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Neutron-rich Helium isotopes based on hyperspherical harmonics

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We present recent results for neutron-rich Helium isotopes obtained from the hyperspherical harmonics method. Ground-state properties, like the binding energy and the point-proton radius are shown for the two-neutron halo nucleus ^{6}He using two-body low-momentum interactions derived from chiral forces. The applicability of the method to the four-neutron halo nucleus ^{8}He is discussed. As an excited-state observable we present a recent calculation of the nuclear electric polarizability of ^{6}He from a semi-realistic potential. A comparison of the calculated quantities to experimental data is performed.

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1. Introduction

The physics of light neutron-rich nuclei is particularly interesting, because of the appearance of their exotic structures, like those of halo nuclei. The lightest nuclei of this kind are found in the Helium isotope chain: ^{6}He as a two-neutron and ^{8}He as a four-neutron halo nucleus. They are both radioactive and undergo β-decay with a half life of $t_{1/2} = 0.8$s and 0.1s, respectively. Despite the short life time, a combination of atomic and nuclear physics techniques, has enabled precise measurements of ground-state observables like the energy and the charge radius [1]. Excited-state properties, like electromagnetic transitions in the continuum have been investigated in the past with Coulomb dissociation experiments by Aumann *et al*. [2]. Tackling the theoretical study of these nuclei is very challenging, because one needs to simultaneously describe the small separation energy of the halo neutrons and the large radius of the whole system. Because they are light-mass nuclei, one can use *ab-initio* techniques to study them. Here, we will discuss the recent results obtained using the hyperspherical harmonics (HH) method.

A major breakthrough in nuclear physics has been the development of chiral effective field theory, which is well routed to Quantum Chromo Dynamics. Even though several light nuclei have been investigated with chiral potentials, we are still missing a prediction of Helium halo nuclei from chiral Hamiltonians. Here, we show a first step taken in this direction by using low-momentum chiral two-nucleon forces.

The paper is organized as follows. In section 2 we will introduce the hyperspherical harmonics method. In section 3 and 4 we will present results for ground-state and excited-state properties, respectively. Finally, in section 5 we will draw some conclusions.

2. Hyperspherical Harmonics

Given the Hamiltonian H we use the HH expansion to solve the Schrödinger equation. The HH method is typically a few-body method used for 3 and 4-body systems. Using the powerful antisymmetrization algorithm introduced in [3], it is possible to extend the method to a larger mass number and tackle Helium halo nuclei. The HH approach starts from the Jacobi coordinates

$$\eta_0 = \frac{1}{\sqrt{A}}\sum_{i=1}^{A}\mathbf{r}_i\,, \qquad \eta_{k-1} = \sqrt{\frac{k-1}{k}}\left(\mathbf{r}_k - \frac{1}{k-1}\sum_{i=1}^{k-1}\mathbf{r}_i\right), k=2,...,A\,, \tag{2.1}$$

where $\mathbf{r}_i$ are the particle coordinates. Using the η_i one can then transform to hyperspherical coordinates composed of one hyperradial coordinate $\rho = \sqrt{\sum_{i=1}^{A-1}\eta_i^2}$ and a set of $(3A-4)$ angles that we denote with Ω (for more details see [3]). Using this coordinates one can recursively construct the hyperspherical harmonics $\mathscr{Y}_{[K]}$ and use them as a complete basis to expand the wave function. Such expansion reads

$$\Psi(\eta_1,...,\eta_{A-1},s_1,...,s_A,t_1,...,t_A) = \sum_{n}^{n_{\max}}\sum_{[K]}^{K_{\max}} C_{[K]n}\,R_n(\rho)\,\mathscr{Y}_{[K]}(\Omega,s_1,...,s_A,t_1,...,t_A), \tag{2.2}$$

where s_i and t_i are the spin and isospin of the nucleon i, respectively; $C_{[K]n}$ is the coefficient of the expansion, labeled by $[K]$, which represents a cumulative quantum number that includes the

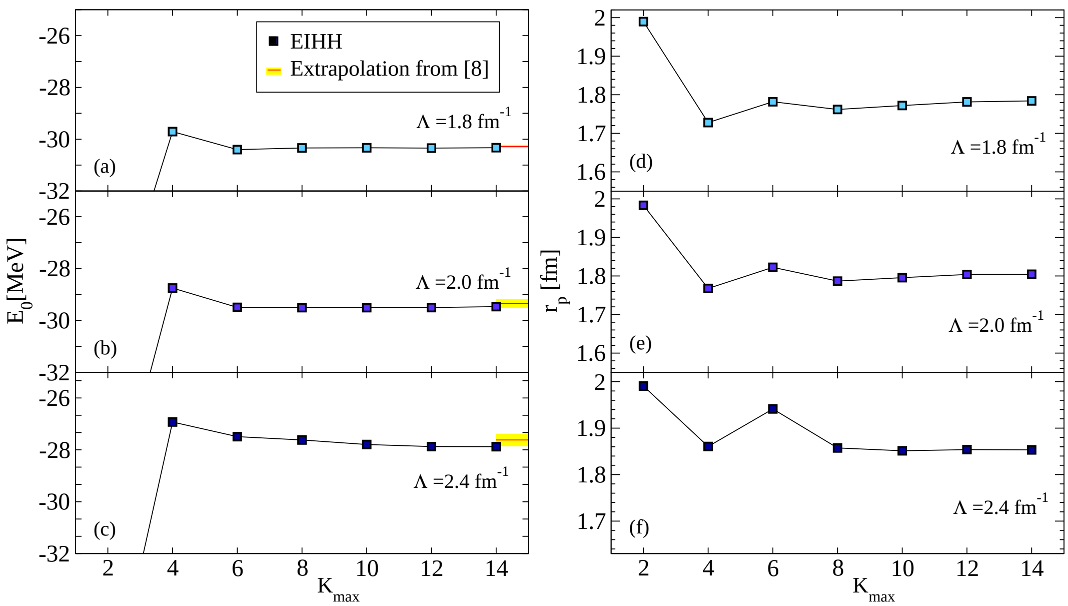

Figure 1: (Color online) The ^{6}He ground-state energy (left) and the point-proton radius (right) as functions of K_{max} obtained for three different values of the cutoff $\Lambda = 1.8, 2.0, 2.4$ fm^{-1} of the $V_{\mathrm{low}\,k}$ chiral potential.

grandangular momentum K; n labels the hyperradial wave function $R_n(\rho)$. The latter is expanded in terms of the generalized Laguerre polynomials times an exponentially falling off function, which is essential to speed up the convergence of halo nuclei, due to their extended tail. To further increase the convergence rate of the calculations, we typically employ an effective interaction in the hyperspherical harmonics (EIHH), as first introduced in [4].

3. Results for ground state properties

In the following, we present our results for the ground state energy and the point-proton radius calculated using the hyperspherical harmonics method. As input Hamiltonian we employ a class of low-momentum potentials, which are obtained applying a $V_{\mathrm{low}\,k}$ procedure [5] on a starting chiral two-body potential [6] at next-to-next-to-next-to leading order (N ^{3}LO).

In Fig. 1, we show the ^{6}He convergence patterns of the EIHH method for the ground-state energy and the point-proton radius as a function of the maximal grandangular momentum [7]. Three different cutoffs $\Lambda = 1.8, 2.0, 2.4$ fm^{-1} of the $V_{\mathrm{low}\,k}$ chiral potential are shown. One observes that the convergence is very nice for all Λ's, even for the largest cutoff. Concerning the energy we also show that the extrapolated results from previous work on hyperspherical harmonics [8] agree nicely with EIHH. The convergence rate is very good also for the point-proton radius, which allows us to provide solid results for ^{6}He.

We have also explored the four-neutron halo nucleus ^{8}He with hyperspherical harmonics. In Fig. 2, we show the ^{8}He ground state energy from a $V_{\mathrm{low}\,k}$ chiral potential with $\Lambda = 1.8$ fm^{-1} as a function of K_{max}. We present both the variational HH expansion (where we do not apply the effective interaction) and the EIHH results. The convergence for ^{8}He is quite slow, as indicated by the fact that the HH and EIHH patterns do not merge yet at $K_{\mathrm{max}} = 10$. Interestingly, an ex-

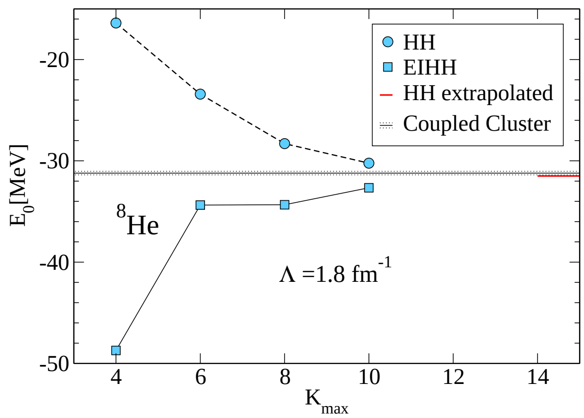

Figure 2: (Color online) The ^{8}He ground-state energy calculated with the HH and EIHH methods as a function of $K_{\rm max}$ obtained for a cutoff of $\Lambda = 1.8$ fm^{-1} of the $V_{{\rm low}\,k}$ chiral potential. The extrapolated HH results are shown as a reference and compared to the Coupled-Cluster results.

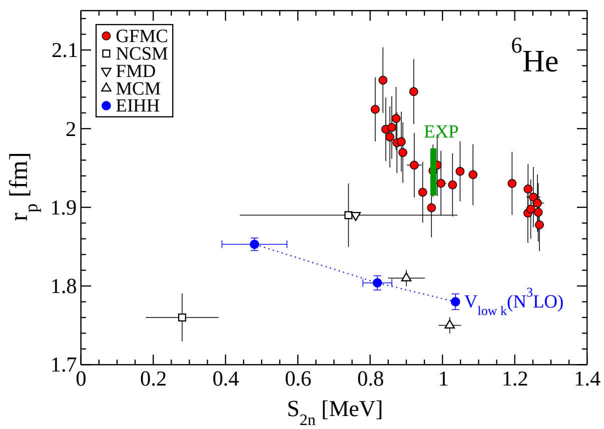

Figure 3: (Color online) Correlation plot of the ^{6}He point-proton radius versus the two-neutron separation energy S_{2n}. The experimental range is compared to theory based on different *ab-initio* methods (see text).

trapolation of the variational HH data lies close to the Coupled-Cluster result from [8] with the same interaction. Nevertheless, at the moment we are not able to provide precise results for ^{8}He from hyperspherical harmonics. Thus, we concentrate on analyzing the correlation between energy and radius just for ^{6}He. In Fig. 3, we plot $r_{\rm p}$ versus the two-neutron separation energy S_{2n} and present a combined comparison of our results to experiment and other *ab-initio* calculations: Green's Function Monte Carlo (GFMC), No Core Shell Model (NCSM), Fermionic Molecular Dynamics (FMD) and Microscopic Cluster Model (MCM) (see also [7] and references therein). The cutoff dependence of our results with $V_{{\rm low}\,k}$ allows us to study the correlation between these observables: the radius increases as the separation energy decreases. Our calculations do not reproduce simultaneously $r_{\rm p}$ and S_{2n}: there exists an optimal value of Λ where S_{2n} is predicted in accordance with experiment, but $r_{\rm p}$ is not reproduced and vice-versa. Also, we would like to note that other calculations which omit $3NF$, (all except from the GFMC) do not go though the experimental band.

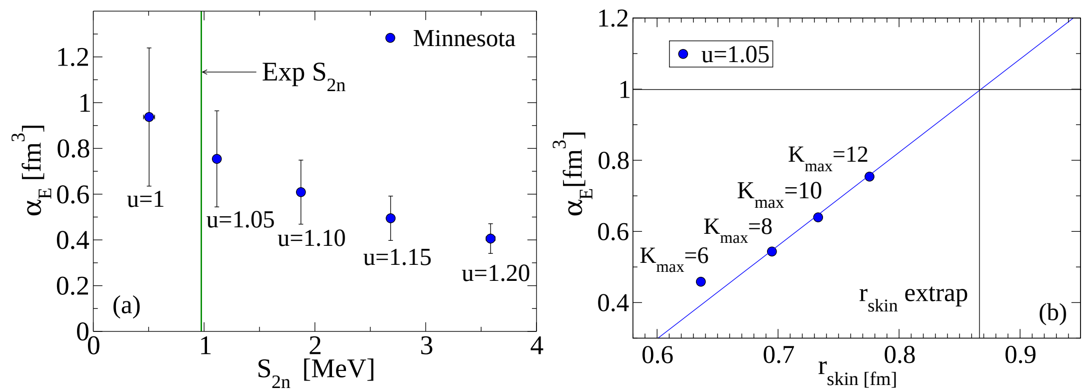

Figure 4: (Color online) Panel (a): The correlation between α_E and S_{2n} in ^{6}He obtained with the the Minnesota potential varying the parameter u. Panel (b): The correlation between $r_{\rm skin}$ and α_E in different model spaces (different $K_{\rm max}$). Shown is also the extrapolated value of $r_{\rm skin}$, which is used to estimate α_E.

This points towards the importance of including three-nucleon forces in the Hamiltonian.

4. Results for excited state properties

As an example of excited-state properties of halo nuclei we report about our recent calculation of the electric dipole polarizability α_E of ^{6}He [9]. α_E is related to the inelastic response of the nucleus to an externally applied electric field and is relevant in the extraction of nuclear quantities from atomic spectroscopic measurements. The atomic energy levels, in fact, are affected by polarization of the nucleus due to the electric field of the surrounding electrons. The polarizability of ^{6}He could be extracted from Coulomb dissociation measurement of the dipole transition by Aumann *et al.* [2] and was reported in Ref. [10] to be much bigger than the polarizability of ^{4}He. The electric dipole polarizability is defined by

$$\alpha_E = 2\alpha \sum_{f\neq 0} \frac{|\langle \Psi_f | E1 | \Psi_0 \rangle|^2}{E_f - E_0}\,, \tag{4.1}$$

where $|\Psi_{0/f}\rangle$ is the ground state and final state of the nucleus and $E1$ is the dipole operator. Because it requires the knowledge of the dipole spectrum of the nucleus, its theoretical evaluation is more involved than a bound-state calculation. We perform our calculation with the EIHH method by using the Lanczos algorithm with a starting dipole pivot as explained in [9]. As nuclear potential we chose the simple semi-realistic Minnesota force, which reproduces the experimental value of the polarizability of ^{4}He reasonably well. Within this force model we can add attractive $P-$wave interactions by changing the parameter u (see [9] for details). This mostly affects ^{6}He, without substantially changing ^{4}He. By varying u we first observe a correlation of α_E vs S_{2n}, as shown in Fig. 4(a). We have chosen u so that the halo feature, represented by S_{2n}, is reproduced. We then study the correlations between α_E and the skin radius $r_{\rm skin} = r_n - r_p$, where r_n is the mean point-neutron radius. By varying the model space we observed that α_E and $r_{\rm skin}$ are correlated linearly for $K_{\rm max} \geq 6$ as $\alpha_E = a + b\, r_{\rm skin}$. From our theoretical data we fit the coefficients a and b and then we used them to estimate the polarizability out of a bound-state calculation of the skin radius. The calculation of $r_{\rm skin}$, in fact, does not require an expansion on the dipole excited states and as such is less computationally demanding and can be performed for larger model spaces ($K_{\rm max} = 16$) and then extrapolated exponentially, leading to $r_{\rm skin} = 0.87(5)$ fm. Using our extrapolated skin radius and the linear dependence, we estimate the theoretical nuclear electric polarizability of ^{6}He to be $\alpha_E = 1.00(14)$ fm^3. The error bar is obtained by propagating the errors on a, b and $r_{\rm skin}$. We observe that our theoretical estimate is about a factor of two smaller than the experimental value of $\alpha_E^{\rm exp} = 1.99(40)$ fm^3 [10]. This points toward a potential disagreement between theory and experiment. Investigations with chiral potentials can possibly help understanding this discrepancy.

5. Conclusions

In conclusion, we have presented our recent results on Helium halo nuclei from hyperspherical harmonics. The binding energy and the radius can be precisely calculated for ^{6}He using chiral low-momentum two-body forces. The obtained cutoff dependence together with a comparison to the experiment serves to highlight the importance of three-nucleon forces. We also discussed the nuclear dipole polarizability of ^{6}He as an excited state observable, which has recently attracted attention. Our estimate from simplified nuclear potentials leads to a disagreement with experimental data, which will be hopefully clarified in the future when realistic chiral potentials will be used.

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