PROCEEDINGS OF SCIENCE

On possible role of scalar glueball-quarkonia mixing and its implications in J/Psi - decays

S. B. Gerasimov*
JINR
E-mail: gerasb@theor.jinr.ru

The next to lowest mass scalar multiplet is treated as the $q\bar{q}$, P-wave nonet, weakly mixed with the lower - mass, presumably $qq\bar{q}\bar{q}$ S-wave nonet and, in principle, with the $J^{PC}=0^{++}$ -glueball. The modified Gell-Mann-Okubo and Schwinger-type mass-formulas are used to derive and discuss the quark-gluon configuration structure of the obtained meson states which are then used to obtain the relations between the ratio $Br(J/Psi \to \omega G)/Br(J/Psi \to \phi G)$ and alike,G being the glueball, and the $f_0(1370), f_0(1506)$ quarkonia decay modes into a pair of the lowest mass pseudoscalar mesons $(\pi\pi, K\bar{K}, \eta\eta, etc)$.

XXI International Baldin Seminar on High Energy Physics Problems
September 10-15, 2012
JINR, Dubna, Russia

*Speaker.

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PoS(Baldin ISHEPP XXI)018

1. Introduction

As is known, the precise understanding of mass and dynamics of the glueball decays is problematic up to now in spite of very large number of works devoted to the problems mentioned (*e.g.* [1],[2], [3] and references therein).

We concentrate on the mass region $1.3 \div 1.7$ GeV occupied by the spin-zero 0^{++} mesons. In this group of mesons there are three isoscalar mesons with similar masses which, in the presence of the nearly lying isotriplet and isodoublet ones, suggest the overpopulated nonet where a possible glueball is hidden within structures of the three isoscalar states. Whether this idea is right or wrong one should deduce from data on the reactions creating them as well as from the relations between the branching ratios of their decays. With this in mind, we present the results of a simple approach enabling one to discuss an acute problem of the existence and properties of glueballs with quantum numbers $I^G J^{PC} = 0^+0^{++}$ (for the different approaches, see the mentioned [1],[2],[3] and [4]).

2. Underlying relations

We define the 3×3 mass-matrix $V\hat{(i)}$ as acting on the basis vectors N, S, G to transform them into one of three vectors of the physical meson states $f_0(i)$:

$$(f_0(i)) = \hat{V}(i) \cdot \begin{pmatrix} N \\ G \\ S \end{pmatrix} \tag{2.1}$$

where

$$N = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \qquad S = s\bar{s},$$

and G is the glueball.

We consider the mass-matrices $\hat{V}(i)$ taking into account explicitly the different appearance of the two types of gluon effects in mixing states of the differing flavor. In a certain sense, we follow the way proposed in old works by Isgur [5] to connect the strong "non-ideality" of the $SU(3)$-singlet-octet mixing angle in the lowest pseudoscalar and scalar meson nonet with the overwhelmingly strong, as compared to the respective term in the vector or tensor meson nonet, annihilation term in the mass-matrix inducing the non-diagonal $q\bar{q} \leftrightarrow s\bar{s}, (q = u, d)$ transitions. We remind that the celebrated Gell-Mann–Okubo[6] formula

$$3m_{f_8}^2 = 4m_{K_0^*}^2 - m_{a_0}^2$$

follows as the mass sum rule after exclusion of parameters introduced into the general mass term of the phenomenological meson lagrangian

$$M^2 \cdot Tr(V_8 V_8) - \mu^2 \cdot Tr(V_8 V_8 \lambda_8).$$

Okubo [7] proposed to replace $V_8 \to V_9$ in the GMO mass operator and drop the term proportional to $Tr(V_9)$. The well-known "ideal mixing " mass relations

$$m^2(\rho) = m^2(\omega), \;\; 2m^2(K^*) - m^2(\rho) = m^2(\phi)$$

PoS(Baldin ISHEPP XXI)018

are fulfilled for the vector and reasonably well for tensor nonet but poor for the pseudoscalar one.

We indicate also the hierarchy of meson masses following from the effective lagrangian GMO

$$m^2(s\bar{s}) \geq m^2(q\bar{s}) \geq m^2(q\bar{q}).$$

The idea to relate the apparently specific situation for the pseudoscalar meson sector with additional strong annihilation mechanism transforming the quark field combinations into each other was put forward phenomenologically by Isgur[5] and is now interpreted as mediated by short-range fluctuations in the quark-gluon vacuum [8].

3. Mass formulas and the mass matrix

We follow these ideas in the further generalized form via introducing the "bare" scalar glueball mass and nondiagonal glueball-quarkonium transition-mass into the spin-zero meson mass-matrices.

Hence, in the $N = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), S = s\bar{s}$ basis our symmetric real mass-matrix acquires the following form:

$$\hat{M}^2 = \begin{pmatrix} M_N^2 + 2A_Q & \sqrt{2}A_G & \sqrt{2}A_Q \\ \sqrt{2}A_G & M_G^2 & A_G \\ \sqrt{2}A_Q & A_G & M_S^2 + A_Q \end{pmatrix} \tag{3.1}$$

After reducing it to the diagonal form we should get the matrix of the eigenvalues $\hat{M}^2{}_{ph}$:

$$\hat{M}^2{}_{ph} = \begin{pmatrix} M_{f_0}^2(1) & 0 & 0 \\ 0 & M_{f_0}^2(2) & 0 \\ 0 & 0 & M_{f_0}^2(3) \end{pmatrix}$$

4. The isoscalar "sub-multiplets" in the scalar sector

We start treating the mass relations with the higher-mass scalar 0^{++}-sector:

$$M_{a_0} = 1474 \pm 19, M_{K^*{}_0} = 1425 \pm 50$$

$$M_{f_0}(1) = 1370 \pm 50, M_{f_0}(2) = 1505 \pm 6,$$

$$M_{f_0}(3) = 1720 \pm 7$$

where all values are in MeV [9].

We define the "bare" mass values M_N and M_S devoid of the strong annihilation contributions via

$$M_N = M_{a_0}, M_S{}^2 = 2M_{K^*{}_0}{}^2 - M_{a_0}{}^2 \tag{4.1}$$

A short digression: the second relation is alike of the S-wave vector quarkonia, but we would like to note the opposite mass hierarchy sequence

$$M^2(s\bar{s}) \leq M^2(q\bar{s}) \leq M^2(q\bar{q})$$

PoS(Baldin ISHEPP XXI)018

which follows from the suggested mixing of two scalar nonets composed of light (u,d,s)-quarks: the low-mass, presumably the two-quark-two-antiquark-nonet with the total orbital angular moment $L = 0$ and $(mass^2) \leq 1GeV^2$ [10][11][12] and the higher-mass, quark-antiquark states with orbital moment $L = 1$. The term A_Q in the mass matrix represents the dynamical self-mass term determined by the short-ranged, quark-flavor changing processes, while M_G and A_G are the "bare" gluon mass and the non-diagonal self-mass term arising in the course of the $SU(3)_{flavor}$-conserving quarkonium-glueball transitions. These three unknown terms have to be found by solution of the system of three non-linear equations representing the equalities of three invariants of the diagonalization process: the trace, the determinant and the sum of main minors of the matrices before and after diagonalization. The diagonalized mass-matrix is assumed to contain only experimentally defined masses of scalar meson resonances. Successively excluding unknown variables A_Q and A_G in favor of M_G, we solve numerically the resulting equation by varying the remaining unknown M_G under constraint ${A_G}^2 \geq 0$. There is trivial "decoupling-solution" $A_G = 0$ and $M_G \simeq M_{f_0(3)}$ and none for the "postulated" ${A_G}^2 > 0$ representing, by convention, the nonzero mixing of the glueball G with the remaining quarkonia. Therefore, we have to accept for the decoupled physical glueball mass our solution practically coinciding with the mass of $M_{f_0}(3)$

$$M_G(ph) \simeq 1730\, MeV \text{ vis-a-vis } M_{f_0}(3) = 1720 \pm 7\, MeV$$

The state vectors of $f_0(1506)$ and $f_0(1370)$ are obtained then by the diagonalization of the rest 2×2 matrix:

$$\begin{aligned} f_0(1506) &= 0.868 \cdot N \pm 0.496 \cdot S \\ f_0(1370) &= \mp 0.496 \cdot N + 0.868 \cdot S \end{aligned} \tag{4.2}$$

The choice of signs remains to be done on the physics ground.

5. A model for $f_0(q\bar{q})$-resonance production and subsequent decays

Out of the different type of parton components appearing in the decays $J/Psi \rightarrow hadrons$, namely, the quarkonium, $R \equiv |q\bar{q}>$, the "hybrid" $H \equiv |gq\bar{q}>$ and the glueball $G \equiv |gg>$ we consider the presumably lower in the mass components - the quarkonium and glueball which can be described by generally mixed state vectors. Further we assume that a pair of the almost massless "current" quarks produced in the "hard" process of the heavy $c\bar{c}$ - annihilation will evolve into massive valence or "constituent" quarks as the result of non-perturbative interaction with soft fluctuating vacuum of the QCD-fields. As to the final stage of the reaction, $f_0(q\bar{q}) \rightarrow \pi\pi(K\bar{K}, etc.)$, we accept a kind of the vacuum $q_{vac}\bar{q}_{vac}$-pair "pick-up" process by the quarks bound in meson resonance to form the colorless hadrons in final state [13]. The vacuum pair participating in the process may appear due to the vacuum auto-ionization by the moving color-charged dipole of quarks in the meson resonance. Otherwise, it can be thought as emerging due to the break of the "string" connecting and binding the valence quarks inside the meson resonance that accompanied by the production of the "extra" $q\bar{q}$-pair .

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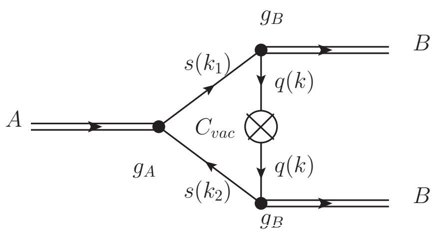

Figure 1: Diagram illustrating the conversion of the bound $q\bar{q}$-state into the pair of pseudoscalar mesons.

Heavier mass of the strange quark $m_s > m_{u(d)}$ invites to foresee some $SU(3)_{flavor}$ violation in the vertex including the effective coupling C_{vac} in the vertex of the $q\bar{q}(^3P_0)$-production off "vacuum"-state [14].

$$\Sigma_{q=u,d,s} C_q^{vac} \cdot q\bar{q} = C_{u(d)}^{vac} \cdot (u\bar{u} + d\bar{d}) + C_s^{vac} \cdot s\bar{s}; C_s^{vac} \leq C_{u(d)}^{vac} \tag{5.1}$$

Defining

$$\Gamma(f_0(i) \to \pi^+\pi^-) = \frac{2}{3}\Gamma(f_0(i) \to \pi\pi) \tag{5.2}$$

$$\Gamma(f_0(i) \to K^+K^-) = \frac{1}{2}\Gamma(f_0(i) \to K\bar{K}) \tag{5.3}$$

$$\frac{\Gamma(f_0(i) \to K^+K^-)}{\Gamma(f_0(i) \to \pi^+\pi^-)} = \frac{\sqrt{m^2_{f_0(i)} - 4m^2_K}}{\sqrt{m^2_{f_0(i)} - 4m^2_\pi}} \times \frac{g^2(f_0(i)K^+K^-)}{g^2(f_0(i)\pi^+\pi^-)} \tag{5.4}$$

$$\frac{g^2(f_0(i)K^+K^-)}{g^2(f_0(i)\pi^+\pi^-)}|_{(orbit)\times(spin)\times(flav)} \simeq \frac{g^2(f_0(i)K^+K^-)}{g^2(f_0(i)\pi^+\pi^-)}|_{(flav)}$$
$$\simeq \frac{[2^{-1/2}C_q(f_0(i)) \cdot C_s^{vac} + C_s(f_0(i)) \cdot C_q^{vac}]^2}{[2^{1/2}C_q(f_0(i)) \cdot C_q^{vac}]^2} \tag{5.5}$$

we take $y = C_s^{vac}/C_q^{vac}$ as a free parameter defined to be $y \simeq .32 \sim \mathcal{O}((m_q/m_s)^2)$ from experimental value $\Gamma(f_0(1500) \to K\bar{K})/\Gamma(f_0(1500) \to \pi\pi) = .246 \pm .026$. With the same y we obtain other ratios for quarkonium states reasonably comparable with the available data. In the quark-flavor basis, the $\eta - \eta^{'}$ mixing-angle was taken $\theta \simeq 39^o$ (*e.g.*,Ref.[15]). For the glueball state $f_0(1720)$, we suggest the (not-quantified yet) model of the breaking of the closed gluonic string resulting in the production of a color- and flavor-neutral $q\bar{q}$ system different from the $f_0(1370)$ and $f_0(1500)$. This $q\bar{q}$ system is directly materialized into the pairs of pseudoscalar meson states. Hence, in Table 1 below, the row corresponding to the glueball-$f_0(1720)$ is still absent. The data in Table 1 are taken from [9] and the literature collected there.

P_1P_2	$\pi\pi$	$\eta\eta$	$\eta^{'}\eta$
$f_0(mass)$	$\frac{BR(f_0(mass)\to P_1P_2)}{BR(f_0(mass)\to K\bar{K})}$		
$f_0(1370)\vert_{mod}$	.9	.09	—
$f_0(1370)\vert_{exp}$	1.0 ± 0.2	—	—
$f_0(1506)\vert_{mod}$	input	.6	—
$f_0(1506)\vert_{exp}$	4.1 ± 0.4	$.6 \pm .1$	—

PoS(Baldin ISHEPP XXI)018

6. A model for $f(0)$-resonance production and decays in processes including the J/Psi(3006)

The sensitive check of our results can also provide the radiative and hadronic decays of the J/Psi-resonance. In the radiative transitions, it is natural to accept the dominance of diagrams of the annihilation of bound $c\bar{c}$)-quarks to photon and the pair of intermediate gluon followed by the hadronization process $J/Psi \to \gamma gg \to \gamma + hadrons$. Then

$$\frac{\Gamma(J/\Psi \to \gamma + f_0(1370))}{\Gamma(J/\Psi \to \gamma + f_0(1506))} = \frac{|\vec{k}(f_0(1370))|}{|\vec{k}(f_0(1506))|} \cdot (\frac{-.868 \cdot sin\theta_{id} + .496 \cdot cos\theta_{id}}{.868 \cdot cos\theta_{id} + .496 \cdot sin\theta_{id}})^2 \approx .009 \qquad (6.1)$$

that is a tiny value for accepted (lower) signs of quark amplitudes of the $f_0(1500)$- and $f_0(1370)$-resonances and for the angle $\theta_{id} \simeq 35^o$ of the "ideal" singlet-octet mixing in $SU(3)_{flav}$ assumed to be valid except in masses. The ratio of the 3-momenta stands for the ratio of the phase space factors accepted for the decays of the type $A \to B + C$ and giving a minimal kinematic dependence in terms of particle masses in the initial and final states. Under these assumptions, we turn to several processes with participation of the vector $\phi(1.02)$- and $\omega(.783)$-mesons that serving to be very good flavor "filters" for the state vectors of scalars participating in a particular reaction. The matrix elements of the process $(J/\Psi \to V + f_0(1720))$ include a series of virtual transitions $J/\Psi \to 3g \to V + gg \to V + f_0(1720)$ that are proportional to the well-known SU(3)-singlet component of the ω- and ϕ- meson and to the form-factors of the $3gV$-vertex. As gluons are assumed to be effectively-massive vector quanta [16], we replace approximately the ratios of the full ω- and $\phi-$ form-factors by the respective ratios of their radial "functions-at-zero-distance", entering into the ratios of the widths of $V \to e^- e^+$ - decays, according to the Van Royen - Weisskopf relation[17]. Thus,

$$R_{\omega\phi}(f_0(1720) = \frac{|\vec{k}_{\omega f_0(1720)}|}{|\vec{k}_{\phi f_0(1720)}|} \cdot (tan\theta_V)^{-2} \cdot \frac{R^2_\omega(0)}{R^2{}_\phi(0)} \simeq 1.71(1.33 \pm .34) \qquad (6.2)$$

Fully analogously

$$R_{\omega\phi}(f_0(1370) = \frac{|\vec{k}_{\omega f_0(1370)}|}{|\vec{k}_{\phi f_0(1370)}|} \cdot (tan\theta_V)^{2} \cdot \frac{R^2_\omega(0)}{R^2{}_\phi(0)} \simeq .7 \qquad (6.3)$$

7. Concluding remarks

- Large amount of strange quarks in $f_0(1370)$ is favored by the experimental observation that the decay $J/\Psi \to \phi\pi\pi$ cannot be fitted without excitation of the $f_0(1370)$-resonance unlike the reaction with ϕ replaced by ω [18] *cf* our relation for $\omega f_0(1370)$-to-$\phi f_0(1370$ yields.

- Dominance of the pion(s) decay mode of $f_0(1506)$ [9] is in agreement with a large amount of the nonstrange $q\bar{q}$-quarks in the state-vector of this resonance.

- The data of the BES-Collaboration [18] seem to signal on a hierarchy of the radiative decay modes :

$$BR(J/\Psi \to \gamma f_0(1370)) < BR(J/\Psi \to \gamma f_0(1506)) < BR(J/\Psi \to \gamma f_0(1720))$$

PoS(Baldin ISHEPP XXI)018

that is in accord with the lower sign ("minus")in the state-vector of $f_0(1506)$ and with the glueball nature of $f_0(1720)$.

- The color-averaged, gluon-exchange contributions operating in transitions $c\bar{c} \to gg \to gg|_{bound}$ are proportional to the 3-gluon couplings and are at least $3/(4/3)$ times larger than in the corresponding $c\bar{c} \to gg \to q\bar{q}|_{bound}$ -transitions. Furthermore, the factor of the "wave-function-at-zero" squared which is natural in the transition of the gluon-gluon S-wave state, has further advantage over the derivative of "wave-function-at-zero" squared connected with the transition matrix element of two-quark P-wave state. Hence the ratio $BR(J/\Psi \to \gamma + f_0(1506)) = (1.01 \pm .32) \cdot 10^{-4}$ is markedly smaller than $BR(J/\Psi \to \gamma + f_0(1720) \to \gamma K\bar{K}) = (8.5^{+1.2}{}_{-.9}) \cdot 10^{-4}$ [9] and especially than the ratio

$$\frac{BR(J/\Psi \to \gamma + f_0(1720) \to \gamma\bar{K}K)}{BR(f_0(1710) \to K\bar{K})} \geq (8.5^{+1.2}{}_{-.9}) \cdot 10^{-4}$$

- We note in conclusion that the glueball dominance of the $f_0(1720)$-structure was inferred also in a number of earlier works [19] [20], though with a different *N*- and *S*-composition of the $f_0(1370)$ and $f_0(1506)$ mesons.

8. Acknowledgments

The author acknowledges the support by the RFBR grant No.10-02-00368a.

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PoS(Baldin ISHEPP XXI)018

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PoS(Baldin ISHEPP XXI)018