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Calculations of the pion-nucleus elastic scattering differential cross sections using the microscopic optical potential

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The calculations are made using the pion-nucleus microscopic optical potential (OP), constructed basing on the pion-nucleon scattering amplitude and the density distribution function of a target nucleus. With this OP, the relativistic wave equation was numerically solved to get the scattering cross sections, and thus both the relativistic effects and distortions of the relative motion wave functions are taken into account. The data are analyzed of the $\pi^{\pm}$-meson scattering on the nuclei ^{28}Si,^{58}Ni,^{208}Pb at T_{lab}=291 MeV and the in-medium parameters of the pion-nucleon amplitude are established and compared with those for pions scattered on free nucleons.

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1. Introduction

It is customary to use two approaches in calculations of the pion-nucleus elastic scattering cross sections in the region of the (3,3) resonanse energies T_{lab} ∼100-300 MeV. The one is based on the Kisslinger optical potential (OP) [1] transformed in [2] to the simple local form. Such OP contains information on the density distribution of a target nucleus and also on the $s-, p-$ and d-phases of the pion-nucleon amplitude of scattering, and thus the potential depends on several (7 and more) parameters including the phenomenological parameter of nuclear correlations ξ [3]. This OP was studied thoroughly in [4], and also in [5] (see refs therein) where the wide applications were made. But if one interested in the in-medium effect on the pion-nucleon amplitude then the problem arises to fit too many parameters used to describe the πN phases. The other approach is based on the high-energy multiple scattering theory developed by Glauber and Sitenko [6, 7] where the pion-nucleon amplitude and nuclear density distribution functions are also utilized. In one of the first paper [8], the 3-parameter amplitudes were taken separately for π^+- and π^--meson scattering on protons and neutrons. However in succeeding years the most applications make use of the joint ("averaged"over isospin) 3-parameter pion-nucleon amplitudes (see, e.g., [9]). As to the Glauber approach as a whole there exist difficulties when accounting for the Coulomb and nuclear distortions of the incident and exit waves and also in calculations of rather complicated multiple scattering terms of phases. The latter problems usually force one to apply the simple gaussian forms for the target nucleus wave functions, and therefore the approach is working well only for light nuclei.

In the present paper, for analysis of the pion-nucleus scattering, we apply a simple microscopic optical potential (OP) [10] derived by comparisons of the eikonal phase of a potential and that obtained in the optical limit of the Glauber theory. This PO is applied to solve the relativistic wave equation and to get the respective elastic scattering differential cross sections. Below we present the main formulas. Then calculations are made where the parameters of the pion-nucleon amplitude, entered into OP, are fitted to get reasonable agreements to the experimental pion-nucleus cross sections. Thus, the in-medium effect on the $\pi^{\pm}N$ scattering amplitude is estimated and discussed.

2. The model and test calculations

The microscopic model of OP was developed in [10] and based on the eikonal phase of the Glauber theory in its optical approximation. It yields the following complex potential:

$$U_{opt}(r) = -\frac{(\hbar c)\beta_c}{(2\pi)^2} \sum_{N=p,n} \sigma_{\pi N}\,[i+\alpha_{\pi N}] \cdot \int_0^\infty j_0(qr)\,\rho_N(q)\,f_{\pi N}(q)\,q^2 dq. \qquad (2.1)$$

Here $\rho_N(q)$ is the form factor of a density distribution of bare nucleons in a nucleus normalized to Z for the nuclear proton density and to $A-Z$ for the neutron one, while $\sigma_{\pi N}$ and $\alpha_{\pi N}$ are the total πN cross sections and the ratio of real to imaginary part of the pion-nucleon amplitude at forward scattering. The amplitude itself has the form

$$F_{\pi N}(q) = \frac{k}{4\pi}\,\sigma_{\pi N}\,[i+\alpha_{\pi N}] \cdot f_{\pi N}(q), \qquad f_{\pi N} = e^{-\beta_{\pi N} q^2/2}, \qquad (2.2)$$

where parameters $\sigma_{\pi N}$, $\alpha_{\pi N}$ and the slope parameter $\beta_{\pi N}$ are done in a number of works where the phase shift analysis of the pion-nucleon scattering data has been made at different energies. Relativistic velocity $\beta_c = v_{c.m.}/c$ in the c.m. pion-nucleus system is expressed through the total laboratory energy $E_{lab} = (k_{lab}^2 + m_\pi)^{1/2} = T_{lab} + m_\pi$ and momentum k_{lab}[1] as follows

$$\beta_c = \frac{k_{lab}}{E_{lab} + m_\pi^2/M_A}, \tag{2.3}$$

with M_A, the mass of a target nucleus.

The microscopic OP (2.1) is then inserted into the Klein-Gordon-Fock relativistic wave equation in the form (see, e.g., [11])

$$(\Delta + k^2)\psi(\mathbf{r}) = 2\mu\, U_{eff}(r)\psi(\mathbf{r}), \tag{2.4}$$

where relativistic momentum k in c.m. system

$$k = \frac{M_A k_{lab}}{\sqrt{(m_\pi + M_A)^2 + 2M_A T_{lab}}} = \frac{M_A \sqrt{T_{lab}(T_{lab} + 2m_\pi)}}{\sqrt{(m_\pi + M_A)^2 + 2M_A T_{lab}}}, \tag{2.5}$$

and the effective potential consists of nuclear OP and the Coulomb potential of charged sphere with the radius $R_c = r_c A^{1/3}$, where r_c=1.3 fm:

$$U_{eff}(r) = \gamma^{(r)} \cdot [U_{opt}(r) + U_c(r)], \qquad \gamma^{(r)} = \frac{\bar{\mu}}{\mu} = \frac{\bar{m}_\pi}{m_\pi} \cdot \frac{m_\pi + M_A}{\bar{m}_\pi + M_A}. \tag{2.6}$$

Here, the relativization factor $\gamma^{(r)}$ is the ratio of the relativistic reduce mass $\bar{\mu} = \bar{m}_\pi M_A/(\bar{m}_\pi + M_A)$ to non-relativistic one $\mu = m_\pi M_A/(m_\pi + M_A)$, where $\bar{m}_\pi = \sqrt{k^2 + m_\pi^2} = T_{c.m.} + m_\pi$. Finally, the transformed wave equation (2.4) is computed using the programm DWUCK4 [12], and thus one obtains differential and total cross sections of elastic scattering. So, this approach automatically accounts for effects of relativization and also distortions of the relative motion wave functions in the field of a target nucleus.

Calculations and comparisons to experimental data from [13] were made for differential elastic scattering cross sections of $\pi^{\pm}$-mesons on nuclei ^{28}Si,^{58}Ni,^{208}Pb at kinetic energy T_{lab}=291 MeV. The density distribution functions of the protons and neutrons in nuclei are taken to be the same fermi functions form of $\rho(r)$ normalized to Z and $A-Z$ respectively with parameters R and a (in fm) equal to 3.134 and 0.477 for ^{28}Si [14]; 4.2 and 0.475 for ^{58}Ni [15]; 6.654 and 0.475 for ^{208}Pb [16]. The parameters of the amplitudes of πp and πn scattering (2.2) were taken from [8], where they were presented and based on the existing data of the phase shift analysis and then applied in calculations of the π^-+^{12}C scattering at T_{lab}=180 and 260 MeV, and π^++^{16}O at 270 MeV using the Glauber theory. In our calculations at 291 MeV both the amplitude parameters at 260 and 270 MeV and also their linear extrapolation to the energy 291 MeV (Table 1) were applied and the results were compared to the existing experimental data.

[1] In eq.(1), it is useful to use units MeV and fm, and then $\hbar c$ = 197.327 MeV·fm. In the other cases we use the natural system of units where $\hbar = c = 1$, and thus E, T, k, m have the same dimension [MeV].

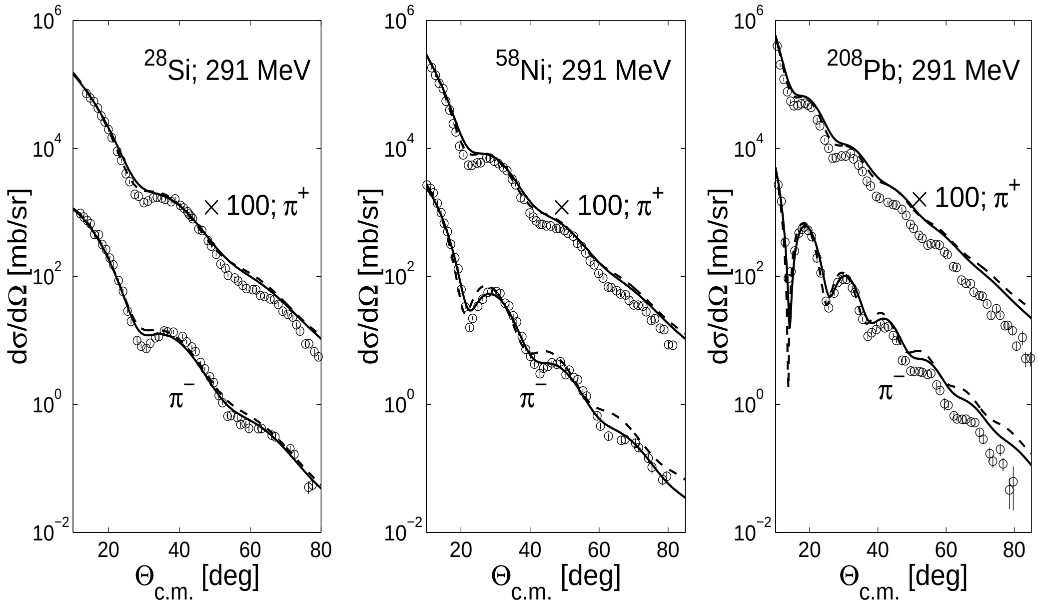

Figure 1: Comparisons with experimental data of the calculated cross sections for scattering of $\pi^{\pm}$- mesons on nuclei ^{28}Si, ^{58}Ni, ^{208}Pb at T_{lab}=291 MeV. The parameters of the free πN amplitudes are in Table 1. Dashed curves are for the π^--meson parameters at 260 MeV and for π^+-mesons at 270 MeV, respectively. Solid curves are for the parameters extrapolated to the energy 291 MeV.

Table 1: Parameters of the amplitude of scattering of pions on free nucleons.

T_{lab}, MeV	$\sigma_{\pi^- p}$,fm^2	$\alpha_{\pi^- p}$	$\beta_{\pi^- p}$,fm^2	$\sigma_{\pi^- n}$, fm^2	$\alpha_{\pi^- n}$	$\beta_{\pi^- n}$,fm^2	paper
260	3.5	-0.28	0.436	9.3	-0.91	0.463	[8]
291	2.5	-0.46	0.37	6	-1.23	0.42	extrapolation
	$\sigma_{\pi^+ p}$,fm^2	$\alpha_{\pi^+ p}$	$\beta_{\pi^+ p}$,fm^2	$\sigma_{\pi^+ n}$,fm^2	$\alpha_{\pi^+ n}$	$\beta_{\pi^+ n}$,fm^2	
270	8.5	-1.0	0.436	3.2	-0.28	0.405	[8]
291	7.7	-1.1	0.4	1.0	-0.58	0.37	extrapolation

In Fig.1 these cross sections are shown by dashed and solid lines, correspondingly, and difference between them are seen mainly at comparably larger angles. It is seen that the tested microscopic OP and the developed scheme of calculations work rather well, and that the usage of free πn and πp amplitudes reproduce the general behavior of the pion scattering on the nuclei ^{28}Si, ^{58}Ni. Moreover, in the case of the Coulomb attraction for the π^--mesons when the more deep penetration of them into a nucleus takes place, the diffraction features are revealed more significant as compared to the repulsion mechanism for the π^+-mesons. In the case of scattering on the heavy nucleus ^{208}Pb, the disagreement to experimental data became more obvious, and this result we can relate to the effect of nuclear matter onto πN interaction. In the whole one can estimate the level of disagreement of theoretical predictions on the respective experimental data by the χ^2/point values suggesting that the experimental errors consist 10 percent on the respective experimental value. In the considered examples these values occur to be for π^- mesons in limits from 4.675 (^{58}Ni) to 85.32 (^{208}Pb), and for π^+-mesons from 24.05 (^{58}Ni) to 173.33 (^{208}Pb). At the same time deviations

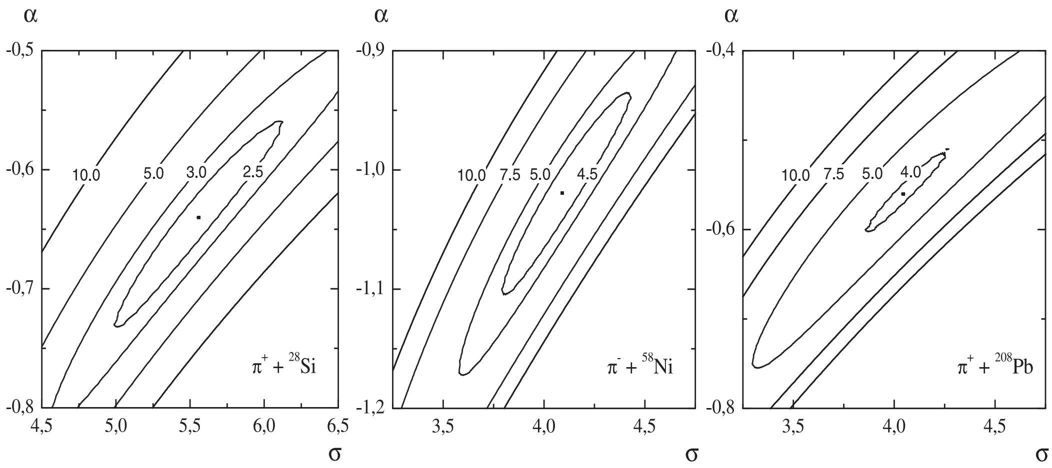

Figure 2: The numbers on lines in the σ, α planes show the respective χ^2/point values obtained when fitting the calculated to experimental differential cross sections of the pion scattering on ^{28}Si, ^{58}Ni, ^{208}Pb at T_{lab}=291 MeV.

of dashed curves are more significant. Thus, if one fits the parameters $\sigma,\ \alpha,\ \beta$ to the data on the pion-nucleus scattering then this provide to estimate the in-effect on the pion scattering from the bounded nucleons in a target nucleus.

3. Analysis of in-medium πN scattering amplitudes

With the object to do the fitting procedure more effective one can reduce the number of parameters to be fit by using the isospin symmetry relations

$$\sigma_{\pi^\pm n} = \sigma_{\pi^\mp p}, \quad \alpha_{\pi^\pm n} = \alpha_{\pi^\mp p}, \quad \beta_{\pi^\pm n} = \beta_{\pi^\mp p}. \tag{3.1}$$

Then the pion-nucleus amplitude takes the form

$$U_{opt}(r) = -\frac{(\hbar c)\beta_c}{(2\pi)^2}\,\sigma\,[i+\alpha]\cdot\int_0^\infty j_0(qr)\,\rho(q)\,f(q)\,q^2 dq, \tag{3.2}$$

where the average parameters of the pion-nucleon amplitude of scattering occur as follows

$$\sigma = \frac{1}{2}\,[\sigma_{\pi^+p} + \sigma_{\pi^-p}], \quad \alpha = \frac{1}{2}\,[\alpha_{\pi^+p} + \alpha_{\pi^-p}], \quad f(q) = e^{-\beta q^2/2}. \tag{3.3}$$

In (3.2), the density distributions of protons and neutrons in nuclei are suggested to be of the same form $\rho_p = \rho_n = (1/2)\rho$ with $\rho(r)$ normalized to the atomic number A. So, in the following, we will use the isotospin averaged πN scattering amplitude with only 3 parameters instead of the 6 parameters when applying the separate πn and πp amplitudes.

Now the optical potential (3.2) depends on the three parameters σ, α and β of the πN amplitude of scattering on free nucleons, and they are expressed through the respective parameters of scattering on free protons. These averaged parameters are presented in tables, e.g., in refs. [17] and [18] at energies T_{lab} from 90 to 280 MeV. Their magnitudes at 291 MeV we obtained by the extrapolations, and they are done in the last column of Table 2. Starting from these values for free

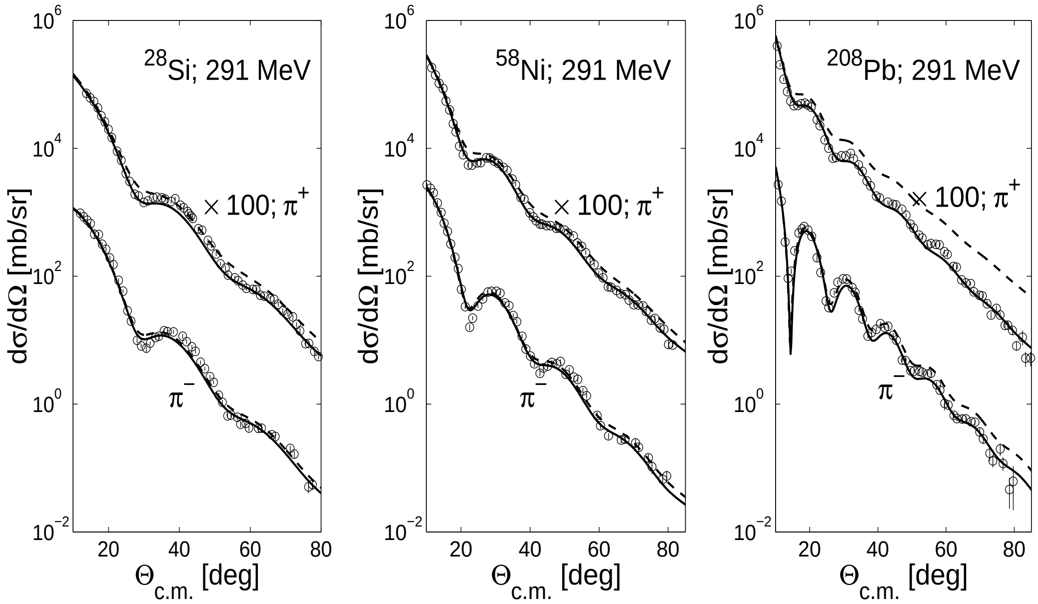

Figure 3: Comparisons of calculated differential cross sections of $\pi^{\pm}$-mesons scattered on nuclei ^{28}Si, ^{58}Ni, ^{208}Pb at T_{lab}=291 MeV to experimental data when using the parameters from Table 2. Dashed curves are for parameters of free πN amplitudes, solid lines are for the fitted parameters.

amplitudes we obtained the respective "in-medium" best-fit parameters σ and α by adjusting the calculated pion-nucleus differential cross sections to the experimental data leaving the parameter β=0.434 fm^2 to be the same as for scattering on free protons. The middle-squared deviations χ^2 are estimated using the following expression

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\frac{\left|d\sigma^{exp}/d\Omega(\theta_i) - d\sigma^{theor}/d\Omega(\theta_i)\right|}{\Delta d\sigma^{exp}/d\Omega(\theta_i)} \tag{3.4}$$

where Δ is suggested to define the 10% error of the experimental cross section in a point i.

The magnitudes of the obtained "in-medium" parameters and corresponding χ^2 per one point are done in Table 2. As an illustration, Fig.2 shows dynamics of the fitting procedure when χ^2 values come to their minima in three processes π^++^{28}Si, ^{58}Ni, ^{208}Pb. In Fig. 3, solid curves show the adjusted cross sections in comparison with the experimental data. The dashed curves are calculations using the free amplitudes with parameters given in the last column of Table 2, and the respective χ_0^2 values are done in its last line.

4. Conclusions

Analyzing the obtained results one can conclude that the usage of free πN amplitudes does not allow one to get successful agrement of the calculated pion nucleus differential cross sections to the respective experimental data. This conclusion is right both for the case when one applies the separate free π-proton and π-neutron amplitudes and also for calculations with the joint pion-nucleon amplitude of scattering. The good agrement can be achieved if one fit the two parameters σ and α of three to the data to demonstrate the typical transformation of the free πN amplitude to

Table 2: Parameters of fitted amplitude of scattering of pions on nucleons in nuclei.

	^{28}Si		^{58}Ni		^{208}Pb		πN
	π^+	π^-	π^+	π^-	π^+	π^-	free
σ, fm^2	5.55	4.81	5.43	4.09	4.04	4.23	4.76
α	-0.64	-0.88	-0.68	-1.02	-0.56	-0.92	-0.95
β, fm^2	0.434		0.434		0.434		0.434
χ^2	2.30	3.56	1.73	4.25	3.94	6.95	
χ^2_0	23.5	5.46	17.7	9.92	676.4	28.1	

the amplitude of scattering in nuclear matter. This in-medium effect can be clearly demonstrated if the parameters of the free πN amplitude will be compared with the fitted parameters averaged over all six cases of the $\pi^{\pm}$ scattering on three considered target nuclei. Doing so, we obtain from the respective values of Table 2 the averaged in-medium $\pi^{\pm}$ scattering cross sections $\bar{\sigma} = 4.69$ fm^2, that coincides fairly well with the free πN scattering cross section σ = 4.76 fm^2 in limits of accuracy of analysis of the data. At the same time, the average deflection parameter $\bar{\alpha}$ = -0.78 occurs in about 20% larger than the respective free α = -0.95. As to the cross section parameters, one can note that at 291 MeV the πN system takes place in a boundary of existence of the 33-resonance region. At this energy the mechanism of the process is determined by two competed factors, that turn out to be comparable to one another. One of them is the "swelling" of nucleons in nuclei [19], that leads to increase of its geometric dimensions and increasing the respective cross section, while the other one is the suppression of the interaction of a pion with the bounded nucleon in a nucleus because of the Pauli blocking effect (see, e.g., [20], [21]). With increasing the energy this latter effect becomes weaker, that is seen, e.g., from analysis of the pion scattering on the ^{12}C nucleus [22] at comparably larger energy 672 MeV. In this case, the first factor plays the decisive role and therefore the corresponding total πN cross section occurs in about 10% larger than that for a free nucleon.

In conclusion one can note that analysis in [22] was also made by adjusting only two parameters σ and α while β was taken to be equal as for the scattering on free protons. By the way one should note that the slope β is also connected to the radius of interaction. Therefore the actual task is retained to fit the all three parameters σ, α, β simultaneously and also to move the investigations in the 33-resonance region of energies with the aim to study the role of different factors on the mechanism of scattering of pions on nuclei.

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