $\mathcal{F}_{\pi^0 \gamma^* \gamma^*}((q_1 + q_2)^2, q_1^2, q_2^2)$ with an off-shell pion enters. In general, measurements of the transition form factor $\mathcal{F}_{\pi^0 \gamma^* \gamma}(Q^2) \equiv \mathcal{F}_{\pi^0 \gamma^* \gamma^*}(m_\pi^2, -Q^2, 0)$ are only sensitive to a subset of the model parameters. Thus, having a good description for $\mathcal{F}_{\pi^0 \gamma^* \gamma}(Q^2)$ is only necessary, not sufficient, to determine $a_\mu^{\text{LbyL}; \pi^0}$.

Simulations have shown that planned measurements at KLOE-2 should be able, within one year of data taking, to determine the $\pi^0 \rightarrow \gamma\gamma$ decay width to 1% statistical precision and the $\gamma^*\gamma \rightarrow \pi^0$ transition form factor $\mathcal{F}_{\pi^0 \gamma^* \gamma}(Q^2)$ for small space-like momenta, $0.01 \text{ GeV}^2 \leq Q^2 \leq 0.1 \text{ GeV}^2$, to 6% statistical precision in each bin. Note that in the two-loop integral for the pion-exchange contribution the relevant regions of momenta are in the range 0 – 1.5 GeV.

With the decay width $\Gamma_{\pi^0 \rightarrow \gamma\gamma}^{\text{PDG}}$ [$\Gamma_{\pi^0 \rightarrow \gamma\gamma}^{\text{PrimEx}}$] and current data for the transition form factor $\mathcal{F}_{\pi^0 \gamma^* \gamma}(Q^2)$, the error on $a_\mu^{\text{LbyL}; \pi^0}$ is $\pm 4 \times 10^{-11}$ [$\pm 2 \times 10^{-11}$], not taking into account the uncertainty related to the off-shellness of the pion. Including the simulated KLOE-2 data reduces the error to $\pm(0.7 - 1.1) \times 10^{-11}$. For models like VMD, which have only few parameters that are completely determined by measurements of $\mathcal{F}_{\pi^0 \gamma^* \gamma}(Q^2)$, this represents the total error. But maybe such models are too simplistic. In other models, e.g. those based on large- N_c QCD, parameters describing the off-shell pion dominate the uncertainty in $a_{\mu; \text{large-}N_c}^{\text{LbyL}; \pi^0} = (72 \pm 12) \times 10^{-11}$.

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1. Introduction

The anomalous magnetic moment of the muon a_μ provides an important test of the Standard Model (SM) and is potentially sensitive to contributions from New Physics. For some time now a deviation is observed between the experimental measurement and the SM prediction, $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \sim (250 - 300) \times 10^{-11}$, corresponding to 3 – 3.5 standard deviations [1, 2]. Hadronic effects dominate the uncertainty in the SM prediction of a_μ . In contrast to the hadronic vacuum polarization in the $g-2$, which can be related to data, the estimates for the hadronic light-by-light (LbyL) scattering contribution $a_\mu^{\text{had. LbyL}} = (105 \pm 26) \times 10^{-11}$ [3] and $a_\mu^{\text{had. LbyL}} = (116 \pm 40) \times 10^{-11}$ [4, 1] rely entirely on calculations using hadronic models which employ form factors for the interaction of hadrons with photons. Some papers [5] yield a larger central value and a larger error of $(150 \pm 50) \times 10^{-11}$, see also further analyses and partial evaluations of hadronic LbyL scattering in Refs. [6, 7]. To fully profit from future planned $g-2$ experiments with a precision of 15×10^{-11} , these large model uncertainties have to be reduced. Maybe lattice QCD will at some point give a reliable number, see Ref. [8]. Meanwhile, experimental measurements and theoretical constraints of the relevant form factors can help to constrain the models and to reduce the uncertainties in $a_\mu^{\text{had. LbyL}}$.

In most model calculations, pion-exchange gives the numerically dominant contribution. The relevant momentum regions¹ for all the light pseudoscalars, π^0, η, η' , can be inferred from Table 1, where we list, for different models of the form factor, the results obtained for a given UV cutoff Λ in the 3-dimensional integral representation derived in Ref. [1]. The cutoff bounds the length of the two Euclidean momenta, $|Q_i| < \Lambda, i = 1, 2$ in the two-loop integral. The third integration variable is the angle between the two 4-vectors Q_i . The off-shell LMD+V model [11, 4] is based on large- N_c QCD matched to short-distance constraints from the operator product expansion. For the vector-meson dominance (VMD) model, the vector meson mass has been obtained by fitting data [12] for the pseudoscalar-photon transition form factors. The model parameters are the same as in Ref. [4].

For the pion the bulk of the contribution comes from the region below $\Lambda = 1$ GeV, about 82% for the LMD+V form factor and about 92% for the VMD form factor. For the VMD form

Λ [GeV]	π^0			η	η'
	LMD+V ($h_3 = 0$)	LMD+V ($h_4 = 0$)	VMD	VMD	VMD
0.25	14.8 (20.6%)	14.8 (20.3%)	14.4 (25.2%)	1.76 (12.1%)	0.99 (7.9%)
0.5	38.6 (53.8%)	38.8 (53.2%)	36.6 (64.2%)	6.90 (47.5%)	4.52 (36.1%)
0.75	51.9 (72.2%)	52.2 (71.7%)	47.7 (83.8%)	10.7 (73.4%)	7.83 (62.5%)
1.0	58.7 (81.7%)	59.2 (81.4%)	52.6 (92.3%)	12.6 (86.6%)	9.90 (79.1%)
1.5	64.9 (90.2%)	65.6 (90.1%)	55.8 (97.8%)	14.0 (96.1%)	11.7 (93.2%)
2.0	67.5 (93.9%)	68.3 (93.8%)	56.5 (99.2%)	14.3 (98.6%)	12.2 (97.4%)
5.0	71.0 (98.8%)	71.9 (98.8%)	56.9 (99.9%)	14.5 (99.9%)	12.5 (99.9%)
20.0	71.9 (100%)	72.8 (100%)	57.0 (100%)	14.5 (100%)	12.5 (100%)

Table 1: The pseudoscalar exchange contribution, $a_\mu^{\text{LbyL:PS}} \times 10^{11}$, PS = π^0, η, η' , for different models with a cutoff for the two Euclidean momenta, $|Q_i| < \Lambda, i = 1, 2$ in the two-loop integral. In brackets, the relative contribution of the total obtained with $\Lambda = 20$ GeV.

¹For attempts to visualize the relevant momentum regions in hadronic LbyL scattering for the pseudoscalars and for other contributions, see Refs. [9, 10, 6].

factor, the small contribution from the region with momenta higher than 1 GeV can be understood from the weight-functions in the integrals derived in Ref. [9], which peak around 0.5 GeV, and the strong fall-off of the VMD form factor at large momenta. For the off-shell LMD+V form factor, the region with larger momenta is more important as the form factor drops off less quickly and there is no damping at the external vertex, see Ref. [4]. For the η and η' , the peaks of the relevant weight functions in the integrals are shifted to higher values of $|Q_i|$ and the saturation effect only sets in around $\Lambda = 1.5$ GeV, with about 95% of the contribution to the total.

In Ref. [13] it was shown that planned measurements at KLOE-2 could determine the $\pi^0 \rightarrow \gamma\gamma$ decay width to 1% statistical precision and the $\gamma^*\gamma \rightarrow \pi^0$ transition form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(Q^2)$ for small space-like momenta, $0.01 \text{ GeV}^2 \leq Q^2 \leq 0.1 \text{ GeV}^2$, to 6% statistical precision in each bin. The simulations have been performed with the Monte-Carlo program EKHARA [14] for the process $e^+e^- \rightarrow e^+e^-\gamma^*\gamma^* \rightarrow e^+e^-\pi^0$, followed by the decay $\pi^0 \rightarrow \gamma\gamma$ and combined with a detailed detector simulation. The results of the simulations are shown in Figure 1. The KLOE-2 measurements will allow to almost directly measure the slope of the form factor at the origin and check the consistency of models which have been used to extrapolate the data from larger values of Q^2 down to the origin.

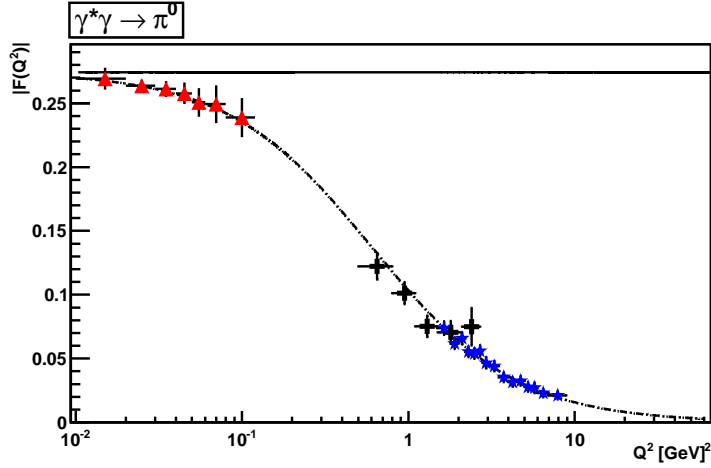


Figure 1: Simulation of KLOE-2 measurement of $F(Q^2)$ (red triangles) with statistical errors for 5 fb^{-1} , corresponding to one year of data taking. The dashed line is the $F(Q^2)$ form factor according to the LMD+V model, the solid line is $F(0) = 1/(4\pi^2 F_\pi)$ given by the Wess-Zumino-Witten term. Data [12] from CELLO (black crosses) and CLEO (blue stars) at high Q^2 are also shown for illustration.

2. Impact of KLOE-2 measurements on $a_\mu^{\text{LbyL};\pi^0}$

Any experimental information on the neutral pion lifetime and the transition form factor is important in order to constrain the models used for calculating the pion-exchange contribution. However, having a good description, e.g. for the transition form factor, is only necessary, not sufficient, in order to uniquely determine $a_\mu^{\text{LbyL};\pi^0}$. As stressed in Ref. [15], what enters in the calculation of $a_\mu^{\text{LbyL};\pi^0}$ is the fully off-shell form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2)$ (vertex function), where also the pion is off-shell with 4-momentum (q_1+q_2) . Such a (model dependent) form factor can for instance be defined via the QCD Green's function $\langle VVP \rangle$, see Ref. [4] for details. The form

factor with on-shell pions is then given by $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) \equiv \mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2, q_1^2, q_2^2)$. Measurements of the transition form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(Q^2) \equiv \mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2, -Q^2, 0)$ are in general only sensitive to a subset of the model parameters and do not allow to reconstruct the full off-shell form factor.

For different models, the effects of the off-shell pion can vary a lot. In Ref. [4] the off-shell LMD+V form factor was proposed and the estimate $a_{\mu;\text{LMD+V}}^{\text{LbyL};\pi^0} = (72 \pm 12) \times 10^{-11}$ was obtained. The error estimate comes from the variation of all model parameters, where the uncertainty of the parameters related to the off-shellness of the pion completely dominates the total error. In contrast to the off-shell LMD+V model, many other models, e.g. the VMD model or constituent quark models, do not have these additional sources of uncertainty related to the off-shellness of the pion. These models often have only very few parameters, which can all be fixed by measurements of the transition form factor or from other observables. Therefore, for such models, the precision of the KLOE-2 measurement can dominate the total accuracy of $a_{\mu}^{\text{LbyL};\pi^0}$.

Essentially all evaluations of the pion-exchange contribution use for the normalization of the form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2, 0, 0) = 1/(4\pi^2 F_\pi)$, as derived from the Wess-Zumino-Witten (WZW) term. Then the value $F_\pi = 92.4$ MeV is used without any error attached to it, i.e. a value close to $F_\pi = (92.2 \pm 0.14)$ MeV, obtained from $\pi^+ \rightarrow \mu^+ \nu_\mu(\gamma)$ [16]. If one uses the decay width $\Gamma_{\pi^0 \rightarrow \gamma\gamma}$ for the normalization of the form factor, an additional source of uncertainty enters, which has not been taken into account in most evaluations [17]. We account for this by using in the fits:

- $\Gamma_{\pi^0 \rightarrow \gamma\gamma}^{\text{PDG}} = 7.74 \pm 0.48$ eV from the PDG 2010 [16]
- $\Gamma_{\pi^0 \rightarrow \gamma\gamma}^{\text{PrimEx}} = 7.82 \pm 0.22$ eV from the PrimEx experiment [18]
- $\Gamma_{\pi^0 \rightarrow \gamma\gamma}^{\text{KLOE-2}} = 7.73 \pm 0.08$ eV for the KLOE-2 simulation (assuming a 1% precision).

The assumption that the KLOE-2 measurement will be consistent with the LMD+V and VMD models, allowed us in Ref. [13] to use the simulations as new “data” and evaluate the impact on the precision of the $a_{\mu}^{\text{LbyL};\pi^0}$ calculation. We fit the models to the data sets [12] from CELLO, CLEO and BaBar for the transition form factor and the values for the decay width given above:

A0 : CELLO, CLEO, PDG	B0 : CELLO, CLEO, BaBar, PDG
A1 : CELLO, CLEO, PrimEx	B1 : CELLO, CLEO, BaBar, PrimEx
A2 : CELLO, CLEO, PrimEx, KLOE-2	B2 : CELLO, CLEO, BaBar, PrimEx, KLOE-2

The BaBar measurement does not show the $1/Q^2$ behavior as expected from theoretical considerations [19] and as seen in the data of CELLO, CLEO and Belle. The VMD model always shows a $1/Q^2$ fall-off and therefore is not compatible with the BaBar data. The LMD+V model has another parameter, h_1 , which determines the behavior of the transition form factor for large Q^2 . To get the $1/Q^2$ behavior, one needs to set $h_1 = 0$. However, one can simply leave h_1 as a free parameter and fit it to the BaBar data [17]. Since VMD and LMD+V with $h_1 = 0$ are not compatible with the BaBar data, the corresponding fits are very bad and we will not include these results in the current paper. We use two ways to calculate $a_{\mu}^{\text{LbyL};\pi^0}$: the Jegerlehner-Nyffeler (JN) approach [4, 1] with the off-shell pion form factor and the Melnikov-Vainshtein (MV) approach [20] with the on-shell pion form factor at the internal vertex and a constant (WZW) form factor at the external vertex.

Table 2 shows the impact of the PrimEx and the future KLOE-2 measurements on the model parameters and on the $a_{\mu}^{\text{LbyL};\pi^0}$ uncertainty. The other parameters of the (on-shell and off-shell) LMD+V model have been chosen as in the papers [4, 1, 20]. We stress that our estimate of the

Model	Data	$\chi^2/d.o.f.$	Parameters			$a_\mu^{\text{LbyL};\pi^0} \times 10^{11}$
VMD	A0	6.6/19	$M_V = 0.778(18)$ GeV	$F_\pi = 0.0924(28)$ GeV		$(57.2 \pm 4.0)_{JN}$
VMD	A1	6.6/19	$M_V = 0.776(13)$ GeV	$F_\pi = 0.0919(13)$ GeV		$(57.7 \pm 2.1)_{JN}$
VMD	A2	7.5/27	$M_V = 0.778(11)$ GeV	$F_\pi = 0.0923(4)$ GeV		$(57.3 \pm 1.1)_{JN}$
LMD+V, $h_1 = 0$	A0	6.5/19	$\bar{h}_5 = 6.99(32)$ GeV ⁴	$\bar{h}_7 = -14.81(45)$ GeV ⁶		$(72.3 \pm 3.5)_{JN}^*$ $(79.8 \pm 4.2)_{MV}$
LMD+V, $h_1 = 0$	A1	6.6/19	$\bar{h}_5 = 6.96(29)$ GeV ⁴	$\bar{h}_7 = -14.90(21)$ GeV ⁶		$(73.0 \pm 1.7)_{JN}^*$ $(80.5 \pm 2.0)_{MV}$
LMD+V, $h_1 = 0$	A2	7.5/27	$\bar{h}_5 = 6.99(28)$ GeV ⁴	$\bar{h}_7 = -14.83(7)$ GeV ⁶		$(72.5 \pm 0.8)_{JN}^*$ $(80.0 \pm 0.8)_{MV}$
LMD+V, $h_1 \neq 0$	A0	6.5/18	$\bar{h}_5 = 6.90(71)$ GeV ⁴	$\bar{h}_7 = -14.83(46)$ GeV ⁶	$h_1 = -0.03(18)$ GeV ²	$(72.4 \pm 3.8)_{JN}^*$
LMD+V, $h_1 \neq 0$	A1	6.5/18	$\bar{h}_5 = 6.85(67)$ GeV ⁴	$\bar{h}_7 = -14.91(21)$ GeV ⁶	$h_1 = -0.03(17)$ GeV ²	$(72.9 \pm 2.1)_{JN}^*$
LMD+V, $h_1 \neq 0$	A2	7.5/26	$\bar{h}_5 = 6.90(64)$ GeV ⁴	$\bar{h}_7 = -14.84(7)$ GeV ⁶	$h_1 = -0.02(17)$ GeV ²	$(72.4 \pm 1.5)_{JN}^*$
LMD+V, $h_1 \neq 0$	B0	18/35	$\bar{h}_5 = 6.46(24)$ GeV ⁴	$\bar{h}_7 = -14.86(44)$ GeV ⁶	$h_1 = -0.17(2)$ GeV ²	$(71.9 \pm 3.4)_{JN}^*$
LMD+V, $h_1 \neq 0$	B1	18/35	$\bar{h}_5 = 6.44(22)$ GeV ⁴	$\bar{h}_7 = -14.92(21)$ GeV ⁶	$h_1 = -0.17(2)$ GeV ²	$(72.4 \pm 1.6)_{JN}^*$
LMD+V, $h_1 \neq 0$	B2	19/43	$\bar{h}_5 = 6.47(21)$ GeV ⁴	$\bar{h}_7 = -14.84(7)$ GeV ⁶	$h_1 = -0.17(2)$ GeV ²	$(71.8 \pm 0.7)_{JN}^*$

Table 2: KLOE-2 impact on the accuracy of $a_\mu^{\text{LbyL};\pi^0}$ in case of one year of data taking (5 fb^{-1}). The values marked with asterisk (*) do not contain additional uncertainties coming from the “off-shellness” of the pion.

$a_\mu^{\text{LbyL};\pi^0}$ uncertainty is given only by the propagation of the errors of the fitted parameters in Table 2. We can clearly see from Table 2 that for each given model and each approach (JN or MV), there is a trend of reduction in the error for $a_\mu^{\text{LbyL};\pi^0}$ by about half when going from A0 (PDG) to A1 (including PrimEx) and by about another half when going from A1 to A2 (including KLOE-2):

- Sets A0, B0: $\delta a_\mu^{\text{LbyL};\pi^0} \approx 4 \times 10^{-11}$ (with $\Gamma_{\pi^0 \rightarrow \gamma\gamma}^{\text{PDG}}$)
- Sets A1, B1: $\delta a_\mu^{\text{LbyL};\pi^0} \approx 2 \times 10^{-11}$ (with $\Gamma_{\pi^0 \rightarrow \gamma\gamma}^{\text{PrimEx}}$)
- Sets A2, B2: $\delta a_\mu^{\text{LbyL};\pi^0} \approx (0.7 - 1.1) \times 10^{-11}$ (with simulated KLOE-2 data)

This is mainly due to the improvement in the normalization of the form factor, related to the decay width $\pi^0 \rightarrow \gamma\gamma$, controlled by the parameters F_π or \bar{h}_7 , respectively, but more data also better constrain the other model parameters M_V or \bar{h}_5 . This trend is also visible in the last part of the Table (LMD+V, $h_1 \neq 0$), when we fit the sets B0, B1 and B2 which include the BaBar data.

Note that both VMD and LMD+V with $h_1 = 0$ can fit the data sets A0, A1 and A2 for the transition form factor very well with essentially the same χ^2 per degree of freedom for a given data set. Nevertheless, the results for the pion-exchange contribution differ by about 20% in these two models. For VMD the result is $a_\mu^{\text{LbyL};\pi^0} \sim 57.5 \times 10^{-11}$ and for LMD+V with $h_1 = 0$ it is 72.5×10^{-11} with the JN approach and 80×10^{-11} with the MV approach. This is due to the different behavior, in these two models, of the fully off-shell form factor $\mathcal{F}_{\pi^0^* \gamma^* \gamma^*}((q_1 + q_2)^2, q_1^2, q_2^2)$ on all momentum variables, which enters for the pion-exchange contribution. The VMD model is known to have a wrong high-energy behavior with too strong damping. For the VMD model, measurements of the neutral pion decay width and the transition form factor completely determine the model parameters F_π and M_V and the error given in Table 2 is the total model error. Note that a smaller error does not necessarily imply that the VMD model is better, i.e. closer to reality. Maybe it is too simplistic.

We conclude that the KLOE-2 data with a total integrated luminosity of 5 fb^{-1} will give a reasonable improvement in the part of the $a_\mu^{\text{LbyL};\pi^0}$ error associated with the parameters accessible via the $\pi^0 \rightarrow \gamma\gamma$ decay width and the $\gamma^* \gamma \rightarrow \pi^0$ transition form factor. Depending on the modelling of the off-shellness of the pion, there might be other, potentially larger sources of uncertainty which cannot be improved by the KLOE-2 measurements.

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