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Determination of CP and CPT violation parameters in the neutral kaon system using the Bell-Steinberger relation and WA data

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Mario Antonelli*
LNF-INFN
E-mail: mario.antonelli@lnf.infn.it

I present an improved determination of the *CP* and *CPT* violation parameters $\mathrm{Re}(\varepsilon)$ and $\mathrm{Im}(\delta)$ based on the unitarity condition (Bell-Steinberger relation) and on world average results from kaon experiments. We find $\mathrm{Re}(\varepsilon) = (161.1 \pm 0.5) \times 10^{-5}$ and $\mathrm{Im}(\delta) = (-0.7 \pm 1.4) \times 10^{-5}$, consistent with no *CPT* violation. Assuming no CPT violation in decays one can derive: $|m_{K^0} - m_{\overline{K}^0}| < 4 \times 10^{-19}$ GeV at 95 % C.L.

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*Speaker.

The three discrete symmetries of quantum mechanics, charge conjugation (C), parity (P) and time reversal (T), are known to be violated in nature, both singly and in bilinear combinations. Only CPT appears to be an exact symmetry of nature. Exact CPT invariance holds in quantum field theory, which assumes Lorentz invariance (flat space), locality and unitarity [1]. Testing the validity of CPT invariance therefore probes the most fundamental assumptions of our present understanding of particles and their interactions. These hypotheses are likely to be violated at very high energy scales, where quantum effects of the gravitational interaction cannot be ignored [2]. On the other hand, since we still lack a consistent theory of quantum gravity, it is hard to predict at which level violation of CPT invariance might become experimentally observable.

The neutral kaon system offers unique possibilities for the study of CPT invariance. From the requirement of unitarity, Bell and Steinberger have derived a relation, the so-called Bell-Steinberger relation [3]. The Bell-Steinberger relation relates a possible violation of CPT invariance ($m_{K^0} \neq m_{\overline{K}^0}$ and/or $\Gamma_{K^0} \neq \Gamma_{\overline{K}^0}$) in the time-evolution of the K^0–$\overline{K}^0$ system to the observable CP-violating interference of K_L and K_S decays into the same final state f. Strictly speaking, evidence of CPT violation found via the Bell-Steinberger relation could just be a failure of the unitarity assumption. However, unitarity is also one of the main hypotheses of the CPT theorem; thus the Bell-Steinberger relation allows a test of the basic assumptions of quantum field theories.

In this work we use recent results from the Kaon experiments to improve the determination of the phenomenological CP- and CPT-violating parameters $\mathrm{Re}(\varepsilon)$ and $\mathrm{Im}(\delta)$ by means of the BSR.

Strangeness oscillation in $K^0 - \overline{K}^0$ system, described by the equation

$$i\frac{d}{dt}\begin{bmatrix} K^0 \\ \overline{K}^0 \end{bmatrix} = [M - i\Gamma/2]\begin{bmatrix} K^0 \\ \overline{K}^0 \end{bmatrix} ,$$

where M and Γ are hermitian matrices (see PDG review[4], references [5][6], and KLOE paper[7] for notations and previous literature), allows a very accurate test of CPT symmetry; indeed since CPT requires $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$, the mass and width eigenstates, $K_{S,L}$, have a CPT-violating piece, δ, in addition to the usual CPT-conserving parameter ε:

$$K_{S,L} = \frac{1}{\sqrt{2\,(1+|\varepsilon_{S,L}|^2)}}\left[(1+\varepsilon_{S,L})\,K^0 + (1-\varepsilon_{S,L})\,\overline{K}^0\right]$$

$$\varepsilon_{S,L} = \frac{-i\mathrm{Im}\,(M_{12}) - \frac{1}{2}\mathrm{Im}\,(\Gamma_{12}) \mp \frac{1}{2}\left[M_{11} - M_{22} - \frac{i}{2}\,(\Gamma_{11} - \Gamma_{22})\right]}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2}$$

$$\equiv \varepsilon \pm \delta.$$

Using the phase convention $\mathrm{Im}(\Gamma_{12}) = 0$, we determine the phase of ε to be $\varphi_{SW} \equiv \arctan\frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L}$. Imposing unitarity to an arbitrary combination of K^0 and $\overline{K}^0$ wave functions, we obtain the Bell-Steinberger relation[8] connecting CP and CPT violation in the mass matrix to CP and CPT violation in the decay; in fact, neglecting $\mathscr{O}(\varepsilon)$ corrections to the coefficient of the CPT-violating parameter, δ, we can write[7]

$$[\frac{\Gamma_S + \Gamma_L}{\Gamma_S - \Gamma_L} + i\tan\phi_{\mathrm{SW}}][\frac{\mathrm{Re}(\varepsilon)}{1+|\varepsilon|^2} - i\mathrm{Im}(\delta)] = \frac{1}{\Gamma_S - \Gamma_L}\sum_f A_L(f)A_S^*(f)$$

where $A_{L,S}(f) \equiv A(K_{L,S} \to f)$. We stress that this relation is phase-convention-independent. The advantage of the neutral kaon system is that only a few decay modes give significant contributions to the r.h.s. in Bell-Steinberger relation; in fact, defining for the hadronic modes

$$\alpha_i \equiv \frac{1}{\Gamma_S}\langle \mathscr{A}_L(i)\mathscr{A}_S^*(i)\rangle = \eta_i\, \mathscr{B}(K_S \to i),$$

$$i = \pi^0\pi^0, \pi^+\pi^-(\gamma), 3\pi^0, \pi^0\pi^+\pi^-(\gamma),$$

the recent data from CPLEAR, KLOE, KTeV, and NA48 have led to the following determinations the analysis described in Ref [7] has been updated by using the recent measurements of K_L branching ratios from KTeV[9], NA48[10] and the recent result obtained by KTeV on the CP phases[**?**].

$$\alpha_{\pi^+\pi^-} = ((1.112 \pm 0.010) + i(1.061 \pm 0.010)) \times 10^{-3},$$

$$\alpha_{\pi^0\pi^0} = ((0.493 \pm 0.005) + i(0.471 \pm 0.005)) \times 10^{-3}\ ,$$

$$\alpha_{\pi^+\pi^-\pi^0} = ((0 \pm 2) + i(0 \pm 2)) \times 10^{-6},$$

$$|\alpha_{\pi^0\pi^0\pi^0}| < 7 \times 10^{-6} \text{ at 95\% CL}$$

The semileptonic contribution to the right-handed side of Bell-Steinberger relation requires the determination of several observables: we define[5][6]

$$\mathscr{A}(K^0 \to \pi^- l^+ \nu) = \mathscr{A}_0(1-y)\ ,$$

$$\mathscr{A}(K^0 \to \pi^+ l^- \nu) = \mathscr{A}_0^*(1+y^*)(x_+ - x_-)^*\ ,$$

$$\mathscr{A}(\overline{K}^0 \to \pi^+ l^- \nu) = \mathscr{A}_0^*(1+y^*)\ ,$$

$$\mathscr{A}(\overline{K}^0 \to \pi^- l^+ \nu) = \mathscr{A}_0(1-y)(x_+ + x_-)$$

where x_+ (x_-) describes the violation of the $\Delta S = \Delta Q$ rule in CPT-conserving (violating) decay amplitudes, and y parametrizes CPT violation for $\Delta S = \Delta Q$ transitions. Taking advantage of their tagged $K^0(\overline{K}^0)$ beams, CPLEAR has measured $\mathrm{Im}(x_+)$, $\mathrm{Re}(x_-)$, $\mathrm{Im}(\delta)$, and $\mathrm{Re}(\delta)$[13]. These determinations have been improved in Ref [7] by including the information $A_S - A_L = 4[\mathrm{Re}(\delta) + \mathrm{Re}(x_-)]$, where $A_{L,S}$ are the K_L and K_S semileptonic charge asymmetries, respectively, from the PDG[14] and KLOE[15]. Here we are also including the T-violating asymmetry measurement from CPLEAR[16].

The value $A_S + A_L$ in Table 1 can be directely included in the semileptonic contributions to the Bell Steinberger relations in Bell-Steinberger relation

$$\begin{aligned}\Sigma_{\pi\ell\nu}\langle \mathscr{A}_L(\pi\ell\nu)\mathscr{A}_S^*(\pi\ell\nu)\rangle &= 2\Gamma(K_L \to \pi\ell\nu)\left(\mathrm{Re}(\varepsilon) - \mathrm{Re}(y) - i(\mathrm{Im}(x_+) + \mathrm{Im}(\delta))\right)\\ &= 2\Gamma(K_L \to \pi\ell\nu)\left((A_S + A_L)/4 - i(\mathrm{Im}(x_+) + \mathrm{Im}(\delta))\right)\ .\end{aligned} \tag{1}$$

$$\alpha_{\pi\ell\nu} \equiv \frac{1}{\Gamma_S}\sum_{\pi\ell\nu}\langle \mathscr{A}_L(\pi\ell\nu)\mathscr{A}_S^*(\pi\ell\nu)\rangle + 2i\frac{\tau_{K_S}}{\tau_{K_L}}\mathscr{B}(K_L \to \pi\ell\nu)\mathrm{Im}(\delta)\ ,$$

we find:

$$\alpha_{\pi\ell\nu} = ((-0.2 \pm 0.5) + i(0.1 \pm 0.5)) \times 10^{-5}\ .$$

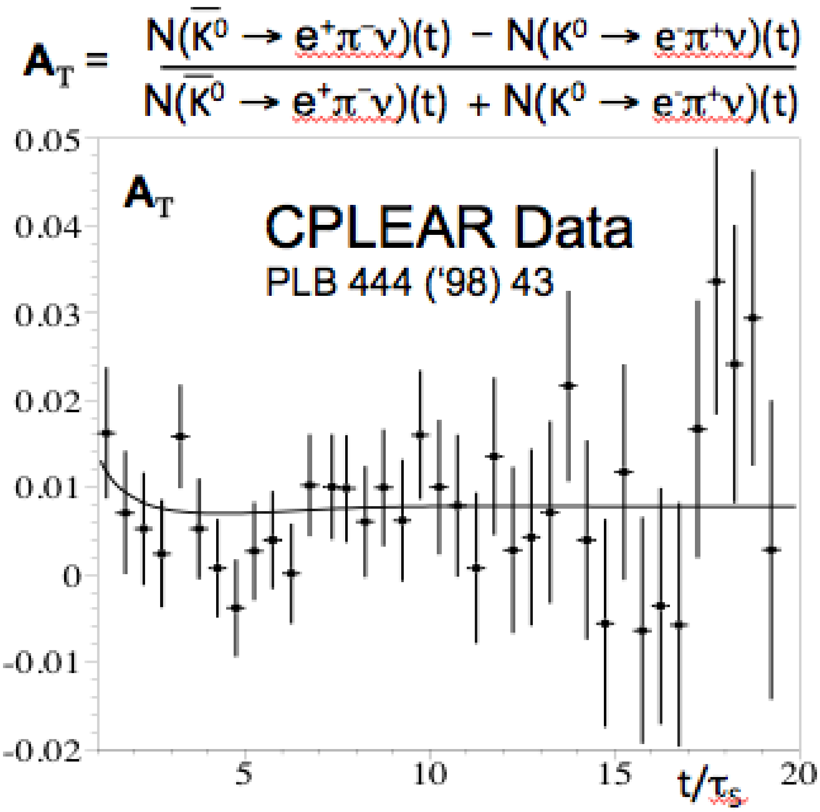

Figure 1: CPLEAR A_T measurement and our fit result.

	value	Correlation coefficients				
$\mathrm{Re}(\delta)$	$(3.0 \pm 2.3) \times 10^{-4}$	1				
$\mathrm{Im}(\delta)$	$(-0.66 \pm 0.65) \times 10^{-2}$	-0.21	1			
$\mathrm{Re}(x_-)$	$(-0.30 \pm 0.21) \times 10^{-2}$	-0.21	-0.60	1		
$\mathrm{Im}(x_+)$	$(0.02 \pm 0.22) \times 10^{-2}$	-0.38	-0.14	0.47	1	
$A_S + A_L$	$(-0.40 \pm 0.83) \times 10^{-2}$	-0.10	-0.63	0.99	0.43	1

Table 1: Values, errors, and correlation coefficients for $\mathrm{Re}(\delta)$, $\mathrm{Im}(\delta)$, $\mathrm{Re}(x_-)$, $\mathrm{Im}(x_+)$, and $A_S + A_L$ obtained from a combined fit, including KLOE [7] and CPLEAR [16].

Inserting the values of the α parameters into Bell-Steinberger relation, we find

$$\mathrm{Re}(\varepsilon) = (161.1 \pm 0.5) \times 10^{-5},$$

$$\mathrm{Im}(\delta) = (-0.7 \pm 1.4) \times 10^{-5}$$

The complete information is given in Table 2.

Now the agreement with *CPT* conservation, $\mathrm{Im}(\delta) = \mathrm{Re}(\delta) = \mathrm{Re}(x_-) = 0$, is at 20% C.L.

	value	Correlation coefficients			
$\mathrm{Re}(\varepsilon)$	$(161.1 \pm 0.5) \times 10^{-5}$	+1			
$\mathrm{Im}(\delta)$	$(-0.7 \pm 1.4) \times 10^{-5}$	+0.09	1		
$\mathrm{Re}(\delta)$	$(2.4 \pm 2.3) \times 10^{-4}$	+0.08	-0.12	1	
$\mathrm{Re}(x_-)$	$(-4.1 \pm 1.7) \times 10^{-3}$	+0.14	0.22	-0.43	1

Table 2: Summary of results: values, errors, and correlation coefficients for $\mathrm{Re}(\varepsilon)$, $\mathrm{Im}(\delta)$, $\mathrm{Re}(\delta)$, and $\mathrm{Re}(x_-)$.

The allowed region in the $\mathrm{Re}(\varepsilon) - \mathrm{Im}(\delta)$ plane at 68% CL and 95% C.L. is shown in the top panel of Fig 2.

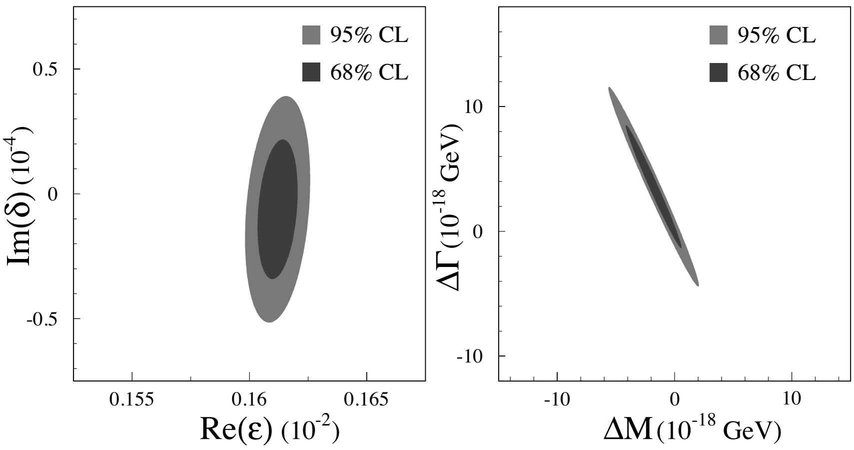

Figure 2: Left: allowed region at 68% and 95% CL in the $\mathrm{Re}(\varepsilon)$, $\mathrm{Im}(\delta)$ plane. Right: allowed region at 68% and 95% CL in the $\Delta M, \Delta\Gamma$ plane.

The process giving the largest contribution to the size of the allowed region is $K_L \to \pi^+\pi^-$, through the uncertainty on ϕ_{+-}.

The limits on $\mathrm{Im}(\delta)$ and $\mathrm{Re}(\delta)$ can be used to constrain the $K^0 - \overline{K}^0$ mass and width difference

$$\delta = \frac{i(m_{K^0} - m_{\overline{K}^0}) + \frac{1}{2}(\Gamma_{K^0} - \Gamma_{\overline{K}^0})}{\Gamma_S - \Gamma_L} \cos\phi_{SW} e^{i\phi_{SW}} \left[1 + \mathscr{O}(\varepsilon)\right].$$

The allowed region in the $\Delta M = (m_{K^0} - m_{\overline{K}^0}), \Delta\Gamma = (\Gamma_{K^0} - \Gamma_{\overline{K}^0})$ plane is shown in the bottom panel of Fig 2.

As a result, we improve on the previous limits (see for instance, P. Bloch in Ref. [14]) and in the limit $\Gamma_{K^0} - \Gamma_{\overline{K}^0} = 0$ we obtain

$$-4 \times 10^{-19}\ \mathrm{GeV} < m_{K^0} - m_{\overline{K}^0} < 4 \times 10^{-19}\ \mathrm{GeV\ at\ 95\ \%\ C.L.}$$

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