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A new approach to chiral two-nucleon dynamics

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A novel scheme developed recently for analyzing pion-nucleon scattering using chiral Lagrangian is applied to the nucleon-nucleon interaction close to threshold. Partial-wave amplitudes are obtained by an analytic extrapolation of subthreshold reaction amplitudes computed in chiral perturbation theory, where the constraints set by analyticity and unitarity are used to stabilize the extrapolation. Phase shifts are analyzed up to laboratory energies $T_{\rm lab} \simeq 250$ MeV in terms of the parameters relevant at next-to-next-to-leading order. The convergence properties of the method are discussed.

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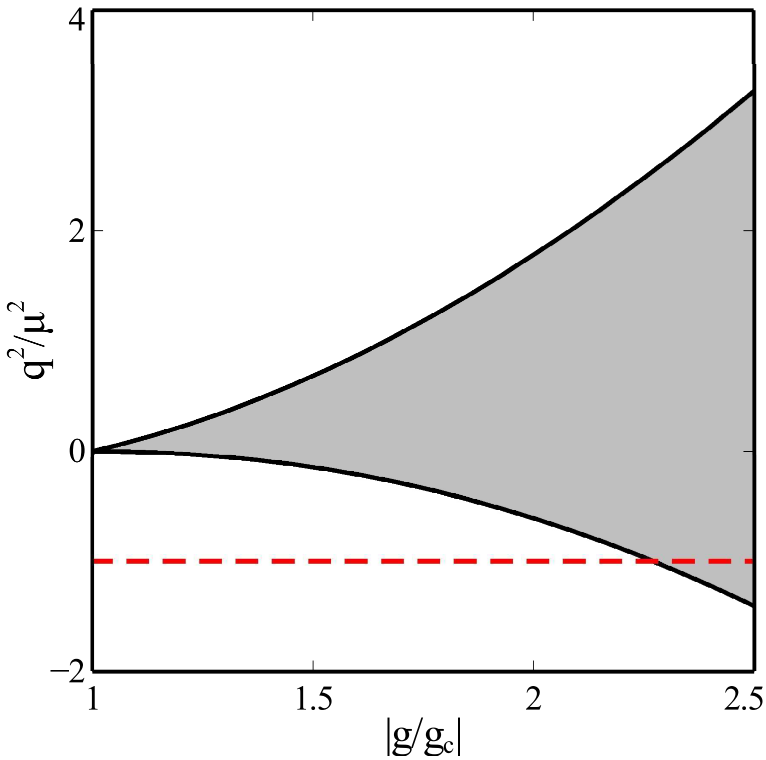

Figure 1: Convergence domain of the Born series for the *s*-wave scattering in the Yukawa potential as a function of the coupling strength a/g_c.

1. Introduction

Chiral perturbation theory is a powerful tool for studying low-energy hadron dynamics. It turned out to be quite successful in particular in describing the $\pi\pi$ and πN scattering processes in the threshold region. On the other hand in the nucleon-nucleon sector a strictly perturbative treatment is impossible even close to threshold. Large *S*-wave scattering lengths and the deuteron bound state are obvious manifestations of this fact. One of the solutions of this problem within ChPT based on the ideas of Weinberg [1] is to implement a potential approach. In such a scheme, the scattering amplitude is generated by solving a dynamical equation in a non-perturbative manner with a potential calculated from the chiral Lagrangian. A remarkable progress in this direction has been achieved in the recent years [2, 3].

We suggest an alternative approach [4] based on analytic properties of the scattering amplitude and the assumption that the interaction turns perturbative in some energy region below the two-nucleon threshold. One of the motivations for such an assumption can be a toy model with Yukawa potential $ge^{-\mu r}/r$ between two particles in non-relativistic quantum mechanics [5]. Fig. 1 shows the regions in the parameter plane of the coupling constant g (in some units) and the energy (in terms of $\frac{q^2}{\mu^2}$) where the Born series for the *s*-wave amplitude does not converge (shaded area) and where it converges (the rest of the plane). One can see that for sufficiently large values of the coupling constant the threshold region is non-perturbative, whereas at some energies below threshold one can use the standard perturbation theory. Of course, this toy model is just an illustration and the

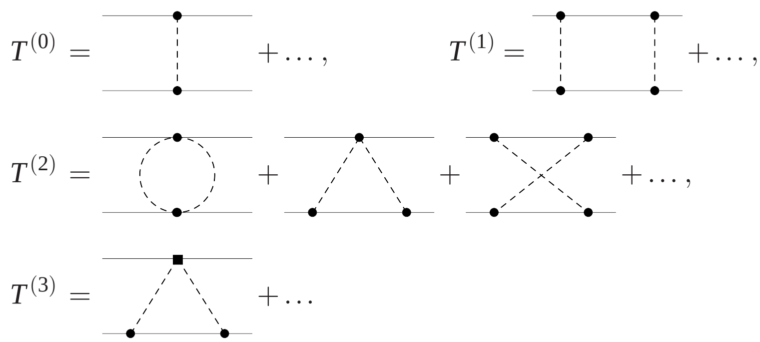

Figure 2: Feynman diagrams contributing to the NN scattering amplitude $T^{(i)}$ at order Q^i in the chiral expansion. Solid dots (filled squares) denote the lowest-order vertices in the chiral Lagrangian (subleading vertices $\propto c_i$). The ellipses denote diagrams leading to shorter-range contributions.

realistic picture differ from it. This picture can be justified only after an application to the real NN system is performed, and an analytic continuation of the subthreshold amplitude into the above-threshold region is carried out. This procedure was worked out in detail for the case of πN [10] and $\pi\pi$ [11] scattering and turned out to be quite successful in describing the data.

2. Analytic continuation of the subthreshold amplitude

The starting point of our approach is the nucleon-nucleon scattering amplitude calculated from the chiral Lagrangian using perturbation theory (see e.g. [6, 7]). We follow here the regular dimensional power counting that leads to the contributions up to the chiral order Q^3 as shown in Fig. 2. Renormalization of the loop diagrams is performed using the strategy analogous to the ones proposed in [6, 8, 9].

The next step consists in the analytic continuation of the subthreshold partial wave amplitudes calculated perturbatively into the physical region taking into account the singularity structure of the amplitude in the complex s-plane. The cut structure of the NN partial-wave amplitude is well-known. In particular, the nearest left-hand cuts correspond to 1(2,3)-pion exchange contributions and are located at $s = 4m_N^2 - (nM_\pi)^2$, $n = 1,2,3$, whereas the nearest right-hand cuts are located at $s = (2m_N + nM_\pi)^2$ with $n = 0,1,2$ and correspond to the elastic and pion-production thresholds.

The partial wave dispersion relation separates right- and left-hand singularities of the amplitude and reads

$$T(s) = U(s) + \int_{4m_N^2}^{\infty} \frac{ds'}{\pi}\, \frac{s-\mu_M^2}{s'-\mu_M^2}\, \frac{T(s)\,\rho(s')\,T^*(s')}{s'-s-i\varepsilon}\,, \tag{2.1}$$

where μ_M is the matching point where the perturbation theory is assumed to converge most rapidly. Its exact position is not important but it must be sufficiently far away from s- and t-channel unitarity cuts (excluding the one-pion exchange cut). We chose it, therefore, to be $\mu_M^2 = 4m_N^2 - 2M_\pi^2$.

The generalized potential $U(s)$ possesses only the left hand cuts (we neglect the contributions from the inelastic channels) and can be represented as the sum [10]

$$U(s) = U_{\text{inside}}(s) + U_{\text{outside}}(s), \tag{2.2}$$

where $U_{\text{inside}}(s)$ is calculated in chiral perturbation theory through the discontinuity $\Delta T(s)$

$$U_{\text{inside}}(s) = \int_{\Lambda^2}^{4m_N^2-M_\pi^2} \frac{ds'}{\pi} \frac{\Delta T(s')}{s'-s}, \tag{2.3}$$

whereas $U_{\text{outside}}(s)$ is analytically continued to some domain Ω using conformal mapping technique [10]. This domain includes the energy region we are interested in and defines the separation of the 'inside' and 'outside' contributions to the generalized potential. Outside of the Ω-region determined by the values of Λ^2 and Λ_s^2 the generalized potential $U(s)$ cannot be calculated reliably. We choose $\Lambda^2 = 4m_N^2 - 9M_\pi^2$ at the beginning of the 3π-cut separating the nonperturbative short range dynamics and $\Lambda_s^2 = (2m_N + 2M_\pi)^2$ corresponding to energies where inelastic channels become important.

Finally, one restores the amplitude above the threshold by solving Eq. (2.1) with $U(s)$ as an input.

3. Results

There are several free parameters that we fit to the proton-neutron scattering data by means of the partial wave analysis [12]. Another PWA [13] is used for comparison. These parameters are the coefficients of the conformal mapping transformation and are in one-to-one correspondence with the low-energy constants of the chiral Lagrangian. We consider only the lowest (S and P) partial waves where the non-perturbative dynamics is most prominent. For the fit the energy points below $T_{\text{lab}} = 100$ MeV are taken into account in order to correctly reproduce the threshold physics.

The results of our fit at different chiral orders are shown in Fig. 3(4) for the uncoupled (coupled) partial waves.

We show phase shifts and mixing angles at energies up to $T_{\text{lab}} = 250$ MeV where neglecting by the influence of inelastic channels is still justified. At orders Q^0 and Q^1, there is only one free parameter in each of the S-waves and none in the other partial waves. At orders Q^2 and Q^3, there is one more free parameter in the S-waves and one free parameter in each P-wave and in ε_1. The quality of the fit is comparable with the one obtained in the potential approach. In particular, the convergence pattern from the order Q^0 to Q^3 looks very reasonable. This supports our assumption of the perturbative nature of the nucleon-nucleon interaction in the subthreshold region. Note that in our scheme, the box diagram contributes at order Q^1. On the other hand, in the potential approach it is generated through an iteration of the leading order one-pion exchange potential. It is, therefore, difficult to disentangle the contributions of one- and two-pion-exchange left-hand cuts. Such a separation appears naturally in our method, and one can see the relative importance of the leading (order-Q^1) two-pion-exchange cut. In particular, for the coupled partial waves, its contribution significantly improves the agreement with the data, see Fig. 4, without introducing additional free parameters.

We also studied the sensitivity of our results to a particular choice of the values of the parameters μ_M and Λ_s and found no significant dependence on these values within the physically reasonable range.

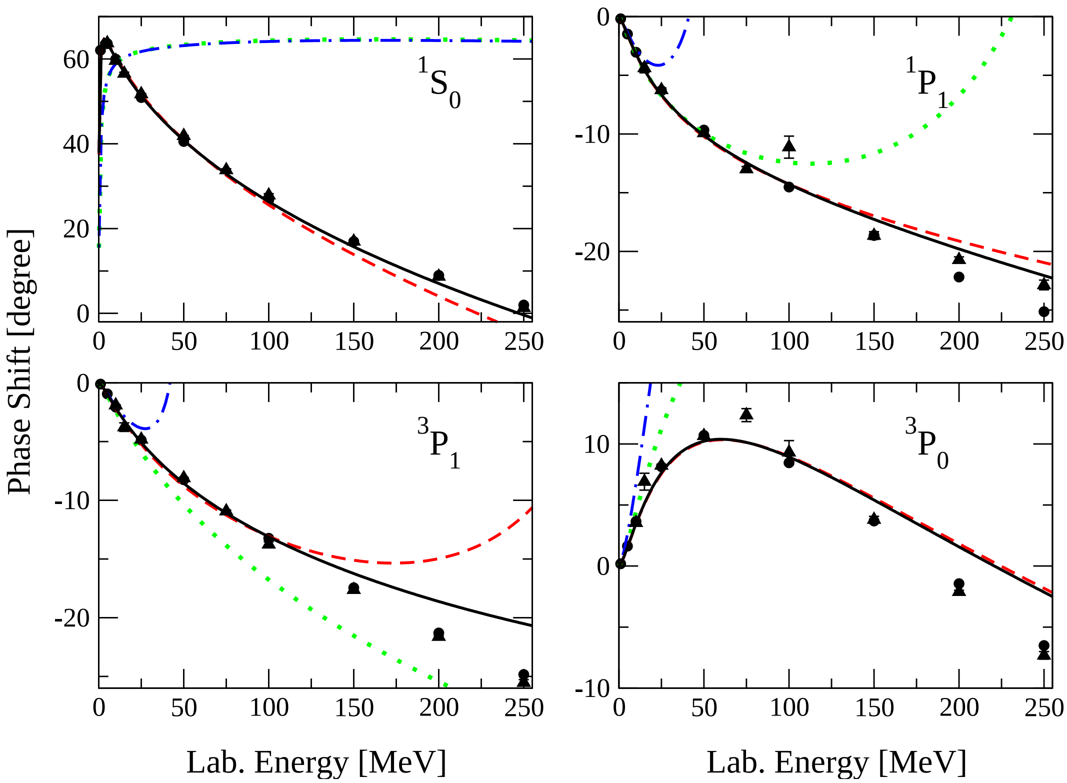

Figure 3: Phase shifts in uncoupled S- and P-waves calculated at orders Q^0 (dotted lines), Q^1 (dash-dotted lines), Q^2 (dashed lines) and Q^3 (solid lines) in comparison with the Nijmegen [12] (filled circles) and SAID [13] (filled triangles) PWAs.

4. Summary

We studied nucleon-nucleon interaction from threshold up to $T_{\rm lab} = 250$ MeV with a novel approach developed in [10] based on an analytic extrapolation of subthreshold amplitudes calculated in ChPT by means of partial-wave dispersion relations combined with a conformal mapping technique. The free parameters, which are related with the low energy constants of the chiral Lagrangian, were adjusted to the nucleon-nucleon phase shifts using existing partial-wave analyses. The obtained fits are in a reasonable agreement with the empirical PWAs. The quality of the fit at order Q^3 is comparable with the one obtained in conventional approaches based on the chiral expansion of the NN potential. In all partial waves, the expansion for the phase shifts converges when going from the order Q^0 to Q^3. This supports our assumption of the validity of a perturbative expansion of the NN amplitude in the subthreshold region. We also found our results to be rather weakly dependent on the parameters μ_M and Λ_s provided these parameters are varied within a physically acceptable range.

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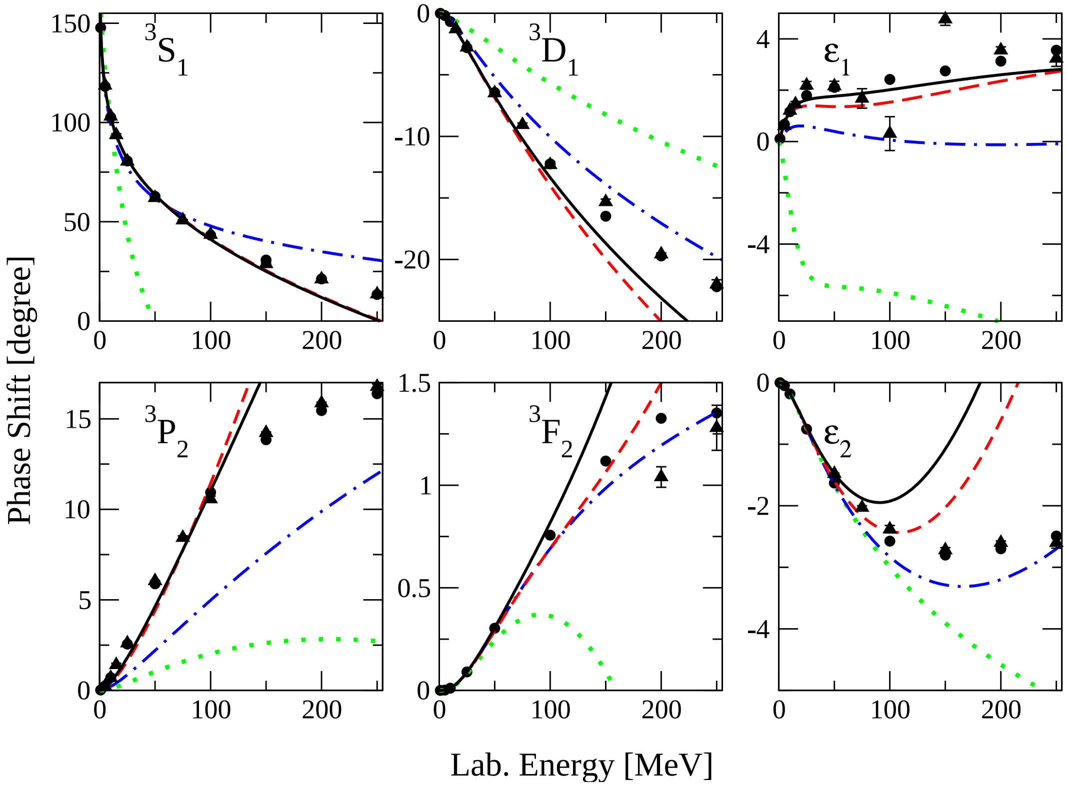

Figure 4: Phase shifts and mixing angles in the coupled $^3S_1$-$^3D_1$ and $^3P_2$-$^3F_2$ channels. For notation see Fig. 4.

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