Study of the phase transition in $\pi^- C$ interactions at 40 GeV/c

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In this report we are proposed to study the phase transition process to use the new pair for variables (T, n_c) instead of (T, ρ) which is mainly used in theoretical calculations and considered the transverse energy spectra of protons and π^--mesons produced in π^-C interactions at 40 GeV/c as a function of the cumulative number n_c (or the four dimensional momentum transfer t). Analysis carried out in this paper indicates about the possible appearance of the phase transition of nuclear matter.

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[1] Speaker

1   Introduction

The investigation of the multiparticle production process in hadron-nucleus and nucleus-nucleus interactions at high energies and large momentum transfers is very important for understanding the strong interaction mechanism and inner quark-gluon structure of nuclear matter.

During the last years the possibility of the observing of the collective phenomena such as the cumulative particle production [1], the production of nuclear matter with high densities, the phase transition from the hadronic matter to the quark-gluon plasma state is widely discussed in the literature [2-5,11-14].

According to the different ideas and models, if exist these phenomena in the nature, then they will be observed in the hadron-nucleus and nucleus- nucleus interactions at high energies and large momentum transfers and should be influenced to the dynamics of interaction process and would be reflected in the angular and momentum characteristics of the reaction products.

In hadron-nucleus and nucleus-nucleus interactions, in difference from hadron-nucleon interactions, the secondary particles may be produced as a result of multi-nucleon interactions, in other word, the particles are produced in the region kinematically forbidden for hadron-nucleon interactions. This fact is one of the reasons of interest to study the nucleus collision at high energies.

In this paper we are considered the next reactions:

$$\pi^- C \to p + X \qquad\qquad (1)$$
$$\pi^- C \to \pi^- + X \qquad\qquad (2)$$

This paper is a continuation of our previous publications [6, 7].

2   Variables Used:

2.1 Cumulative number

The cumulative number n_c in the fixed target experiment is determined by the next formula.

$$n_c = \frac{P_a \cdot P_c}{P_a \cdot P_b} \cong \frac{E_c - P_{II}^c}{m_p} \qquad\qquad (3)$$

Here P_a, P_b and P_c are the four dimensional momentum of the incident particle, target and the considering secondary particles correspondingly.

E_c is the energy and P_{II}^c is the longitudinal momentum of the considering particle, m_p is the proton mass. From this formula we see that this variable is a **relativistic invariant**.

This variable (n_c) may be interpreted as minimal target mass, which is required for producing of the given secondary particle, because at summarizing by all secondary particles should be obtained the value of the target mass determined on the basis of the energy-momentum conservation law, i.e.

$$M_t \cong \frac{\sum_{i=1}^{n}(E - P_{II})_i}{m_p} \qquad (4)$$

So, n_c distribution gives the **ordered mass values** from the target required for producing of the considering secondary particles.

The connection of the variable n_c and momentum transfer t is determined by the next formula [7]:

$$t = -Q^2 = -(p_a - p_c)^2 = -m_\pi^2 - m_c^2 + 2 \cdot E_a \cdot \frac{m_p}{m_p} \cdot (E - P_{II}) \approx S_{\pi^- p} \cdot n_c \quad (5)$$

Where $S_{\pi^- p}$ is the total energy square of $\pi^- p$ interaction. In this experiment $S_{\pi^- p} \cong 2E_a \cdot m_p \cong 75\, GeV^2/c^2$ is constant. To study the phase transition of the nuclear matter we must choose the variables corresponding to this physical process. In the theoretical calculations are mainly used the effective temperature T, the density of nuclear matter ρ/ρ_0 or the quark chemical potential μ. But the variables μ and ρ are not fully determined experimentally. So for studying the phase transition process we must select the other appropriate variable. With this goal let us consider the formula (5) which gives the connection between the variables n_c and t. From this formula one can see that with increasing t the minimal target mass value n_c which is required from the target for producing of the considering secondary particle increases or vice versa. This means that if any one particle with $n_c > 1$ (cumulative particle) is produced in hadron-nucleus interactions at high energies then this particle should be produced at large momentum transfers more than the total energy square of $\pi^- p$ interaction $S_{\pi^- p}$, i.e. $t > S_{\pi^- p}$, for example, if $n_c = 1.5$ then only for this particle $t \cong S_{\pi^- p} \cdot n_c = 75\, GeV^2/c^2 \cdot 1.5 = 112.5\, GeV^2/c^2$ which is not allowed for $\pi^- p$ interactions.

From the other hand side, if two particles are produced at two different values of $n_c (Q_1^2 < Q_2^2)$, then the size of the particle production region for every particle is different and may be estimated by the formula $r_1 \sim 1/Q_1$, $r_2 \sim 1/Q_2$ and satisfies the condition $r_1 > r_2$, i.e. the size of the particle emission region at high Q^2 is smaller than the case at low Q^2.

As a result of these two features (increasing of the mass $(n_c \cdot m_p)$ and decreasing of the distance r) of the variable n_c (or t), the density of the nuclear matter ρ increases with increasing of n_c. **In this sence n_c (or t) is the more appropriate variable to study the phase transition process of nuclear matter**. In this case the density of nuclear matter ρ may be determined by the next formula:

$$\rho = \frac{m}{V} = \frac{n_c \cdot m_p}{V} \qquad (6)$$

Where V is the volume from which the particle is emitted. The volume V may be estimated as a sphere, i.e.

$$V = \frac{4\pi}{3} r^3 \qquad (7)$$

2.2 The effective temperature T

The transverse energy spectra of the secondary particles in the different n_c intervals are approximated by exponential functions of the next form:

$$\frac{1}{2 \cdot E_t} \frac{\Delta N}{\Delta E_t} \sim e^{-b \cdot E_t}, \quad E_t = \sqrt{p_t^2 + m^2} \tag{8}$$

The effective temperature T is determined as the inverse of the slope parameters b,

$$T = \frac{1}{b} \tag{9}$$

3   Experimental method

The experimental material was obtained with the help of Dubna 2-meter propane (C_3H_8) bubble chamber exposed by π^--mesons with momentum 40 GeV/c from Serpukhov accelerator. According to the advantage of the bubble chamber experiment, all distributions in this paper are obtained in the condition of 4π geometry of secondary particles.

The average error of the momentum measurements is ~ 12% and the average error of the angular measurements is ~0.6°.

All secondary negative particles are taken as π^--mesons. The average boundary momentum from which π^--mesons were well identified in the propane bubble chamber is ~70 MeV/c. In connection with the identification problem between energetic protons and π^+-mesons, protons with momentum more than ~1 GeV/c are included to the π^+-mesons.

The other experimental details are described in [8, 9].

4   Statistics

8791 $\pi^- C$ interactions are used in this analysis.
1. The total number of protons is 12441.
2. The total number of π^--mesons is 30145.

- $p + C \rightarrow \pi^- + X$ at 10 GeV/c

$$N_{event} = 22510$$
$$N_{\pi^-} = 21470$$

- $p + C \rightarrow \pi^- + X$ at 4.2 GeV/c

$$N_{event} = 8340$$
$$N_{\pi^-} = 3396$$

5 $\pi^- + C \rightarrow p + X$ analysis

It is interesting to stress that in this experiment the incident particles are π^--mesons. This means that the secondary protons in $\pi^- C$ interactions at 40 GeV/c should be produced only in the target fragmentation region, in other words, there is no overlapping from the projectile fragmentation region in the rapidity distribution of protons.
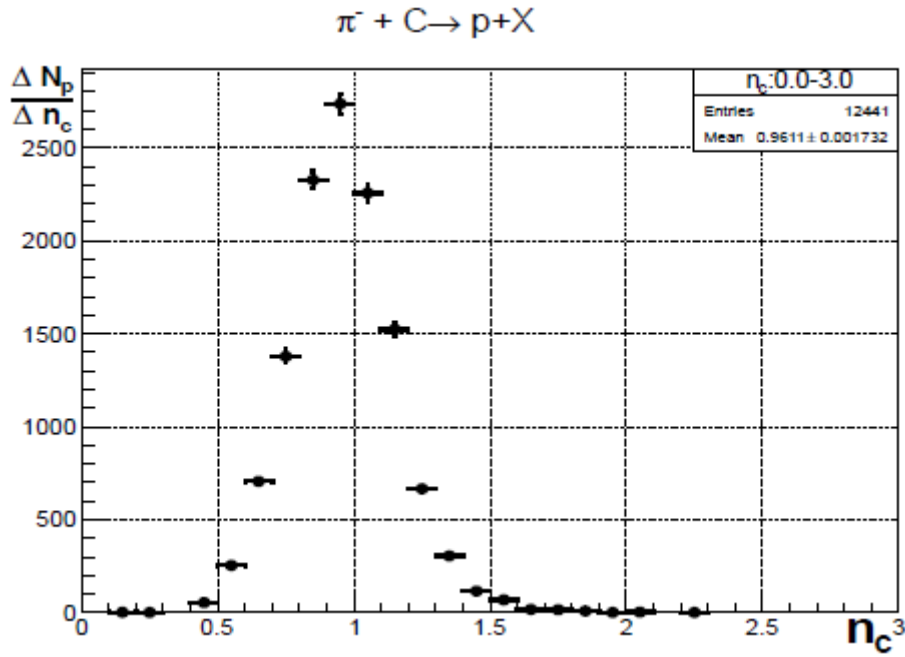
Figure 1: Cumulative number (n_c) distribution of protons.

Fig.1 shows the n_c distribution of the secondary protons produced in $\pi^- C$ interactions at 40 GeV/c. From this distribution we see that many protons are produced in the region with $n_c > 1$ (cumulative protons). This distribution continues until $n_c \approx 2.3$.

Fig.2 gives the rapidity (Y_{lab}) distribution of protons. This distribution shows the protons from $\pi^- C$ interactions are produced in the target fragmentation region.
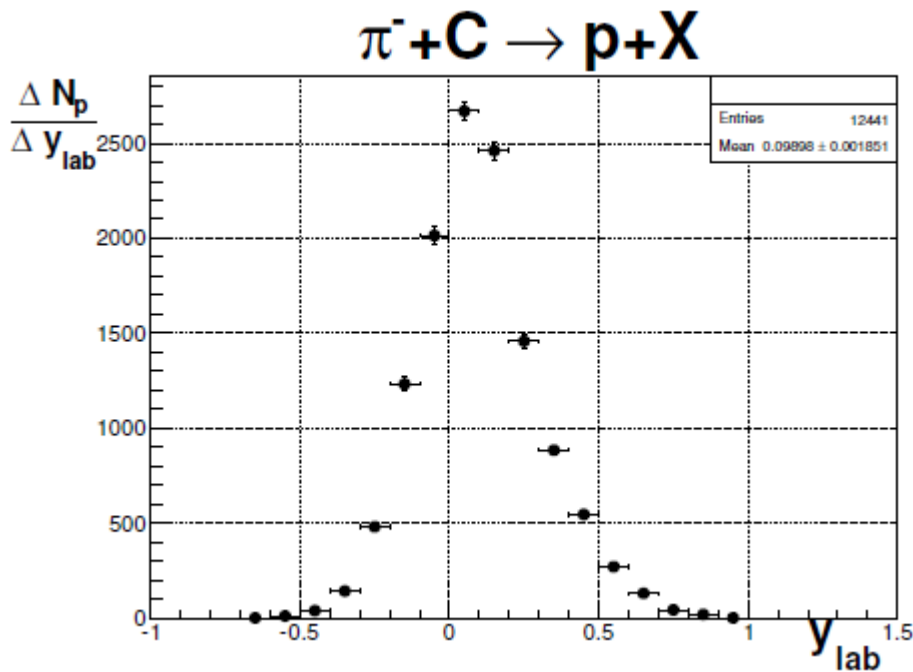

Figure 2: Rapidity distribution of protons.

The transverse energy (or transverse momentum) spectrum of the secondary particles produced in hA and AA interactions at high energies may reflect the dynamics of the interaction process more clearly. This is connected with the fact that the transverse effects are mainly generated during the interaction process.

In connection with this on Fig. 3(a, b) are shown the transverse energy (E_t) spectrum of protons in different n_c intervals.
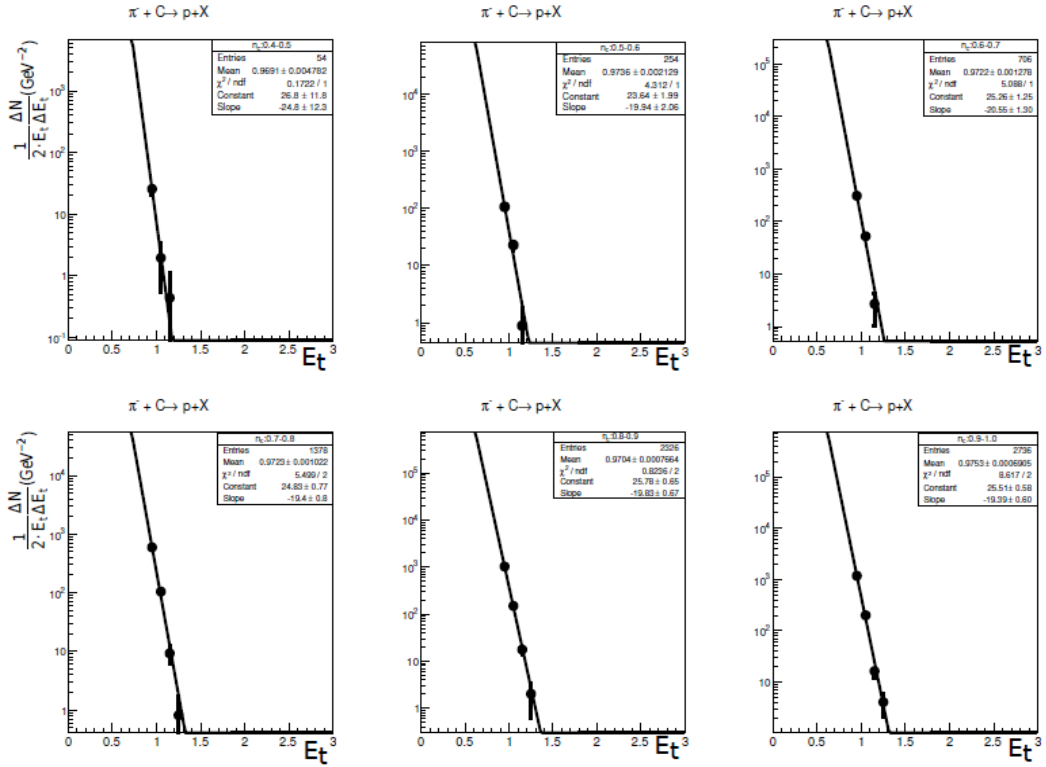

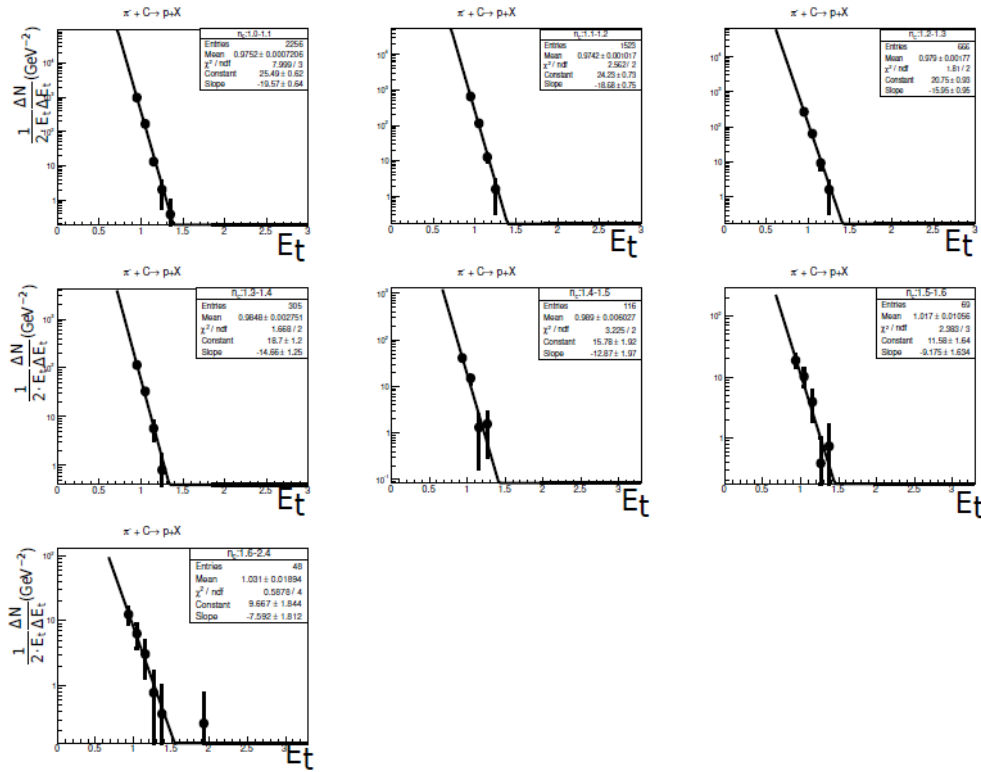Figure 3(a): Transverse energy E_t spectrum of protons as a function of different n_c intervals.

Figure 3(b): Transverse energy E_t spectrum of protons as a function of different n_c intervals.

Table 1: The values of the slope parameter b and the effective temperatures T on the variable n_c of protons from $\pi^- C$ interactions.

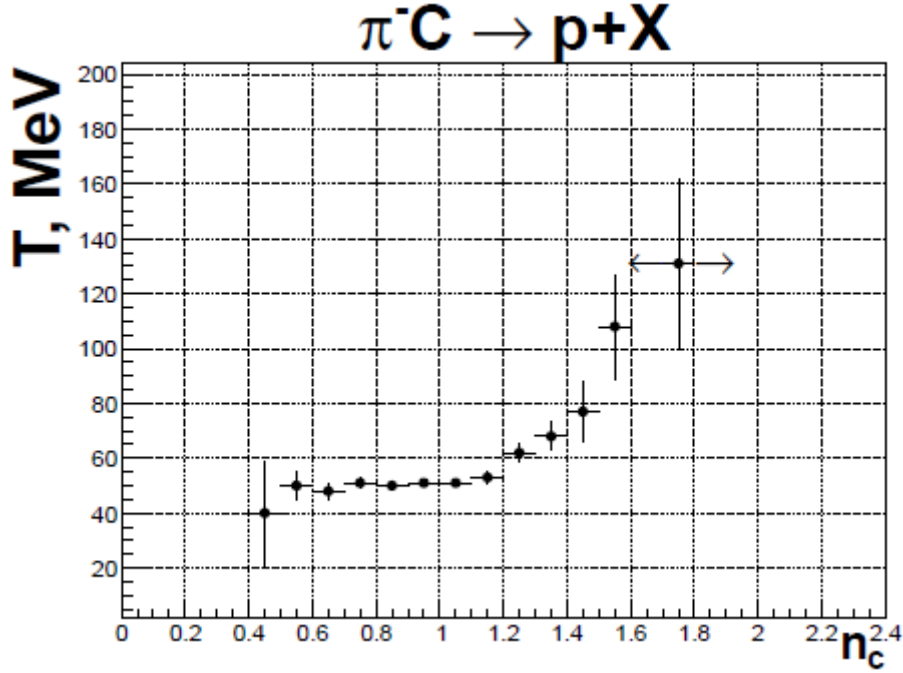| Δn_c	N	b	$T(GeV)$
$0.4 \div 0.5$	54	-24.8 ± 12.3	0.040 ± 0.019
$0.5 \div 0.6$	254	-19.94 ± 2.06	0.050 ± 0.005
$0.6 \div 0.7$	706	-20.55 ± 1.30	0.048 ± 0.003
$0.7 \div 0.8$	1378	-19.4 ± 0.8	0.052 ± 0.002
$0.8 \div 0.9$	2326	-19.83 ± 0.67	0.050 ± 0.001
$0.9 \div 1.0$	2736	-19.39 ± 0.60	0.051 ± 0.001
$1.0 \div 1.1$	2256	-19.57 ± 0.64	0.051 ± 0.001
$1.1 \div 1.2$	1523	-18.68 ± 0.75	0.053 ± 0.002
$1.2 \div 1.3$	666	-15.95 ± 0.95	0.062 ± 0.003
$1.3 \div 1.4$	305	-14.66 ± 1.25	0.068 ± 0.005
$1.4 \div 1.5$	116	-12.87 ± 1.97	0.077 ± 0.011
$1.5 \div 1.6$	69	-9.17 ± 1.16	0.108 ± 0.019
$1.6 \div 2.4$	48	-7.59 ± 1.80	0.131 ± 0.031

Figure 4: The effective temperature T of the secondary protons as a function of the variable n_c.

The experimental spectrum obtained in the every n_c interval is approximated by the exponential function (8) and the values of the slope parameters b and the effective temperatures T on the variable n_c are shown Fig.4. Figure 3(a, b), 4 and Table 1 show that the effective temperatures T are remained practically constant on the level $T \cong 50 MeV$ until $n_c \cong 1.2 \div 1.3$ and then essentially increase. T may be expressed by Kelvin temperature, then $T^p_{plateau} = 50\ MeV = 0.58 \cdot 10^{12} K$, $T^p_{max} = 131\ MeV = 1.52 \cdot 10^{12} K$ where $T^p_{plateau}$ and T^p_{max} are the effective temperatures corresponding to region $n^p_c \le 1.2$ and $n^p_c = 1.75 m_p$. For QGP state QCD predicts $T \cong 2 \cdot 10^{12} K$. Now if we suppose the particle is emitted from the spherical volume V which is determined by the formula (7), then we can give the estimation of the density of the nuclear matter in the hA interactions at high energies using the formula (6), $\rho^p_{n_c=1.2} \to \frac{1.2 m_p}{V} = \frac{1.2 m_p}{0.082 fm^3} = 14.6\ m_p/fm^3$, $\rho^p_{n_c=1.75} \to \frac{1.75 m_p}{V} = \frac{1.75 m_p}{0.039 fm^3} = 44.9\ m_p/fm^3$. Where $\rho^p_{n_c=1.2}$ and $\rho^p_{n_c=1.75}$ are the estimations of the densities corresponding to the regions $n_c = 1.2$ and $n_c = 1.75$ [13].

6 $\pi^- + C \to \pi^- + X$ analysis

Now we will consider π^-–meson case.

Fig.5 presents the cumulative number (n_c) distribution of π^-–mesons from $\pi^- C$ interactions at 40 GeV/C. This distribution shows n_c distribution for π^--mesons continues until $n_c \cong 3$ and well approximated by the sum of three exponential

functions with slope parameters $b_1 = 31.84 \pm 0.42$, $b_2 = 6.555 \pm 0.158$ and $b_3 = 1.417 \pm 0.277$.
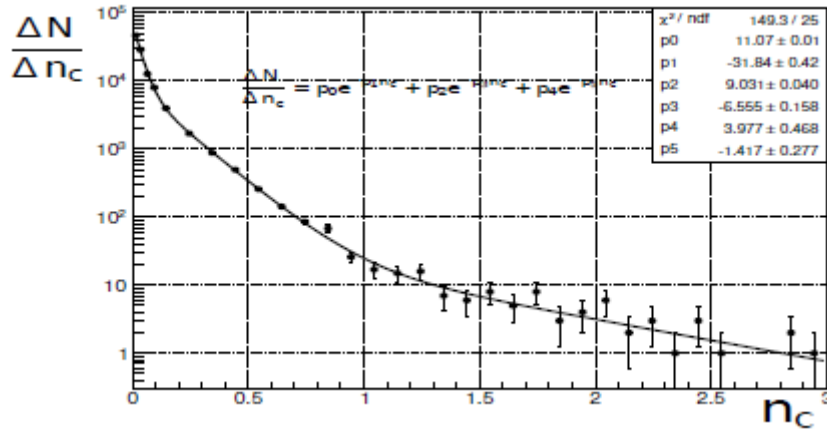

χ²/ndf	149.3 / 25
p0	11.07 ± 0.01
p1	-31.84 ± 0.42
p2	9.031 ± 0.040
p3	-6.555 ± 0.158
p4	3.977 ± 0.468
p5	-1.417 ± 0.277

Figure 5: The cumulative number (n_c) distribution of π^-–mesons from $\pi^- C$ interactions.

The rapidity distribution of all π^--mesons from $\pi^- C$ interactions is shown in Fig.6. In this experiment the secondary π^--mesons are produced in the projectile fragmentation, central and target fragmentation regions.
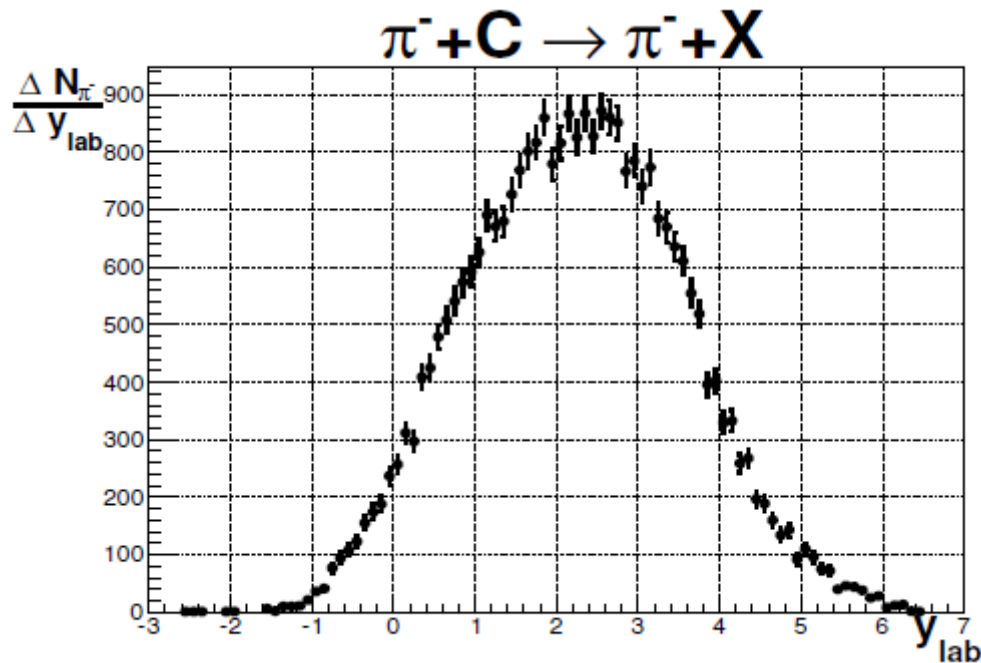

Figure 6: The rapidity distribution of π^--mesons.

Fig.7(a, b, c) shows the transverse energy (E_t) spectrum of π^--mesons as a function of different n_c intervals. The spectrum in the every n_c interval is well approximated by the exponential function (8) and the slope parameters b and the effective temperature T are given in Table 2.
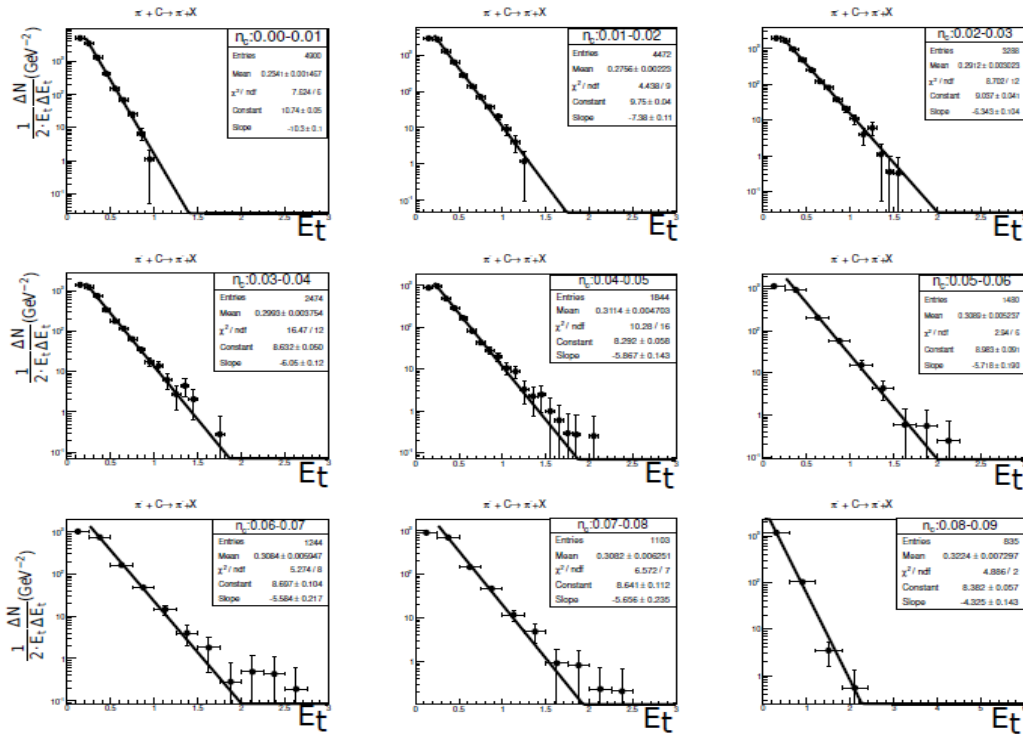
Figure 7(a): Transverse energy E_t spectrum of π^-−mesons as a function of different n_c intervals.
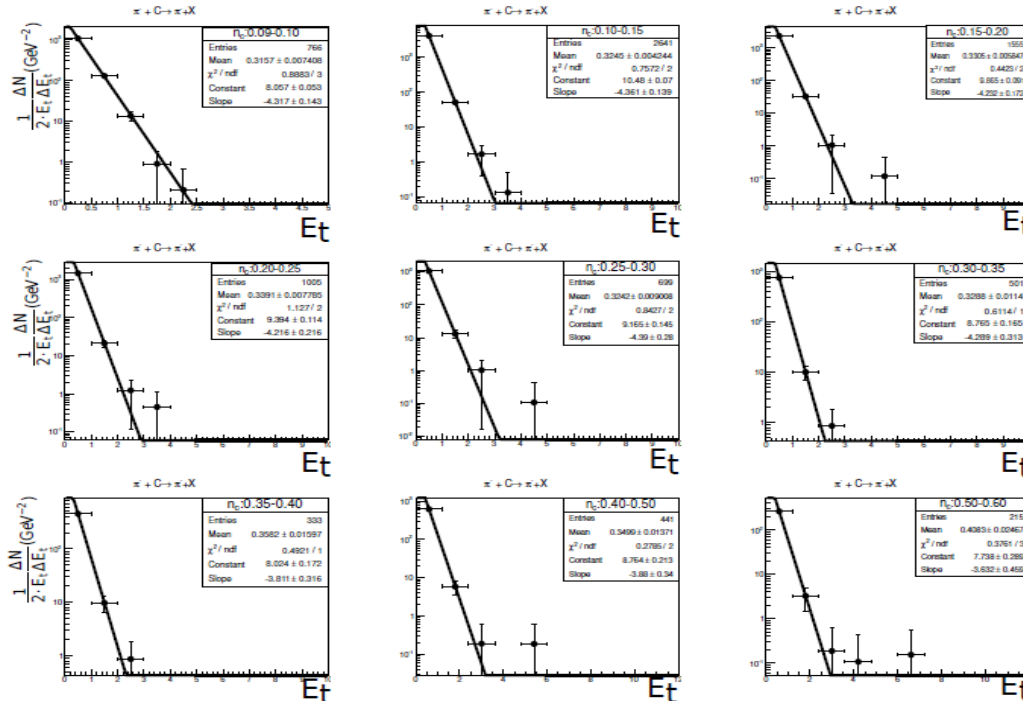

Figure 7(b): Transverse energy E_t spectrum of π^-−mesons as a function of different n_c intervals.
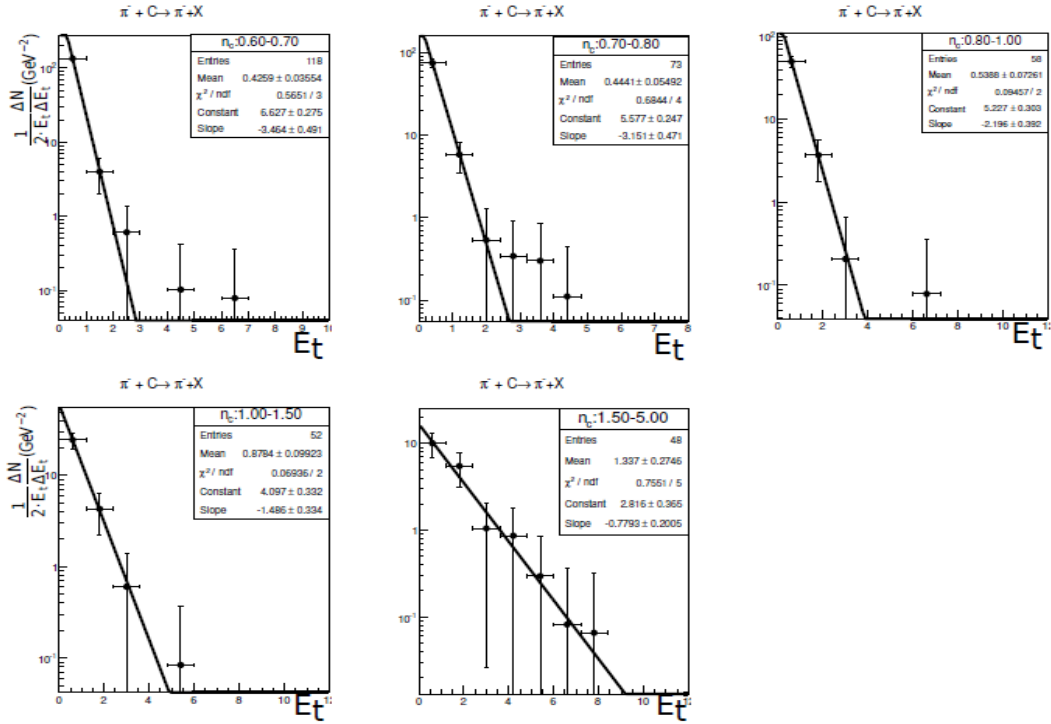
Figure 7(c): Transverse energy E_t spectrum of π^-–mesons as a function of different n_c intervals.

Table 2: The values of the slope parameter b and the effective temperatures T on the variable n_c of π^--mesons from $\pi^- C$ interactions.

| $\triangle n_c$	N	b	T, GeV
$0.0 \div 0.01$	4900	-10.3 ± 0.1	0.097 ± 0.008
$0.01 \div 0.02$	4472	-7.38 ± 0.11	0.135 ± 0.002
$0.02 \div 0.03$	3288	-6.343 ± 0.104	0.157 ± 0.002
$0.03 \div 0.04$	2474	-6.05 ± 0.12	0.165 ± 0.003
$0.04 \div 0.05$	1844	-5.867 ± 0.143	0.176 ± 0.005
$0.05 \div 0.06$	1480	-5.718 ± 0.190	0.181 ± 0.006
$0.06 \div 0.07$	1244	-5.584 ± 0.217	0.199 ± 0.012
$0.07 \div 0.08$	1103	-4.656 ± 0.235	0.240 ± 0.005
$0.08 \div 0.09$	835	-4.325 ± 0.143	0.231 ± 0.007
$0.09 \div 0.10$	766	-4.317 ± 0.143	0.231 ± 0.007
$0.1 \div 0.15$	2641	-4.361 ± 0.139	0.229 ± 0.007
$0.15 \div 0.2$	1555	-4.232 ± 0.172	0.236 ± 0.009
$0.2 \div 0.25$	1005	-4.216 ± 0.216	0.237 ± 0.012
$0.25 \div 0.3$	691	-4.39 ± 0.28	0.227 ± 0.014
$0.3 \div 0.35$	501	-4.289 ± 0.313	0.233 ± 0.017
$0.35 \div 0.4$	333	-3.811 ± 0.316	0.262 ± 0.021
$0.4 \div 0.5$	441	-3.88 ± 0.34	0.257 ± 0.022
$0.5 \div 0.6$	215	-3.632 ± 0.459	0.275 ± 0.034
$0.6 \div 0.7$	118	-3.464 ± 0.491	0.288 ± 0.040
$0.7 \div 0.8$	73	-3.151 ± 0.471	0.317 ± 0.47
$0.8 \div 1.0$	58	-2.196 ± 0.392	0.455 ± 0.081
$1.0 \div 1.5$	52	-1.486 ± 0.334	0.672 ± 0.151
$1.5 \div 5.0$	48	-0.7793 ± 0.2005	1.283 ± 0.330

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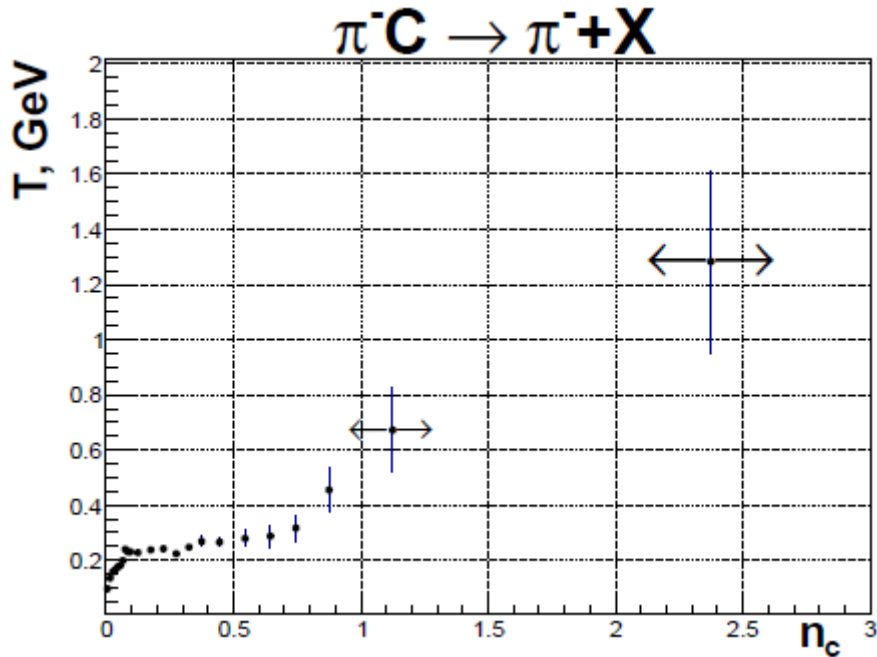
Figure 8: The effective temperature T of the secondary π^--mesons as a function of the variable n_c.

The dependence of T parameters for π^--mesons from $\pi^- C$ interactions as a function of n_c is shown on Fig.8. From this figure we see that with increasing n_c the effective temperatures T in the beginning are increasing until $n_c \cong 0.1$ and then in the $n_c \cong 0.1 \div 0.5$ interval the parameter T is remained practically constant on the level $T \cong 0.220 \div 0.230 \ GeV$ and then they are essentially increased [13].
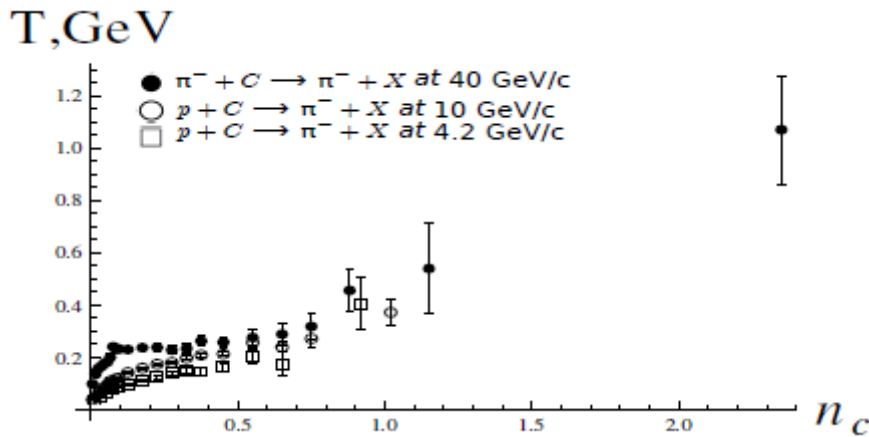
Figure 9: shows the temperature dependence on the variable n_c of π^--mesons produced in $p + C$ -interactions at 4.2 *GeV/c* (□-open square), 10 GeV/c(○-open circle) and $\pi^- + C$ -interactions at 40 *GeV/c* (●-black circle). From this figure we see that with increasing of the projectile energies, the values of T -parameters are increased, but at low energy $p + C$ experiments there are no clearly expressed plateaus which is observed in the case of π^- production from $\pi^- + C$ interactions at 40 *GeV/c*.

Figure 10 shows the one-fourth power of the energy density (x-axis) as a function of temperature (y-axis) for $T_c = 104 \, MeV$ with $B^{1/4} = 0.145 \, GeV$. The transition region is the horizontal line at $T = T_c$. This figure was taken from the literature [14].
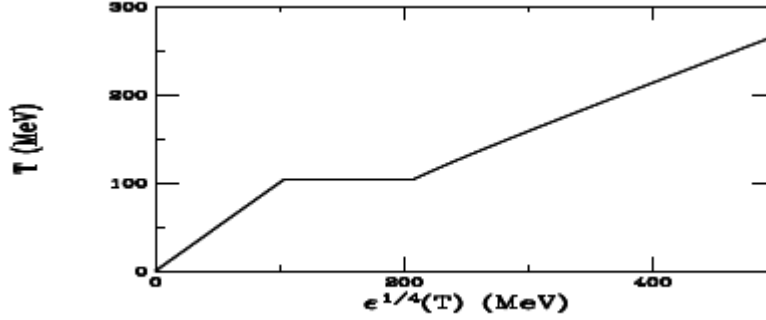

Figure 10: The temperature is shown versus energy density in a first-order phase transition with $T_c = 104 \, MeV$.

We would like to stress that the behavior of the theoretical dependence of temperature T as a function of the energy density $\in^{1/4}$ is very similar to our experimental dependence obtained between temperature T and cumulative number n_c.

7   Discussion

Depending on the temperature T and the variables n_c (or t), the strongly interacting matter may occur in three distinct phases: the hadronic phase, thermodynamical equilibrium and pure QGP.
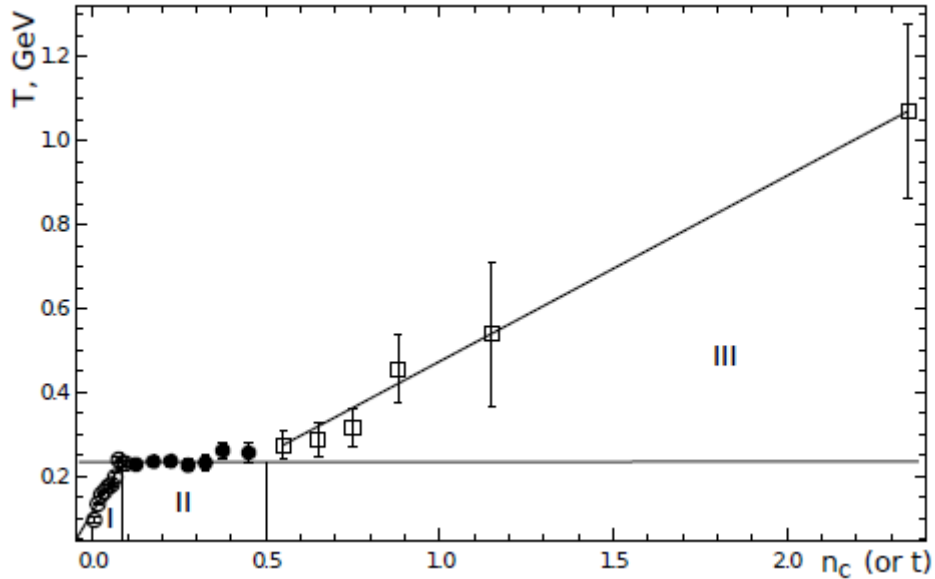

Figure 11: The phase diagram of π^--mesons produced from $\pi^- C$ -interactions at 40 GeV/c.
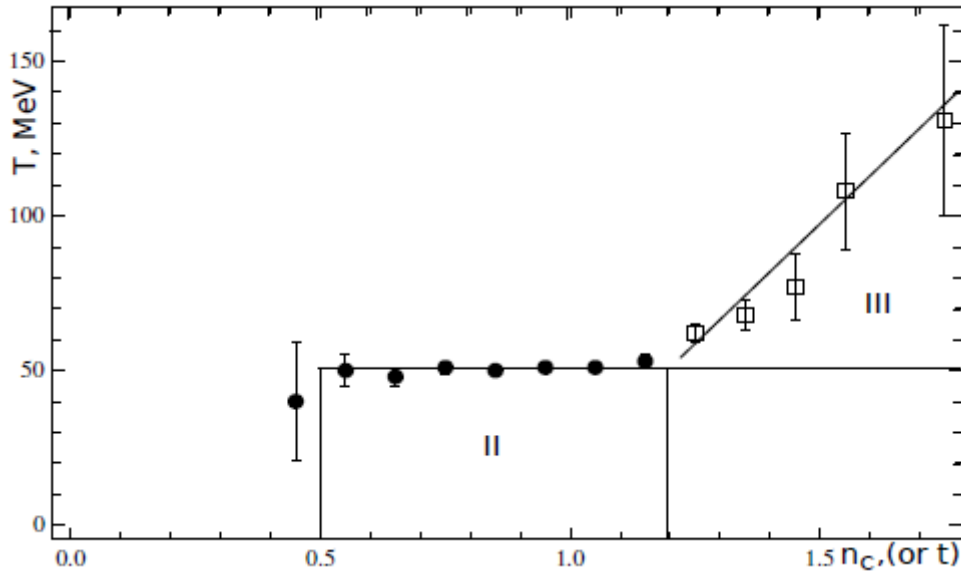
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Figure 12: The phase diagram of protons produced from $\pi^- C$ -interactions at 40 *GeV/c*.

Fig.11 and Fig.12 show the phase structure of strongly interacting matter for π^- -mesons and protons produced from $\pi^- C$ -interaction.

So we observe the essential changing of the behaviors of the parameter T of protons and π^--mesons from $\pi^- C$ interactions at 40 GeV/c on the variable n_c in two different regions for protons and in three different regions for π^--mesons. Such behaviours may indicate about the particle production different mechanism in these regions of the variable n_c (or t) and temperature T. If so, the *I* region (open circles) in which the temperature T is increased for π^--mesons (Fig.11) may be connected with the thermalisation of the strongly interacting objects. In this region the strongly interacting matter is in the hadronic phase.

In the *II* regions (black circles of Fig.11 and Fig.12) where the temperature T is remained practically constant, the strongly interacting matter is in the thermodynamical equilibrium state (or the mixed phase).

In the *III* regions (open quadratics) where the parameter T is again increased, the strongly interacting matter is in the pure QGP phase.

8   Conclusion

- The analysis carried out in this paper gives us the possibility to speak about the possible appearance of the phase transition of nuclear matter.
- To study the phase transition processes in *hA* and *AA* interactions at high energies, the variable n_c (or t) which is used instead of the density of nuclear matter ρ is a more appropriate variable, in other words, we are proposed to study the phase transition process to use the pair of variables (T, n_c) instead of (T, ρ).

- The strong changing of the characters of the above mentioned dependences may indicate about the particle production different mechanism in these regions. If so, the first region with increasing T until $n_c \leq 0.1$ may correspond to the thermalisation of the interacting objects (here the strongly interacting matter is in the hadronic phase), the second region with $0.1 \leq n_c \leq 0.5$ for π^--mesons and with $0.4 \leq n_c \leq 1.2$ for protons may indicate about the equilibrium state formation (hadron+QGP state) and the third region with $n_c \geq 0.5$ for mesons and $n_c \geq 1.2$ for protons can be connected with the production of pure QGP state.

- Our results show that the numerical characteristics (the temperatures in thermodynamical equilibrium and in the pure QGP state) of phase transition processes for protons and pions are different.

- As a result of the interaction process the strongly interacting matter with different densities is produced and then the secondary particles go out from the nuclear matter with corresponding them density.

9   Acknowledgements

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