PROCEEDINGS OF SCIENCE

Model-Independent Form Factor Relations at Large Nc

Vojtech Krejcirik[∗†]

University of Maryland

E-mail: vkrejcir@umd.edu

A new model-independent relation which holds in the large N_c and chiral limits of QCD is demonstrated. It correlates the long distance part of the Fourier transform of the electromagnetic form factors of the nucleon. This relation was previously conjectured based on the analysis of semi-classical chiral soliton models. The main goal of this contribution is to show that the result is truly a model-independent by deriving it directly in large N_c chiral perturbation theory. This new type of model-independent relation is of great value in studying holographic soliton models, since it is a model-independent method how to check whether the large N_c and chiral physics are encoded correctly in these models. The role of the ordering of limits is studied. It is shown that large N_c and chiral limit do not commute in the case of this particular relation.

The 7th International Workshop on Chiral Dynamics
August 6 -10, 2012
Jefferson Lab, Newport News, Virginia, USA

[∗]Speaker.

[†]This work was supported by the U.S. Department of Energy through grant DE-FG02-93ER-40762.

PoS(CD12)072

1. Introduction

The physics of strong interaction is ultimately traceable to Quantum Chromodynamics (QCD). QCD is a gauge theory of quarks and gluons based on $SU(N_c = 3)$ symmetry. However, a problem arises at low energies: the theory is strongly coupled. Thus, the conventional perturbative expansion in powers of a coupling constant, i.e., around a non-interacting theory, is not directly applicable. Alternative schemes have been developed in order to gain insight into various low-energy phenomena. In this work, we study aspects of two such approaches: the $1/N_c$ expansion about the the large N_c limit, and the chiral expansion about the massless quark limit.

In the large N_c limit [1], QCD becomes a weakly interacting theory of mesons; baryons emerge as soliton-like configurations of meson fields [2]. A class of baryon models has been developed following this approach, namely chiral soliton models treated semi-classically. The large N_c limit of QCD is encoded in the very core of these models, in the semiclassical treatment of baryons. The chiral limit, and following chiral symmetry is imposed at a later stage as a constraint on the dynamics of meson fields. A more rigorous formulation sounds: chiral soliton models treated semiclassically are models based on the large N_c and chiral limits of QCD with $N_c \to \infty$ taken first. The Skyrme model [3, 4], where baryons are identified with the quantum states of collective motions of Skyrmions, is an example of such a model.

In principle, an infinite number of models based on large N_c and chiral limits of QCD could be develop. For example, there has been a wide interest in using gauge/gravity duality to develop holographic baryon models recently [5, 6, 7, 8]. These models are also built in a way that large N_c limit is taken first. Obviously, there is more to modeling of QCD than just getting large N_c and chiral physics right; yet, it is particularly important to develop a "simple" method how to test whether the chiral and large N_c behavior have been incorporated correctly, since holographic models are so different from conventional soliton models and are rather nontrivial to implement.

2. A new model-independent relation

To do this a new class of model-independent relation, which uses the long distance behavior of the Fourier transform of the electromagnetic form factors as a model-independent probe [9], is formulated. Traditionally, the long distance behavior in the usual model-independent relation fixes how certain quantities diverge as $m_\pi \to 0$. However, such approach is unusable for holographic models, since to date many of the treatments of these models have been done only for $m_\pi = 0$. Hence one does not have access to how these quantities diverge with m_π. Thus, a conceptually new approach is required.

A particularly useful model-independent relation is for a ratio of the product of the position-space electric form factors to the product of the magnetic form factors of the nucleon in the long distance limit [9]:

$$\lim_{r\to\infty} \frac{r^2\, \widetilde{G}_E^{I=0}\, \widetilde{G}_E^{I=1}}{\widetilde{G}_M^{I=0}\, \widetilde{G}_M^{I=1}} = 18, \tag{2.1}$$

where the position-space isoscalar and isovector electric and magnetic form factors $\widetilde{G}_{E,M}^{I=0,1}$ are Fourier transformed momentum-space form factors electric and magnetic form factors $G_{E,M}$.

This relation has already proven to be of great value in analysis of holographic soliton models. The "bottom up" phenomenological model [5, 6] was shown to satisfy the new model-independent relation, which indicates that it had correctly captured the large N_c and chiral behavior. The "top down" model of Sakai and Sugimoto [7, 8] treating solitons as instantons failed to satisfy the relation indicating that something was wrong. It was recently shown by Cherman and Ishii [10] that the underlying problem appears to be due to a failure of the flat-space instanton approximation.

Coming back to the relation (2.1), the remarkable thing about this particular ratio is that all low-energy constants, normalization of currents, and various sign or Fourier transform conventions cancel. Thus, only a given power of radius r (easily deduced from dimensional analysis) and a universal constant remains. This universal constant, 18, is a model-independent quantity that must be satisfied in all large N_c chiral models. Note that for certain observables the leading behavior depends on the ordering of large N_c and chiral limits [11]. It is the case for the relation (2.1), which is valid if large N_c limit is taken first, prior to the chiral and large r limits.

Equation (2.1) was originally [9] derived in the context of the Skyrme model. It turns out that Eq. (2.1) is satisfied independent of the details of the Lagrangian or the number of degrees of freedom in the problem. Although this seems to be compelling evidence that the result is truly model independent (it was shown that a result derived in Skyrme model agrees with a result obtained in the large N_c chiral perturbation theory, whenever it is independent of model details), it is important to do the next step and verify that Eq. (2.1) is truly model independent, i.e. calculate it directly from large N_c chiral perturbation theory (χPT). Doing so is the principal purpose of this work.

3. Calculation within large N_c chiral perturbation theory

The calculation of electromagnetic form factors appearing in the relation (2.1) is somewhat lengthy and tedious, thus only main ideas and results are presented in this paper. Full calculation with all the details can be found in the paper of Cohen and Krejčiřík [12].

The mass of the baryon is parametrically large, of order N_c, and therefore it is justifiable to work in the heavy baryon approximation of χPT, i.e. to treat the baryons non-relativistically and to neglect their recoil. The long distance hadronic physics is dominated by the pion cloud, since a pion is the lightest particle available. So, the dominant diagrams to consider will consist of currents connected to the pion loops containing the fewest possible number of pions. Also, diagrams suppressed by factors $1/N_c$ need not be taken into account. Additionally, the large N_c consistency relations require nucleon and Δ to be degenerate at large N_c forcing one to include diagrams with Δ in the intermediate state. More specifically, the mass difference $\Delta = m_\Delta - m_N$ serves as a new low-energy constant and is of order $1/N_c$. The need to include the Δ increases the number of Feynman diagrams contributing and has far-reaching consequences. The large N_c consistency relations also specify the coupling at the pion-baryon-baryon$'$ vertex.

Diagrams one must include in the calculation of isoscalar formfactors are shown in Fig.1 and diagrams contributing to isovector formfactors are in Fig.2

Note that the Fourier transformed (i.e. position-space) form factors are finite even in the chiral limit $m_\pi \to 0$. Moreover, sending $m_\pi \to 0$ allows us to perform both the loop momentum integration and the Fourier transform analytically and obtain the form factors in a closed form.

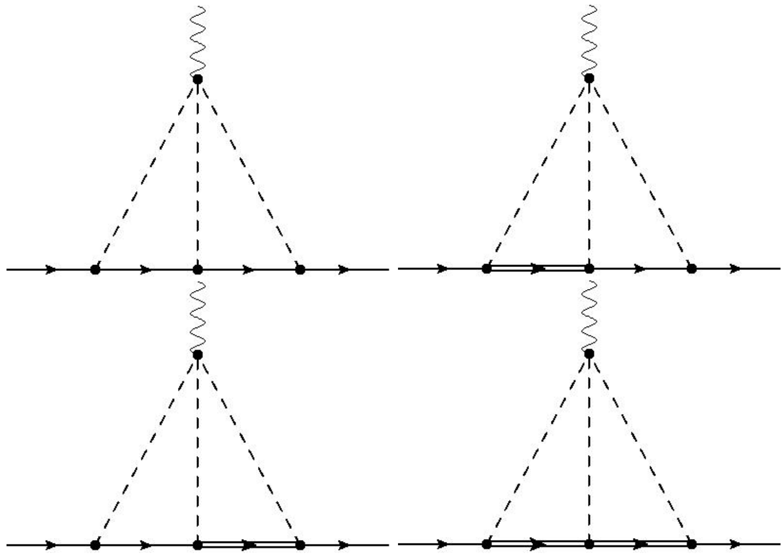

Figure 1: Feynman diagrams contributing to the long distance part of the isoscalar formfactors (double lines in the intermediate states represent the Δ-isobar).

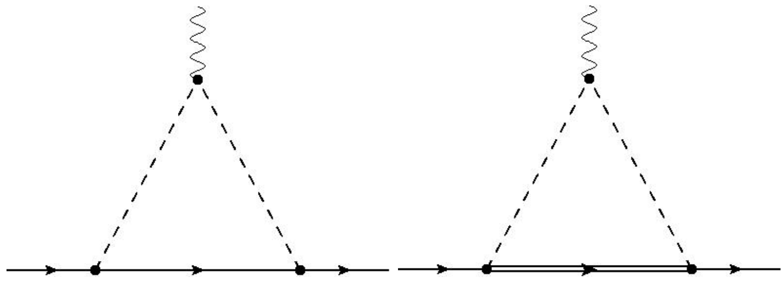

Figure 2: Feynman diagrams contributing to the long distance part of the isovector form factors (double line represents the Δ-isobar).

Evaluating diagrams in Figs.1 and 2, Fourier transforming, setting the pion mass to zero and then extracting the longest distance part of isoscalar electric and magnetic form factors and isovector electric and magnetic form factors yields

$$\lim_{r\to\infty} \widetilde{G}_E^{I=0} = \frac{3^3}{2^9\pi^5}\frac{1}{f_\pi^3}\left(\frac{g_A}{f_\pi}\right)^3\frac{1}{r^9}, \tag{3.1}$$

$$\lim_{r\to\infty} \widetilde{G}_M^{I=0} = \frac{3}{2^9\pi^5}\frac{1}{f_\pi^3}\left(\frac{g_A}{f_\pi}\right)^3\frac{\Delta}{r^7}, \tag{3.2}$$

$$\lim_{r\to\infty} \widetilde{G}_E^{I=1} = \frac{1}{2^4\pi^2}\left(\frac{g_A}{f_\pi}\right)^2\frac{\Delta}{r^4}, \tag{3.3}$$

$$\lim_{r\to\infty} \widetilde{G}_M^{I=1} = \frac{1}{2^5\pi^2}\left(\frac{g_A}{f_\pi}\right)^2\frac{1}{r^4}. \tag{3.4}$$

One can trivially use the results of (3.1)-(3.4) to show that the relation (2.1) is satisfied in the large N_c χPT:

$$\lim_{r\to\infty} \frac{r^2\, \widetilde{G}_E^{I=0}\, \widetilde{G}_E^{I=1}}{\widetilde{G}_M^{I=0}\, \widetilde{G}_M^{I=1}} = 18. \tag{3.5}$$

As advertised, all low energy constants canceled in the ratio of form factors and only universal number 18 remains.

For the preceding analysis, it was crucial that the large N_c limit was taken on the onset of the problem. If it is taken in the end, only diagrams with nucleons in the intermediate states contribute. In this case, the result is:

$$\lim_{N_c\to\infty} \lim_{r\to\infty} \lim_{m_\pi\to 0} \frac{r^2\, \widetilde{G}_E^{I=0}\, \widetilde{G}_E^{I=1}}{\widetilde{G}_M^{I=0}\, \widetilde{G}_M^{I=1}} = 9. \tag{3.6}$$

We proved that the relation (2.1) holds in the large N_c χPT, provided that $N_c \to \infty$ limit is taken at the outset of the problem. Consequently, it may serve as an honest model-independent constraint on baryon models based on large N_c and chiral physics.

Acknowledgement: This work was supported by the U.S. Department of Energy through grant DE-FG02-93ER-40762 and by JSA/Jefferson Lab Graduate Fellowship.

References

[1] G. t'Hooft, Nucl. Phys. B **72** (1974) 461.

[2] E. Witten, Nucl. Phys. B **160** (1979) 57.

[3] T.H.R. Skyrme, Proc. Roy. Soc. Lond. A **260** (1961) 127.

[4] G.S. Adkins, C.R. Nappi, E. Witten, Nucl. Phys. B **228** (1983) 552.

[5] A. Pomarol, A. Wulzer, JHEP **03** (2008) 051.

[6] A. Pomarol, A. Wulzer, Nucl. Phys. B **809** (2009) 347.

[7] T. Sakai, S. Sugimoto, Prog. Theor. Phys. **113** (2005) 843.

[8] T. Sakai, S. Sugimoto, Prog. Theor. Phys. **114** (2005) 1083.

[9] A. Cherman, T.D. Cohen, M. Nielsen, Phys. Rev. Lett. **103** (2009) 022001.

[10] A. Cherman, T.Ishii, Phys. Rev. D 86, 045011 (2012).

[11] T.D. Cohen, W. Broniowski, Phys. Lett. B **292** (1992) 5.

[12] T.D. Cohen, V. Krejcirik, Phys. Rev. C 85, 035205 (2012).