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Theoretical studies of muon capture on light nuclei

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We review the present status of theoretical studies of muon capture reactions on light nuclei. In particular we consider the two reactions ^{2}H(μ^-,ν_μ)nn and ^{3}He(μ^-,ν_μ)^{3}H, and the most recent calculations performed within the chiral effective field theory framework. The unresolved discrepancies among the different calculations and future developments are also discussed.

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1. Introduction

In the past few years there has been a significant body of theoretical work on muon captures on light nuclei, in particular on the reactions

$$\mu^- + d \rightarrow n + n + \nu_\mu \, , \tag{1.1}$$

$$\mu^- + {}^3\mathrm{He} \rightarrow {}^3\mathrm{H} + \nu_\mu \, . \tag{1.2}$$

This large interest is motivated by the fact that these processes are related to astrophysically relevant reactions, such as the weak captures on proton and ^{3}He (the *pp* and *hep* reactions), whose rates cannot be measured experimentally [1].

The observables of interest for the two reactions are the doublet capture rate Γ^D for reaction (1.1), i.e., the rate obtained when the stopped muons are captured from the initial doublet hyperfine state, and the total capture rate Γ_0 for reaction (1.2). The experimental situation for reaction (1.2) is quite clear: a very precise determination yielded $\Gamma_0 = (1496 \pm 4)$ sec^{-1} [2], a value consistent with those of earlier measurements, although these were affected by considerably larger uncertainties. On the other hand, Γ^D is poorly known: the available experimental data are (365 ± 96) sec^{-1} [3], (445 ± 60) sec^{-1} [4], (470 ± 29) sec^{-1} [5] and (409 ± 40) sec^{-1} [6]. These measurements, while consistent with each other, are not very precise, with errors in the $6 - 10$ % range. However, there is hope to have this situation clarified by the MuSun Collaboration [7], which is performing at present an experiment at the Paul Scherrer Institut, with the goal of measuring Γ^D with a precision of 1.5 %. Part of the renewed interest on the capture processes (1.1) and (1.2) has been indeed spurred by this experiment.

Theoretical work on reactions (1.1) and (1.2) is very extensive. The status *ante* 2012 has been recently reviewed in Ref. [8], where the theoretical formalism and the details of the different calculations are clearly discussed. Here, we briefly review again only the latest work of Refs. [9, 10].

In Refs. [9, 10], reactions (1.1) and (1.2) are simultaneously studied within two different schemes: the "potential model approach" (PMA), also known as "standard nuclear physics approach", and the approach known as the "hybrid" chiral effective field theory (χEFT), often indicated with χEFT*. In PMA, Hamiltonians based on conventional two-nucleon (NN) and three-nucleon (NNN) potentials are used to calculate the nuclear wave functions, and the weak transition operator includes, beyond the one-body contribution (the impulse approximation—IA) associated with the basic process $p + \mu^- \rightarrow n + \nu_\mu$, meson-exchange currents as well as currents arising from the excitation of Δ-isobar degrees of freedom [11]. In the χEFT* approach, the weak operators are derived in χEFT, but their matrix elements are evaluated between wave functions obtained from conventional potentials. Typically, the PMA and χEFT* predictions are in good agreement with each other. For example, the PMA and χEFT* calculations for Γ^D give 391 sec^{-1} and (393 ± 1) sec^{-1}, respectively [9]. For Γ_0, the PMA and χEFT* calculations give 1486 sec^{-1} and (1484 ± 4) sec^{-1}, respectively. Note that in Ref. [9] the radiative corrections of Ref. [12] were not included. These would increase the PMA (χEFT*) central values of Ref. [9] to 392 (395) sec^{-1} for reaction (1.1), and 1496 (1494) sec^{-1} for reaction (1.2). In summary, the combined PMA and χEFT* results of Ref. [9] provide for Γ^D and Γ_0 a conservative range of (393.5 ± 3.9) sec^{-1} and (1493 ± 19) sec^{-1}, when radiative corrections with their uncertainties are considered [12].

Note that just one year before the work of Refs. [9, 10], an other calculation of Γ^D was performed in Ref. [13] within the PMA, although the meson-exchange currents were of different origin. The results of Ref. [13] differ from those of Refs. [9, 10] by $7-10$ %. Only very recently, the origin of this discrepancy has been understood, as it will discussed in the next section.

The present paper continues as follows: in Sec. 2 we review and discuss the results of the first "non-hybrid" χEFT calculation of reactions (1.1) and (1.2). Some concluding remarks are given in Sec. 3.

2. The "non-hybrid" χEFT study of muon capture

The first "non-hybrid" χEFT study of reactions (1.1) and (1.2) has been performed in Ref. [14]. The nuclear interaction consists of NN and NNN potentials. The NN potential has been derived in Refs. [15, 16] up to next-to-next-to-next-to leading order (N3LO). It consists of one-pion- and two-pion-exchange and of contact terms. The low-energy constants (LEC's) entering the contact terms have been constrained by accurate fits to the NN scattering database at energies below the pion production threshold. The NNN potential, which first contributes at next-to-next-to leading order (N2LO), includes S- and P-wave two-pion-exchange —its P-wave piece is the familiar Fujita-Miyazawa NNN potential—a one-pion-exchange plus NN contact term with LEC c_D and a NNN contact terms with LEC c_E. The N2LO NNN potential used in Ref. [14] is the local version obtained in Ref. [17].

The vector part of the weak current has been derived in χEFT up to N3LO by several groups. However, only in Refs. [18] and [19] a consistent model for both the vector and axial pieces of the weak current has been constructed up to N3LO. Therefore, this model has been considered in Ref. [14]. In this model, the one-body operators are the same as those obtained in the PMA by retaining, in the expansion of the covariant single-nucleon four-current, corrections up to order $(v/c)^2$ relative to the leading-order term [11]. The two-body operators in the axial current include a one-pion-exchange contribution, involving the known LEC's c_3 and c_4 (determined by fits to the NN data [16]), and one contact current, whose strength is parametrized by the LEC d_R (see below). In the axial charge, only one-pion-exchange contributes, and the associated operator is proportional to g_A/F_π^2, where g_A (F_π) is the single-nucleon axial coupling constant (pion decay constant). The vector weak current is related (via the CVC constraint) to the electromagnetic current, which includes at N3LO one-pion- and two-pion-exchange (i.e., one-loop corrections), as well as isoscalar and isovector contact terms, whose strengths are parametrized by the LEC's denoted, respectively, as g_{4S} and g_{4V} [18, 19]. No two-nucleon vector charge operators are present at N3LO. Finally, we notice that due to the power-law behavior for large momenta of both potentials and currents, these have been regularized by introducing a momentum-cutoff function, with cutoff Λ taken to be 500 MeV and 600 MeV.

The mentioned LEC's d_R, c_D, c_E, g_{4S}, and g_{4V} have been determined as follows: as it has been observed in Refs. [20] and [21], the LEC's d_R and c_D are related to each other via the relation

$$d_R = \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6} \, , \tag{2.1}$$

where M_N is the nucleon mass and $\Lambda_\chi = 700$ MeV is the the chiral-symmetry-breaking scale. Then, the calculation is implemented as follows. The ^{3}H and ^{3}He ground state wave functions are calculated with the hyperspherical-harmonics method (see Ref. [22] for a review) using the chiral NN+NNN potentials of Refs. [15, 16, 17] for $\Lambda = 500$ and 600 MeV. The corresponding set of LEC's $\{c_D, c_E\}$ is determined by fitting the $A = 3$ experimental binding energies, BE(^{3}H)=8.475 MeV and BE(^{3}He)=7.725 MeV, corrected for small contributions (+7 keV in ^{3}H and –7 keV in ^{3}He) due to the *n*-*p* mass difference [23]. Then the range $c_D \in [-3,3]$ is considered, and, in correspondence to each c_D in this range, c_E is determined so as to reproduce either BE(^{3}H) or BE(^{3}He). The resulting trajectories, shown in Fig. 1 of Ref. [14], are nearly indistinguishable. Then, for each set of $\{c_D, c_E\}$, the triton and ^{3}He wave functions are calculated and, using the χEFT weak axial current discussed above, the Gamow-Teller matrix element of tritium β-decay (GT$^{\rm TH}$) is determined. The ratio GT$^{\rm TH}$/GT$^{\rm EXP}$ is considered, for both $\Lambda = 500$ and 600 MeV, with GT$^{\rm EXP} = 0.955 \pm 0.004$ [9, 14], and the range of c_D values, for which GT$^{\rm TH}$ = GT$^{\rm EXP}$ within the experimental error, is found to be $[-0.20, -0.04]$ for $\Lambda = 500$ MeV, and $[-0.32, -0.19]$ for $\Lambda = 600$ MeV. The corresponding ranges for c_E are $[-0.208, -0.184]$ and $[-0.857, -0.833]$, respectively.

For the minimum and maximum values of $\{c_D, c_E\}$ in the selected range, i.e., $\{-0.20, -0.208\}$ and $\{-0.04, -0.184\}$ for $\Lambda = 500$ MeV, and $\{-0.32, -0.857\}$ and $\{-0.19, -0.833\}$ for $\Lambda = 600$ MeV, the isoscalar and isovector LEC's, g_{4S} and g_{4V}, entering the NN contact terms of the electromagnetic current, are determined by reproducing the $A = 3$ magnetic moments. These LEC's are listed in Table I of Ref. [14]. At this point, the potential and current models are fully constrained, and the results for Γ^D and Γ_0 are χEFT predictions. They are found to be $\Gamma^D = (399 \pm 3)$ sec^{-1} and $\Gamma_0 = (1494 \pm 21)$ sec^{-1}, including electroweak radiative corrections [12]. These results are in good agreement with those of Refs. [9, 10], as well as with experimental data, but not with those of Ref. [13], for which the discrepancy remains of the order of $4-9$ %. We now discuss in some detail the origin of this discrepancy. Some of the authors of Ref. [13] have performed in Ref. [24] a χEFT calculation of Γ^D very similar to the one presented in this section and in fact contemporary to it. In this calculation, Γ^D has been found in the range $383.8 - 392.4$ sec^{-1}, depending on the χEFT two-nucleon potential used. This result is very different from the previous PMA calculation of Ref. [13], and it is in much better agreement with the results of Refs. [9, 10, 14]. In fact, the study of Ref. [24] has a very interesting history, according to the two different versions of the manuscript, still present on the archive, `www.arXiv.org`. In the very first version of the manuscript, Γ^D was found in the range $401.2 - 409.8$ sec^{-1}, in good agreement with the results of Ref. [13], and again in disagreement with those of Refs. [9, 10, 14]. The printed version of the manuscript, which corresponds to the second version of the preprint on the archive, however, reported the range quoted above, $383.8 - 392.4$ sec^{-1}. Therefore, we can conclude that one or more computational errors were found before the submission of the second version, suggesting that very likely the results of Ref. [13] were also affected by the same errors. For this reason, they should be disregarded. Finally, it should be pointed out that the authors of Ref. [24] consider, besides the χEFT potential mentioned here of Ref. [15], also two other potentials, labelled EGM(204) and EGM(205), taken from Ref. [25]. The quoted range $383.8 - 392.4$ sec^{-1} is obtained excluding the EGM(205) results. Such range would become $383.8 - 419.1$ sec^{-1} when all the χEFT potential models are considered, with a theoretical uncertainty of about 8 %. The authors, though, do not provide any explanation on why the EGM(205) results should be excluded. Such a large theoretical uncertainty, as argued by the authors themselves, is presumably due to the fact that the $c_D - c_E$ fitting procedure is not always applied including the NNN potential. In any case, the results of

Ref. [24] and the ones discussed here [9, 10, 14] are in agreement within few %.

3. Conclusions

The most recent theoretical studies of the muon capture reactions on deuteron and ^{3}He have been reviewed and discussed. They are performed either using phenomenological potentials and currents, or phenomenological potentials in conjunction with weak currents derived in χEFT, or finally using both potentials and currents obtained consistently within the χEFT approach. The calculated values for the doublet capture rate (total capture rate) for muon capture on deuteron (^{3}He) are in agreement with each other and in agreement with the available experimental data. Furthermore, the theoretical uncertainty is reduced to the level of a %, even for the latest "non-hybrid" χEFT calculation. The differences between the calculations reviewed here and those of Refs. [13, 24] have been discussed. In particular, we have pointed out that the results of Ref. [13] were clearly affected by some computational errors and should be disregarded. We expect the authors of Ref. [13] to publish one day an *erratum*, eventually.

We would like to conclude with the following observations: (i) the present "non-hybrid" χEFT framework can be used to calculate reactions of astrophysical interest, as the pp or hep reactions, as well as the proton-deuteron radiative capture reaction. Work along this line is strongly pursued. (ii) Some small inconsistencies are still present in what we have called "non-hybrid" χEFT framework. In fact, the NN and NNN potentials are derived at two different chiral order (N3LO and N2LO, respectively), and the momentum-cutoff functions used for the NNN potential and nuclear currents are different than those present in the NN potential. However, if these inconsistencies were significant, we would have expected, as a general trend of χEFT calculations, a strong cutoff dependence of the results. Instead, the cutoff-dependence of the presented results is extremely weak.

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