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MEM Analysis of the QCD Sum Rule and its Application to nucleon spectrum

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The nucleon and its negative-parity excited states in vacuum are examined in a maximum entropy method (MEM) analysis of QCD sum rules. The MEM method does not restrict the spectral function to the usual "pole + continuum"-form, which can be useful when applied to analyses at finite temperature or density. We derive the parity projected nucleon QCD sum rules in the vacuum including all known first order α_s corrections to the Wilson coefficients of the operator product expansion (OPE). Since some of these corrections are large, we suppress them by using a phase-rotated Gaussian kernel. Additionally, this phase rotation strongly suppresses the continuum contribution and improves the convergence of the OPE. The resulting sum rule has the interesting feature that it is dominated by the term of the chiral condensate of dimension 3. Analyzing this sum rule by the maximum entropy method, we are able to extract information of both the positive and negative parity states.

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1. Introduction

The QCD sum rule method is a powerful tool for studying hadron properties directly from QCD [1]. In this method, the correlation function of an interpolating field operator, which can be calculated in the deep Euclidean region by the operator product expansion (OPE), is related to the hadronic spectral function in the physical region by a dispersion relation. The non-perturbative contributions in the correlation function are expressed by vacuum condensates such as the chiral condensate $\langle \overline{q}q \rangle$. When evaluating the correlation function quantitatively, one should take the uncertainties of the values of the vacuum condensates into account and carefully check the convergences of the OPE and α_s corrections. As for the nucleon QCD sum rules considered so far, it is known that the α_s corrections and the uncertainties of the value of the four quark condensates $\langle \overline{q}q \rangle^2$ are large.

In the traditional QCD sum rule analysis, it is necessary to assume some specific functional form for the spectral function such as the “pole + continuum"-ansatz, where the pole corresponds to the lowest lying hadronic state and the continuum stands for contributions of other states. Although it is not completely obvious that the actual form of spectral function is accurately described by the “pole + continuum"-ansatz, the properties of many hadrons have been successfully investigated by this method. On the other hand, our approach with the help of the maximum entropy method (MEM) is able to extract the spectral functions without assuming any specific form [2, 3, 4, 5]. We have applied this new analysis method of QCD sum rules to the nucleon and its negative parity excited state [6] and will review our results in these proceedings.

2. Nucleon QCD sum rules

In the QCD sum rules of the nucleon channel, one usually studies the properties of the time ordered correlation function:

$$\begin{aligned}\Pi(q) &= i\int d^4x e^{iqx}\langle 0|T[\eta(x)\overline{\eta}(0)]|0\rangle \equiv \not{q}\Pi_1(q^2)+\Pi_2(q^2) \\ &= -\int_0^\infty \left[\rho_+(m)\frac{\not{q}+m}{q^2-m^2+i\varepsilon}+\rho_-(m)\frac{\not{q}-m}{q^2-m^2+i\varepsilon}\right]dm. \qquad (2.1)\end{aligned}$$

Here, η is a nucleon interpolating field and $\rho_{+(-)}(m)$ denotes the spectral function which contains contributions of only positive (negative) parity states coupling to η. Note that the nucleon interpolating field couples to both positive and negative parity states [7]. Therefore, when only the sum rule constructed from either $\Pi_1(q^2)$ or $\Pi_2(q^2)$ is used, the contributions of positive and negative parity cannot be separated. The contributions of these states can potentially disturb the analysis and especially lower the reliability of the extraction of the negative parity states.

To overcome this problem, we use the old-fashioned correlation function in the rest frame [8]:

$$\begin{aligned}\Pi^{\rm old}(q_0) &= i\int d^4x e^{iqx}\theta(x_0)\langle 0|T[\eta(x)\overline{\eta}(0)]|0\rangle\Big|_{\vec{q}=0} \\ &\equiv \gamma_0\Pi_1^{\rm old}(q_0)+\Pi_2^{\rm old}(q_0), \qquad (2.2)\end{aligned}$$

where the essential difference to Eq.(2.1) is the insertion of the Heaviside step-function $\theta(x_0)$ before carrying out the Fourier transform. This correlator contains contributions only from states

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which propagate forward in time. Using the Heaviside step-function and parity projection operators $P^{\pm} \equiv \frac{1}{2}(\gamma_0 \pm 1)$, one can construct a correlator that contains contributions of only positive or negative parity states, as

$$\frac{1}{2}\mathrm{Tr}\left[P^{\pm}\Pi^{\mathrm{old}}(q_0)\right] = \Pi_1^{\mathrm{old}}(q_0) \pm \Pi_2^{\mathrm{old}}(q_0) \equiv \Pi^{\pm}(q_0)$$
$$= -\int_0^{\infty} \rho_{\pm}(m)\frac{1}{q_0 - m + i\varepsilon}dm. \tag{2.3}$$

Making use of the property that the old-fashioned correlator is analytic in the upper half of the complex q_0 plane, we get the parity projected sum rule:

$$\int_{-\infty}^{\infty} dq_0 \frac{1}{\pi}\mathrm{Im}\left[\Pi^{\pm}_{\mathrm{OPE}}(q_0)\right]W(q_0) = \int_0^{\infty} dq_0 \rho^{\pm}_{\mathrm{Phys.}}(q_0)W(q_0). \tag{2.4}$$

Here, $\Pi^{\pm}_{\mathrm{OPE}}(q_0)$ is calculated by the OPE in the deep Euclidean region, $\rho^{\pm}_{\mathrm{Phys.}}(q_0)$ stands for the physical spectral function of positive and negative parity and $W(q_0)$ is an arbitrary function which is analytic in the upper half of the imaginary plane and real on the real axis.

To construct the final sum rule, we have to the specify the kernel $W(q_0)$. One usually use Borel kernel $W(q_0) = \exp(-\frac{q_0^2}{\mathrm{M}^2})$ or Gaussian kernel $W(q_0) = \frac{1}{\sqrt{4\pi\tau}}\exp(-\frac{(q_0^2-s)^2}{4\tau})$, which correspond to the Borel and Gaussian sum rules, respectively. However, in these sum rules, the α_s corrections for the perturbative and four quark condensate terms and the contributions of the continuum are large, which lower the reliability of the analyses [9]. Following the method proposed by Ioffe and Zyablyuk [10], who have constructed a new class of sum rules by using the phase rotated complex variable $q^2 e^{i\theta}$ instead of the real q^2, we could improve this situation. One advantage of this approach lies in the possibility of suppressing certain terms of the OPE by choosing some specific value of θ. To apply this idea to the parity projected sum rules, we use the phase-rotated kernel:

$$W(q_0, s, \tau, \theta)dq_0 = \frac{1}{\sqrt{4\pi\tau}}\mathrm{Re}\left[q_0 e^{-i\theta}\exp\left(-\frac{(q_0^2 e^{-2i\theta} - s)^2}{4\tau}\right)e^{-i\theta}dq_0\right]. \tag{2.5}$$

We can then obtain the specific form of $G^{\mathrm{old}\,\pm}_{\mathrm{OPE}}(s,\tau,\theta)$ which is defined as the left hand side of Eq.(2.4) [6]:

$$G^{\mathrm{old}\,\pm}_{\mathrm{OPE}}(s,\tau,\theta) = \left(C_0 + C_{0\alpha_s}(\theta)\right)\cos 5\theta + C_4\langle\frac{\alpha_s}{\pi}G^2\rangle\cos\theta + \left(C_6 + C_{6\alpha_s}\right)\langle\bar{q}q\rangle^2\cos\theta + \cdots$$
$$\pm\left[(C_3 + \frac{\alpha_s}{\pi}C_{3\alpha_s})\langle\bar{q}q\rangle\cos 2\theta + C_5\langle\bar{q}g\sigma\cdot Gq\rangle + \cdots\right], \tag{2.6}$$

where C_0, $C_{0\alpha}(\theta)$, C_3,$C_{3\alpha_s}$, C_4, C_5, C_6 and $C_{6\alpha}$ are numerical coefficients. Using the phase rotation, the ratios of α_s corrections to leading terms at dimension 0 ($\frac{C_{0\alpha_s}}{C_0}$) is reduced from 90 % to 5 % at $\theta = 0.108\pi$ ($\alpha_s = 0.5$), which shows that the convergence of the perturbative expansion is significantly improved.

The perturbative, chiral condensate, four quark condensate terms and $G^{\pm}(s,\tau,\theta)$ at $\theta = 0$ and $\theta = 0.108\pi$ are given in Fig. 1. For both value of θ, the contribution of dimension 6 is small since we use a q_0-odd kernel, which eliminates the leading order contribution of this term. Therefore the uncertainty of the four quark condensate does not much affect the results of this sum rule. For $\theta = 0$, the dimension 0 and 3 terms are dominant, which means that not only low lying nucleon

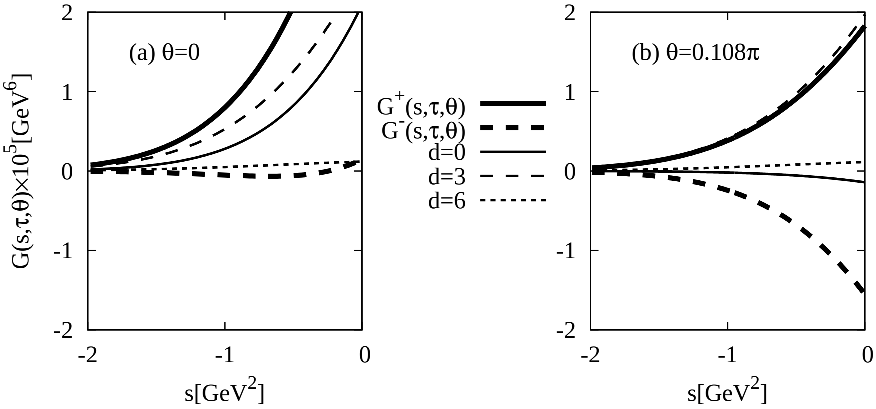

Figure 1: (a) $G^{\text{old}\,\pm}_{\text{OPE}}(s,\tau,\theta)$, dimension 0 (perturbative), dimension 3 (chiral condensate) and dimension 6 (four quark condensate) terms at $\tau = 0.5[\text{GeV}^4]$ and $\theta = 0$. The thick lines denote $G^{\text{old}\,\pm}_{\text{OPE}}(s,\tau,\theta)$ and the thin lines are the dimension 0, 3 and 6 terms. (b) Same as for (a), but for $\theta = 0.108\pi$.

states but also the continuum largely contributes to $G^{\pm}(s,\tau,\theta)$. This situation makes it difficult to extract information on nucleonic properties. On the other hand, from Fig. 1 (b) it can be seen that in the phase rotated sum rule the chiral condensate term clearly gives the dominant contribution to the OPE. Therefore, unlike the sum rule at $\theta = 0$, the disturbance due to the contribution of the continuum is considerably reduced. We also find that the difference of the OPE data between the positive parity and negative parity states is caused by the chiral condensate in Fig. 1.

3. Maximum entropy method

In this section, we briefly introduce the maximum entropy method (MEM) and explain how this approach applies to the analysis of QCD sum rules. The equation to be analyzed can be written down as

$$G_{\text{OPE}}(x) = \int_0^\infty W(x,q_0)\rho(q_0)dq_0. \tag{3.1}$$

Here, x stands for the parameters occurring in the kernel W(x,q_0), such as s and τ. Rigorously solving Eq.(3.1) is an ill-posed problem since only a limited region of x can be used for the analyses and $G_{\text{OPE}}(x)$ is available only with limited precision. Nevertheless, the MEM technique enables us at least to statistically determine the most probable form of $\rho(q_0)$.

In the MEM analyses, we define $P[\rho|GH]$, which is the conditional probability of ρ given G_{OPE} and H, H representing additional knowledge on the spectral function such as positivity and asymptotic values. Using Bayes' theorem, $P[\rho|GH]$ is rewritten as

$$P[\rho|GH] = \frac{P[G|\rho H]P[\rho|H]}{P[G|H]}, \tag{3.2}$$

where $P[\rho|H]$ is the so-called prior probability, and $P[G|\rho H]$ stands for the likelihood function. The most probable form of $\rho(\omega)$ is obtained by maximizing $P[\rho|GH]$. Following [2, 11], we can obtain the specific form of $P[\rho|GH]$:

$$P[\rho|GH] \propto P[G|\rho H]P[\rho|H] \\ = e^{\alpha S[\rho]-L[\rho]} \equiv e^{Q[\rho]}, \tag{3.3}$$

where α is a positive number. $S[\rho]$ is known as the Shannon-Jaynes entropy and can be given as

$$S[\rho] = \int_0^\infty dq_0[\rho(q_0) - m(q_0) - \rho(q_0)\log(\frac{\rho(q_0)}{m(q_0)})], \tag{3.4}$$

where the function $m(\omega)$ is called the default model and is an input of the MEM method. The functional $L[\rho]$ is normally used for χ^2 fitting and can be obtained as

$$L[\rho] = \frac{1}{2(x_{\max} - x_{\min})}\int_{x_{\min}}^{x_{\max}} dx \frac{[G_{\mathrm{OPE}}(x) - \int_0^\infty W(x,q_0)\rho(q_0)dq_0]^2}{\sigma^2(x)}. \tag{3.5}$$

The error $\sigma(x)$ in the above equation is determined from the uncertainty of the vacuum condensates and is evaluated by the statistical method explained in [9]. Therefore, to get the most probable $\rho(\omega)$, we have to solve the numerical problem of obtaining the form of $\rho(\omega)$ that maximizes Q[ρ].

4. Results

Carrying out the analysis using the OPE data $G^{\mathrm{old}\,\pm}_{\mathrm{OPE}}(s,\tau,\theta = 0.108\pi)$ with MEM, we obtain the corresponding spectral functions. The results are shown in Fig. 2. In the positive parity spectral function, peaks are found at 970 MeV and 1930 MeV. As can be inferred from the error bars, the lowest peak which corresponds to the nucleon ground state is statistically significant, while the second one is not. For negative parity, peaks appear at 1540 MeV and 1840 MeV. As in the positive parity case, the second peak is not statistically significant. The lowest peak appears close to the

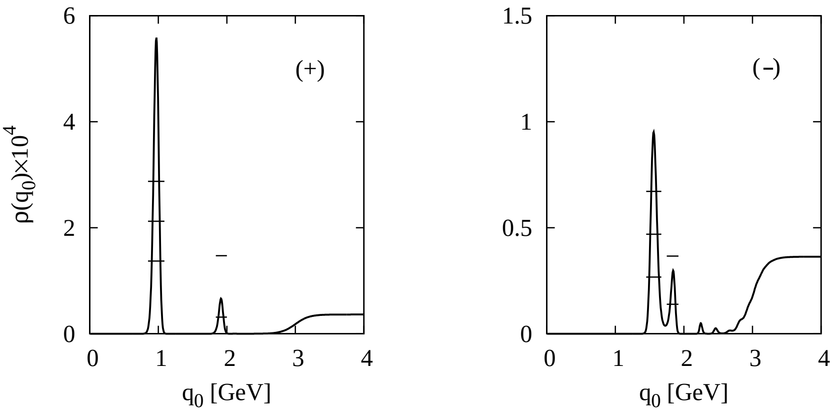

Figure 2: The positive (left) and negative parity (right) spectral functions extracted from MEM analyses of the OPE data $G^{\mathrm{old}\,\pm}_{\mathrm{OPE}}(s,\tau,\theta = 0.108\pi)$. The parity of the corresponding spectral functions is shown on the top right corner of each figure.

experimentally observed lowest negative parity state N(1535). However, we can not conclude that this peak only contains the contribution of the N(1535) due to its large width and the next lying state N(1650). As can be understood from Eq.(2.4), it is difficult to extract information on the physical width of some peak since the analysis uses an integral over the spectral function as an input, which is not very sensitive on the peak width [2, 6]. This fact could cause N(1650) to merge with N(1535) to one single peak. Therefore our conclusion to be drawn from this analysis is that some negative parity excited state exists near 1540 MeV.

5. Summary

We have constructed parity projected nucleon QCD sum rules taking into account first order α_s corrections. Furthermore using the phase-rotated kernel, we have remedied some technical problems of the usual nucleonic sum rules. We have then analyzed the nucleon spectral function in the vacuum from these sum rules using the maximum entropy method and successfully extracted information of both the positive and negative parity states. Because the difference of positive and negative parity is essentially caused by the chiral condensate term of dimension 3, these results provide novel evidence for the scenario in which the spontaneous breaking of chiral symmetry causes the splitting between positive and negative parity baryon states.

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