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Nonperturbative QCD and Transition Form Factors

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We study the transition form factors of π^0, η and η' mesons by means of anomaly sum rule – an exact nonperturbative relation which follows from the dispersive treatment of axial anomaly. The absence of corrections to it allows us to relate the possible corrections to continuum and to lower states contributions within the QCD method which does not rely on factorization hypothesis. The form of the correction is proposed and compared with the available experimental data. We show that both isovector and octet channels of the anomaly sum rule may accommodate such a correction.

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1. Introduction

Recently there was a significant progress in experimental study of the photon-meson transitions. The $\gamma\gamma^* \to \pi^0(\eta,\eta')$ transition form factors (TFFs) were measured by BABAR [1, 2] and BELLE [3] collaborations in the range the photon virtuality $Q^2 \simeq 5-35$ GeV^2 (the other photon was almost real). The data on pion TFF, measured by BABAR [1], BELLE [3] and earlier measurements by CELLO [4] and CLEO [5] collaborations agree well at $Q^2 \lesssim 10$ GeV^2, while at $Q^2 \gtrsim 10$ GeV^2 the BABAR and BELLE data manifest quite different behavior. The BABAR data show a steady rise of the product of the TFF and Q^2, surpassing the pQCD predicted [6] asymptote $Q^2F_{\pi\gamma} \to \sqrt{2}f_\pi$, $f_\pi = 130.7$ MeV at $Q^2 \simeq 10$ GeV^2 and questioning the collinear factorization. On the other hand, the more recent BELLE data do not show such striking behavior: although $Q^2F_{\pi\gamma}$ reaches the pQCD asymptotic value, it does not manifest further growth (within the experimental errors).

In these proceedings, based on [7, 8, 9, 10], we demonstrate the application of the anomaly sum rule to a study of the π^0, η and η' TFFs. This nonperturbative QCD method does not rely on the collinear factorization and can be used even if the factorization is broken.

2. Anomaly sum rule

The phenomenon of axial anomaly [11] leads to a nonconservation of the axial current in the chiral limit. In QCD, for a given flavor q, the divergence of the axial current $J^{(q)}_{\mu 5} = \bar{q}\gamma_\mu\gamma_5 q$ acquires both electromagnetic and gluonic anomalous terms:

$$\partial_\mu J^{(q)}_{\mu 5} = m_q\bar{q}\gamma_5 q + \frac{e^2}{8\pi^2}e_q^2 N_c F\tilde{F} + \frac{\alpha_s}{4\pi}N_c G\tilde{G}, \tag{2.1}$$

where e_q is a quark charge in the units of electron charge e, F and G are electromagnetic and gluonic strength tensors respectively, $\tilde{F}$ and $\tilde{G}$ are their duals, and $N_c = 3$ is a number of colors.

In what follows, we use the definitions of the isovector $J^{(3)}_{\mu 5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$ and octet $J^{(8)}_{\mu 5} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s)$ components the octet of axial currents and the singlet axial current $J^{(0)}_{\mu 5} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s)$. The singlet axial current $J^{(0)}_{\mu 5}$ acquires both electromagnetic and gluonic anomalies, while the isovector $J^{(3)}_{\mu 5}$ and octet $J^{(8)}_{\mu 5}$ currents acquire electromagnetic anomaly only:

$$\partial^\mu J^{(0)}_{\mu 5} = \frac{1}{\sqrt{3}}(m_u\bar{u}\gamma_5 u + m_d\bar{d}\gamma_5 d + m_s\bar{s}\gamma_5 s) + \frac{e^2}{8\pi^2}C^{(0)}N_c F\tilde{F} + \frac{\sqrt{3}\alpha_s}{4\pi}N_c G\widetilde{G}, \tag{2.2}$$

$$\partial^\mu J^{(3)}_{\mu 5} = \frac{1}{\sqrt{2}}(m_u\bar{u}\gamma_5 u - m_d\bar{d}\gamma_5 d) + \frac{e^2}{8\pi^2}C^{(3)}N_c F\tilde{F}, \tag{2.3}$$

$$\partial^\mu J^{(8)}_{\mu 5} = \frac{1}{\sqrt{6}}(m_u\bar{u}\gamma_5 u + m_d\bar{d}\gamma_5 d - 2m_s\bar{s}\gamma_5 s) + \frac{e^2}{8\pi^2}C^{(8)}N_c F\tilde{F}, \tag{2.4}$$

where the electromagnetic charge factors $C^{(a)}$ are defined as follows,

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$$
\begin{aligned}
C^{(3)} &= \frac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \frac{1}{3\sqrt{2}}, \\
C^{(8)} &= \frac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) = \frac{1}{3\sqrt{6}}, \\
C^{(0)} &= \frac{1}{\sqrt{3}}(e_u^2 + e_d^2 + e_s^2) = \frac{2}{3\sqrt{3}}.
\end{aligned} \tag{2.5}
$$

The two-photon transitions are associated with the vector-vector-axial (VVA) amplitude

$$
T_{\alpha\mu\nu}(k,q) = \int d^4x d^4y e^{(ikx+iqy)} \langle 0|T\{J_{\alpha 5}(0)J_\mu(x)J_\nu(y)\}|0\rangle, \tag{2.6}
$$

where J_μ, J_ν are the vector currents with momenta k and q, and $J_{\alpha 5}$ is the axial current with momentum $p = k+q$. This amplitude can be decomposed [12],

$$
\begin{aligned}
T_{\alpha\mu\nu}(k,q) = &F_1\, \varepsilon_{\alpha\mu\nu\rho}k^\rho + F_2\, \varepsilon_{\alpha\mu\nu\rho}q^\rho + F_3\, k_\nu \varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma \\
&+ F_4\, q_\nu \varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma + F_5\, k_\mu \varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma + F_6\, q_\mu \varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma,
\end{aligned} \tag{2.7}
$$

where $F_j = F_j(p^2,k^2,q^2;m^2)$, $j = 1,\dots,6$ are the Lorentz invariant amplitudes constrained by current conservation and Bose symmetry. In what follows, we conform the kinematics of the experiments, restricting ourselves to the case with one virtual photon ($-q^2 = Q^2 > 0$) and one real photon ($k^2 = 0$).

Considering the axial anomaly for the isovector and octet currents in the dispersive approach [13], for the invariant amplitude $F_3 - F_6$, one can derive the anomaly sum rule (ASR) [14]

$$
\int_{4m^2}^{\infty} A_3^{(a)}(s,Q^2;m^2)ds = \frac{e^2}{2\pi}N_c C^{(a)}, \;\; a = 3,8, \tag{2.8}
$$

where $A_3 = \frac{1}{2}Im_{p^2}(F_3 - F_6)$, m is a quark mass.

The ASR (2.8) has a remarkable property – both perturbative and nonperturbative corrections to the integral are absent because of the Adler-Bardeen's theorem [15] and 't Hooft's principle [16, 14].

3. Isovector channel of ASR and pion transition form factor

In order to get the physical applications of the ASR, we need to employ the quark-hadron duality hypothesis.

The $\gamma\gamma^* \to M$ transition form factors ($M = \pi^0, \eta, \eta'$) and meson decay constants f_M^a are defined as

$$
\int d^4x e^{ikx} \langle M(p)|T\{J_\mu(x)J_\nu(0)\}|0\rangle = e^2 \varepsilon_{\mu\nu\rho\sigma}k^\rho q^\sigma F_{M\gamma}, \tag{3.1}
$$

$$
\langle 0|J_{\alpha 5}^{(a)}(0)|M(p)\rangle = ip_\alpha f_M^a. \tag{3.2}
$$

In the case of the *isovector channel*, saturating the lhs of the three-point correlation function (2.6) with the resonances, singling out the first contribution $I_\pi(s_3,Q^2;m^2)$, given by the pion, and

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collecting all the other states into the continuum contribution $I^{(3)}_{cont}(s_3,Q^2;m^2)$, we get the ASR in the form (we suppose $m \equiv m_u = m_d$):

$$I_\pi(s_3,Q^2;m^2) + I^{(3)}_{cont}(s_3,Q^2;m^2) = \frac{1}{2\pi} N_c C^{(3)}, \tag{3.3}$$

where

$$I_\pi(s_3,Q^2;m^2) \equiv \pi f_\pi F_{\pi\gamma}(Q^2,s_3;m^2), \tag{3.4}$$

$$I^{(3)}_{cont}(s_3,Q^2;m^2) \equiv \int_{s_3}^{\infty} A_3^{(3)}(s,Q^2;m^2)ds, \tag{3.5}$$

and s_3 is a continuum threshold; $f_\pi = f_\pi^{(3)} = 130.7$ MeV.

If we employ the one-loop expression for the spectral density [14] (in what follows we take $m=0$)

$$A_3^{(3)}(s,Q^2) = \frac{N_c C^{(3)}}{2\pi} \frac{Q^2}{(Q^2+s)^2}, \tag{3.6}$$

from the Eq. (3.3) we get [7]

$$F_{\pi\gamma}(Q^2,s_3) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_3}{s_3+Q^2}. \tag{3.7}$$

The continuum threshold (pion duality interval) s_3 is determined from the SVZ QCD sum rules $s_3 = 0.75$ GeV^2 [17]. Alternatively, it can be determined from the high-Q^2 limit of (3.7), if we rely on the pQCD predicted value for the pion TFF $Q^2 F_{\pi\gamma} \to \sqrt{2} f_\pi$ [6], $s_3 = 4\pi^2 f_\pi^2 = 0.67$ GeV^2. Then (3.7) reproduces the well-known Brodsky-Lepage interpolation formula [18].

When compared to the experimental data on the pion transition form factor, the equation (3.7) gives a fairly good description of the data of CELLO [4], CLEO [5] and BELLE [3] collaborations, while the data of BABAR collaboration [1] are described much worse (see dashed line in Fig. 1). The BABAR data indicate a log-like growth, and in order to describe them well, one needs to consider the possibility of corrections. As we mentioned above, the integral in the ASR does not have any corrections, but the spectral density $A_3^{(3)}(s,Q^2)$ can acquire corrections (see also [20]), and therefore the continuum and the pion contributions can have the corrections as well. The exactness of the ASR results in an interesting interplay between the corrections to the continuum and pion: they should cancel each other to preserve the ASR, $\delta I^{(3)}_{cont} = -\delta I_\pi$. The form of the correction is not yet known. Nevertheless, we can propose the form the correction relying on the boundary conditions following from the general properties of the ASR:

- $\delta I^{(3)}_{cont} = 0$ at $s_3 \to \infty$ (the continuum contribution vanishes),
- $\delta I^{(3)}_{cont} = 0$ at $s_3 \to 0$ (the full integral has no corrections),
- $\delta I^{(3)}_{cont} = 0$ at $Q^2 \to \infty$ (the perturbative theory works at large Q^2),
- $\delta I^{(3)}_{cont} = 0$ at $Q^2 \to 0$ (anomaly perfectly describes pion decay width).

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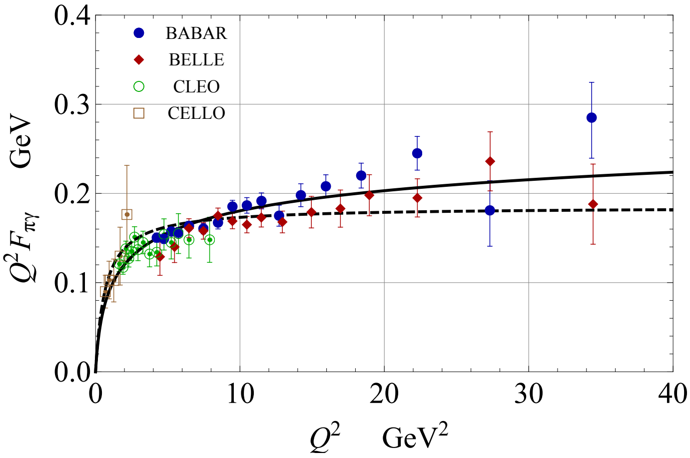

Figure 1: Pion transition form factor compared with experimental data (see explanations in the text).

Supposing the correction contains rational functions and logarithms of Q^2, the simplest form of the correction satisfying those limits results in [10]

$$F_{\pi\gamma}(Q^2) = \frac{1}{\pi f_\pi}(I_\pi + \delta I_\pi) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}\frac{s_3}{s_3+Q^2}\left[1 + \frac{\lambda Q^2}{s_3+Q^2}(\ln\frac{Q^2}{s_3} + \sigma)\right], \tag{3.8}$$

where λ and σ are dimensionless parameters. This kind of correction cannot appear in (a local) OPE and should be attributed, possibly, to instantons or short strings [19] (see more discussion on the origin of the correction in [10]). Note also, that this correction implies that the pion distribution amplitude $\phi(x)$ does not vanish at $x = 0,1$ and violates the QCD factorization (see also [21, 22], where such kind of distribution amplitude was considered after the BABAR data appeared).

The fit of the TFF (3.8) to the combined CELLO, CLEO, BABAR data gives $\lambda = 0.14$, $\sigma = -2.36$, $\chi^2/d.o.f. = 0.94$ $d.o.f. = 35$. The plot of $Q^2 F_{\pi\gamma}$ for these parameters is shown in Fig. 1 as a solid line. The TFF (3.8) with these parameters λ, σ describes well also the combined CELLO, CLEO and BELLE data with $\chi^2/d.o.f. = 0.84$ $(d.o.f. = 35)$. On the other hand, the TFF without correction (3.7) (dashed line in Fig. 1)) gives $\chi^2/d.o.f. = 2.29$ and $\chi^2/d.o.f. = 1.01$ for CELLO, CLEO and BABAR and CELLO, CLEO and BELLE data sets respectively. We can conclude, that although the BABAR data favors the log-like correction, the newly released BELLE data neither confirms, nor excludes the possibility of this correction.

4. Octet channel of ASR and η, η' transition form factors

It is interesting to consider in the same way the ASR in the *octet channel*. Here we should take into account the first two contributions, which are given by the η and η' mesons. Then the ASR in the octet channel is (we use the chiral limit) [8]:

$$f^8_\eta F_{\eta\gamma}(Q^2) + f^8_{\eta'} F_{\eta'\gamma}(Q^2) = \frac{1}{2\sqrt{6}\pi^2}\frac{s_8}{s_8+Q^2}, \tag{4.1}$$

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where s_8 is a continuum threshold in the octet channel, and the decay (coupling) constants f^8_η, $f^8_{\eta'}$ are defined in (3.2). The continuum threshold in the octet channel s_8 can be determined from the large-Q^2 limit of (4.1) and the pQCD predicted expression for the η,η' TFFs [8, 9]:

$$s_8 = 4\pi^2((f^8_\eta)^2 + (f^8_{\eta'})^2 + 2\sqrt{2}[f^8_\eta f^0_\eta + f^8_{\eta'} f^0_{\eta'}]). \tag{4.2}$$

Naturally, if the log-like correction is present in the isovector channel, it should reveal itself in the octet channel too[1]. The similar correction in the octet channel leads to the ASR with the correction term [10]:

$$f^8_\eta F_{\eta\gamma}(Q^2) + f^8_{\eta'} F_{\eta'\gamma}(Q^2) = \frac{1}{2\sqrt{6}\pi^2}\frac{s_8}{s_8+Q^2}\left[1+\frac{\lambda Q^2}{s_8+Q^2}(\ln\frac{Q^2}{s_8}+\sigma)\right]. \tag{4.3}$$

The Eqs. (4.1), (4.2) and (4.3) contain the decay constants f^a_M, which are usually analyzed basing on different mixing schemes or in a scheme independent way (see e.g. [10] and references therein). For the purposes of numerical analysis, we employ the decay constants, obtained in a scheme-independent way in Ref. [10]: $f^8_\eta = 1.11 f_\pi, f^8_{\eta'} = -0.42 f_\pi, f^0_\eta = 0.16 f_\pi, f^8_{\eta'} = 1.04 f_\pi$. Then, the fit of the Eq. (4.3) to the experimental data of BABAR collaboration [2] gives $\lambda = 0.05, \sigma = -2.58$ with $\chi^2/d.o.f. = 0.81$ (see the solid line in Fig. 2), while Eq. (4.1) gives $\chi^2/d.o.f. = 0.85$ (dashed line). At the same time, if the parameters are taken the same as for the pion case $\lambda = 0.14, \sigma = -2.36$, we get $\chi^2/d.o.f. = 1.02$ (dot-dashed line). We see that the current precision of the experimental data on η,η' TFFs can accommodate the log-like correction in the octet channel, although does not require it.

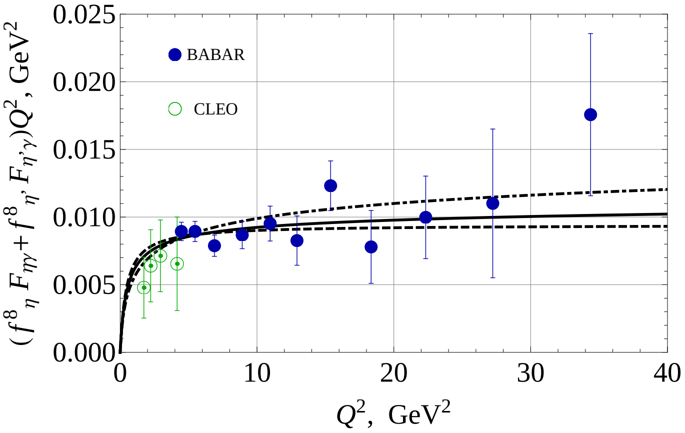

Figure 2: The ASR in the octet channel for different values of fitting parameters compared with the experimental data, see explanations in the text.

5. Conclusions

The current experimental status of the pion TFF in the range of $Q^2 = 10-35 GeV^2$ is rather ambiguous. The BABAR data show an excess over the asymptotic value of the pion TFF, requiring

[1]The possible universality of the corrections is also discussed in [23]

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an essentially nonperturbative correction which is absent in the local OPE and possibly violates the QCD factorization[2]. The more recent BELLE data do not manifest that striking behavior and give more or less consistent with the Brodsky-Lepage interpolation formula description. At the same time, these data do not exclude the considered correction.

The analysis for the octet channel of ASR based on BABAR data on η, η' TFFs leads to the conclusion, consistent with the one made for the isovector channel: within the current experimental errors there is a possibility to accommodate such a correction. However, the correction is not required in this channel by the η, η' BABAR data (contrary to the isovector channel and BABAR data). The further experimental measurements of the π^0, η and η' meson TFFs can enlighten the question whether such a correction exists or not.

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