

## $z$ -Scaling: Inclusive Jet Spectra at RHIC, Tevatron and LHC

---

### **M. Tokarev\***

*Joint Institute for Nuclear Research, Dubna, Russia*

*E-mail: tokarev@jinr.ru*

### **T. Dedovich**

*Joint Institute for Nuclear research, Dubna, Russia*

### **I. Zborovský**

*Nuclear Physics Institute, Academy of Sciences of the Czech Republic*

*Řež, Czech Republic*

*E-mail: zborovsky@ujf.cas.cz*

Self-similarity of jet production in  $pp$  and  $p\bar{p}$  collisions is studied in the framework of  $z$ -scaling. Inclusive jet transverse momentum distributions measured by the STAR Collaboration at RHIC, the CDF and  $D\phi$  Collaborations at Tevatron and the CMS and ATLAS Collaborations at LHC are analyzed. The experimental spectra are compared with next-to-leading order QCD calculations in  $p_T$ - and  $z$ -presentations. It is shown that self-similar features of jet cross sections manifested by  $z$ -scaling give strong restriction on scaling function  $\psi(z)$  at high  $z$ . New results on the energy and angular independence and the asymptotic behavior of  $\psi(z)$  are discussed. The obtained results are considered as confirmation of self-similarity of jet production, fractality of hadron structure and locality of constituent interactions at small scales.

*XXI International Baldin Seminar on High Energy Physics Problems,  
September 10-15, 2012  
JINR, Dubna, Russia*

---

\*Speaker.

## 1. Introduction

Search for scaling regularities in high energy collisions is always a subject of intense investigations [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Such regularities could give experimental indications on new physics phenomena and additional insight into a theory. Production of particles with high transverse momenta from the collisions of hadrons and nuclei at high energies has relevance to constituent interactions at small scales. A universal approach to the description of the processes could provide more detailed understanding of the underlying physical phenomena and impose restrictions on phenomenological ingredients used in comparison of experimental data with a theory. Jets are traditionally considered as best probes of the constituent interactions and most suitable objects for precise test of the perturbative QCD [11, 12, 13, 14, 15, 16, 17]. Study of jets is of interest both for jet production itself and search for new particles identified by the jets.

In this contribution we present results of new analysis of the data on inclusive cross sections of jet production in  $p\bar{p}$  and  $pp$  collisions at the Tevatron [18, 19], RHIC [20], and LHC [21, 22, 23, 24] in the framework of  $z$ -scaling. Results of previous analysis of jet spectra in  $pp$  and  $p\bar{p}$  collisions can be found in [25] and [26]. The method was developed for phenomenological description of differential cross sections of particles in the inclusive reactions at high energies (see [27] and references therein). It is based on the principles of locality, self-similarity and fractality reflecting properties of hadron structure, constituent interactions and processes of particle formation. The  $z$ -scaling is treated as a manifestation of the fractality of the structure of the colliding objects (hadrons or nuclei), locality of the interactions of their constituents, and self-similarity of the particle production process.

For the first time jet production in the  $z$ -presentation was analyzed in [25]. The analysis was based on the experimental data on jet cross sections obtained by the UA1, UA2, CDF and DØ Collaborations. New data on inclusive production of jets at the LHC allow us to verify the  $z$ -scaling in the multi-TeV energy region. Here we report on study of  $z$ -presentation of the jet transverse momentum distributions measured by the CMS [21, 23], ATLAS [22], and ALICE [24] Collaborations in  $pp$  collisions at  $\sqrt{s} = 7$  and 2.76 TeV. The obtained results are compared with data on jet production in  $p\bar{p}$  collisions at the Tevatron [28, 29, 30, 31, 32]. The LHC measurements confirmed the energy independence of  $z$ -scaling for jet production in the new energy domain. The power behavior of scaling function  $\psi(z)$  at high  $z$  is verified. The transverse momentum spectra of jets are calculated in the next-to-next leading order of QCD and are compared with data in  $p_T$ - and  $z$ -presentations.

## 2. $z$ -Scaling

In the present paper we use the  $z$ -scaling in the form considered in [25]. At sufficiently high energies, the collision of extended objects like hadrons and nuclei is considered as an ensemble of individual interactions of their constituents. The constituents are partons in the parton model or quarks and gluons in the theory of QCD. Multiple interactions are assumed to be similar. This property represents a self-similarity of the hadron interactions at a constituent level. The structures of the colliding objects with masses  $M_1$  and  $M_2$  are characterized by parameters  $\delta_1$  and  $\delta_2$ . The interacting constituents carry fractions  $x_1$  and  $x_2$  of incoming momenta  $P_1$  and  $P_2$ . The inclusive

particle with mass  $m_1$  carries momentum  $p$ . The produced recoil with mass  $m_2$  ensures conservation of the additive quantum numbers.

The elementary sub-process is considered as a binary collision of the constituents with masses  $x_1 M_1$  and  $x_2 M_2$  resulting in the scattered and recoil objects with masses  $m_1$  and  $M_X = x_1 M_1 + x_2 M_2 + m_2$ , respectively. The momentum conservation law of the sub-process is connected with recoil mass  $M_X$  which is written as follows

$$(x_1 P_1 + x_2 P_2 - p)^2 = M_X^2. \quad (2.1)$$

This equation is expression of the locality of the hadron interactions at a constituent level. It represents a constraint on fractions  $x_1$  and  $x_2$ . Structural parameters  $\delta_1$  and  $\delta_2$  are connected with the corresponding momentum fractions by the function

$$\Omega(x_1, x_2) = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2}. \quad (2.2)$$

The quantity  $\Omega$  is proportional to a relative number of all such constituent configurations which contain the configuration defined by fractions  $x_1$  and  $x_2$ . Parameters  $\delta_1$  and  $\delta_2$  are interpreted as fractal dimensions in the space of momentum fractions  $\{x_1, x_2\}$ . Fractions  $x_1$  and  $x_2$  are determined in a way to maximize function  $\Omega$  under the condition (2.1). The maximal value of  $\Omega$  is used in the definition of scaling variable  $z$  which has the form

$$z = z_0 \Omega^{-1}. \quad (2.3)$$

Quantity  $z_0 = s_{\perp}^{1/2} / (dN/d\eta|_0) m_N$  is proportional to the transverse kinetic energy  $s_{\perp}^{1/2}$  of the sub-process consumed on the production of the inclusive particle with mass  $m_1$  and its counterpart with mass  $m_2$ . Multiplicity density  $dN/d\eta|_0$  is taken in center-of-mass  $N - N$  system at pseudorapidity  $\eta = 0$ . Constant  $m_N$  is the nucleon mass. We set  $\delta_1 = \delta_2 \equiv \delta$ ,  $M_1 = M_2 \equiv m_N$ , and  $m_2 = m_1 \equiv 0$  for jet production in proton-(anti)proton interactions. Here we use the value  $\delta = 1$  of the nucleon fractal dimension which was found in the analysis of jet production at lower energies [25]. The scaling function

$$\psi(z) = -\frac{\pi s}{(dN/d\eta)\sigma_{in}} J^{-1} E \frac{d^3 \sigma}{dp^3}. \quad (2.4)$$

is expressed in terms of the inclusive cross section, multiplicity density  $dN/d\eta$ , and total inelastic cross section  $\sigma_{in}$ . Symbols  $s$  and  $J$  stand for the square of the center-of-mass energy and the corresponding Jacobian, respectively.

As known from numerous analyses of jet production, different procedures of jet reconstruction are correctly defined only at a relatively high transverse energy [33]. At low  $E_T$ , the main problem is connected with jet overlapping and energy redistribution among multiple jets. It follows that a jet multiplicity density cannot be determined directly from experimental data on jet transverse momentum distributions. Therefore, instead of  $dN/d\eta$  we use the scaled multiplicity density  $\rho(s)/\rho_0$  normalized to unity at energy  $\sqrt{s} = 1800$  GeV. Here we exploit parametrization  $\rho(s) \sim s^{\Delta}$  with  $\Delta = 0.163$  which corresponds to the results found in [25]. As a consequence, the absolute normalization of scaling function  $\psi(z)$  is obtained up to a constant.

### 3. Inclusive jet production

An inclusive jet cross section is a measure of probability of observing a hadron jet with a given transverse energy  $E_T$  and pseudorapidity  $\eta$ . In terms of these variables, the cross section is expressed as follows  $d^2\sigma/dE_T d\eta$ . The measured variables are the transverse energy and pseudorapidity, and the jets are usually assumed to be massless. For most measurements, the cross section is averaged over some range of pseudorapidity. The invariant cross section is written as follows

$$E \frac{d^3\sigma}{dp^3} = \frac{1}{2\pi E_T} \frac{d^2\sigma}{dE_T d\eta}. \quad (3.1)$$

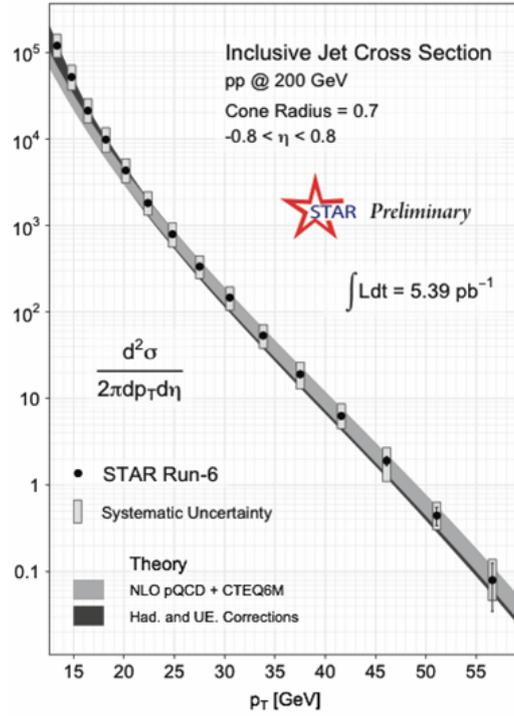
Jets are experimentally defined as the amount of energy deposited in the cone of radius  $R = [(\Delta\eta)^2 + (\Delta\phi)^2]^{1/2}$  in the space  $\{\eta, \phi\}$ , where  $\Delta\eta$  and  $\Delta\phi$  specify the extent of the cone in the pseudorapidity and azimuth. Pseudorapidity  $\eta$  is determined via center mass angle  $\theta$  by formula  $\eta = -\ln(\tan \theta/2)$ . Different algorithms (cone,  $k_T$ , anti- $k_T$ ) of jet reconstruction are developed (see [34] and references therein) and applied for RHIC, Tevatron and LHC data analysis. The algorithms cluster together objects like particles or energies deposited in calorimetric measurements which define single jets. The cone algorithm is based on proximity of the objects in coordinate space. The  $k_T$  algorithm defines jets in a momentum space. The anti- $k_T$  algorithm behaves in many aspects like an optimal jet cone clusterization.

#### 3.1 Jets at RHIC

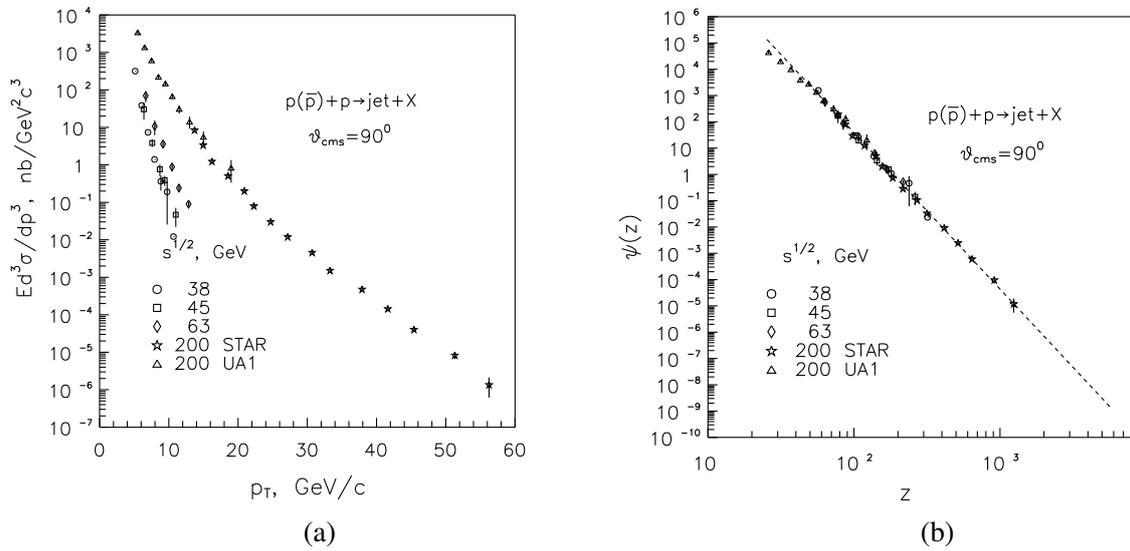
The STAR Collaboration measured the inclusive cross sections of jet production in  $pp$  collisions at energy  $\sqrt{s} = 200$  GeV at RHIC [20]. The experimental data correspond to the integrated luminosity of  $5.39 \text{ pb}^{-1}$ . The measurements cover the kinematic range of the pseudorapidity  $|\eta| < 0.8$  and the transverse momentum  $p_T = 13 - 57$  GeV/c. Figure 1 shows the dependence of the spectra on  $p_T$ . As seen from Fig. 1 the NLO QCD calculations with the CTEQ6M parton distribution functions at the factorization and renormalization scales equal to the transverse momentum ( $\mu_R = \mu_F = p_T$ ) demonstrate a satisfactory agreement with the data.

Figure 2(a) shows comparison of the inclusive cross sections measured by the STAR Collaboration with the FNAL [35] and ISR [36] data obtained at lower energies,  $\sqrt{s} = 38.8, 45, 63$  GeV. As seen from this Figure, the strong dependence of the cross sections on collision energy  $\sqrt{s}$  increases with transverse momentum  $p_T$ . The STAR data are in a good agreement with the UA1 data [37] obtained for jet production in  $p\bar{p}$  collision at energy  $\sqrt{s} = 200$  GeV. The same data are shown in  $z$ -presentation in Fig. 2(b). The energy independence and power law (shown by the dashed line) of scaling function  $\psi(z)$  are observed. The value of slope parameter  $\beta$  is in an agreement with  $\beta = 5.95 \pm 0.21$  found in [25].

The obtained results confirm  $z$ -scaling of jet production in  $pp$  collisions at energy  $\sqrt{s} = 200$  GeV at the RHIC. Some deviation of the UA1 data from the asymptotic behaviour of  $\psi(z)$  is seen at  $z < 10^2$ . Further measurements of the jet spectra in  $pp$  collisions at  $\sqrt{s} = 200$  and 500 GeV at higher  $p_T$  are of interest for precise test of QCD and for verification of the asymptotic behavior of  $\psi(z)$  predicted by  $z$ -scaling in  $pp$  and  $p\bar{p}$  interactions.



**Figure 1:** Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 200$  GeV and  $|\eta| < 0.8$  measured by the STAR Collaboration [20] at the RHIC.



**Figure 2:** Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 38, 45, 63$  and  $200$  GeV in the middle rapidity range in (a)  $p_T$ - and (b)  $z$ -presentation. The experimental data are measured by the E557 [35], AFS [36], and STAR [20] Collaborations. The UA1 data [37] for  $p\bar{p}$  collisions are shown for comparison.

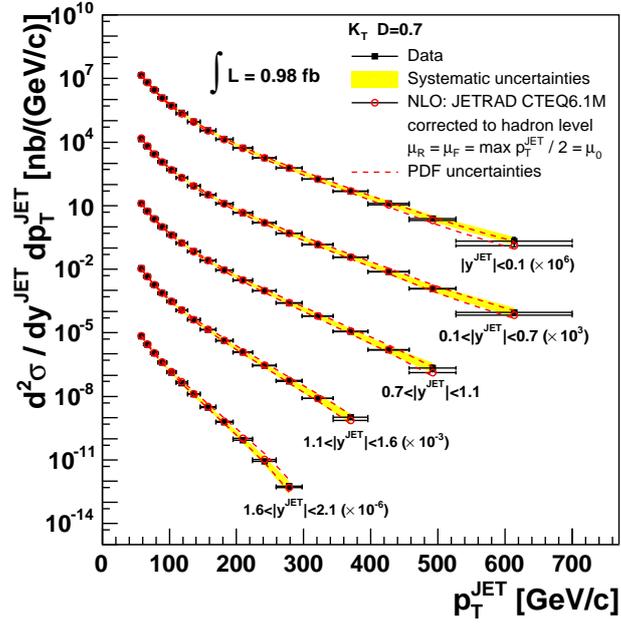
### 3.2 Jets at Tevatron

The collision energy reached at the Tevatron is much higher than maximal energy at the Sp $\bar{p}$ S and also essentially exceeds the energy of  $pp$  interactions at the ISR, SPS and RHIC. Jet production has been successfully measured at the proton-antiproton collider at the FNAL over a range  $\sqrt{s} = 630 - 1960$  GeV in Run I and Run II by the CDF and D $\emptyset$  Collaborations [38]. The experimental data on inclusive cross sections of jet production at different collision energies allow us to study the energy independence of the scaling function and compare it with the similar analysis in  $pp$  collisions. The angular independence of  $\psi(z)$  is tested with the CDF [18] and D $\emptyset$  [19] data measured in a wide range of pseudorapidity.

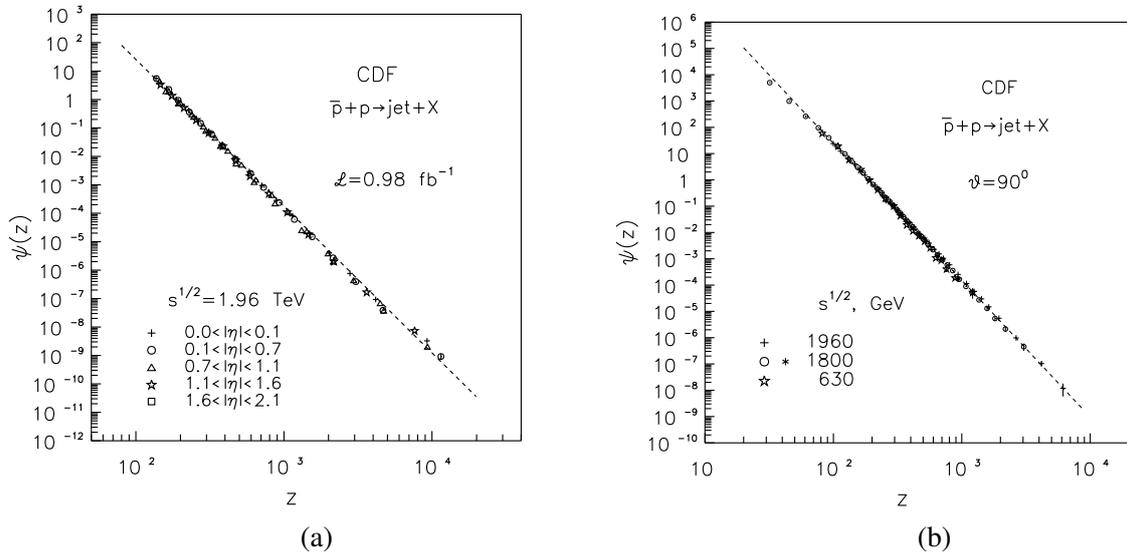
The CDF Collaboration measured the transverse momentum spectra of jets [18] produced in  $p\bar{p}$  collisions at energy  $\sqrt{s} = 1960$  GeV and different angles. The differential cross sections are shown in Fig. 3. The data cover the pseudorapidity range  $|\eta| < 2.1$ . The highest measured transverse energy carried by one jet was about 600 GeV. As seen from Fig. 3, the  $p_T$ -spectra demonstrate the strong dependence on the pseudorapidity of the produced jet. The same data are shown in  $z$ -presentation in Fig. 4(a). The points correspond to the mean values of the transverse momentum in the respective  $p_T$ -bins. The data demonstrate the angular independence and power behavior of scaling function  $\psi(z)$  over the range  $z \simeq (0.15 - 10) \cdot 10^3$ . Figure 4(b) shows  $z$ -presentation of the jet spectra measured by the CDF Collaboration in the central region at energies  $\sqrt{s} = 630, 1800$  and 1960 GeV [30, 31, 32]. All data demonstrate the energy independence of the scaling function. As follows from Fig. 4(b),  $\psi(z)$  shows a power law behavior over a wide range of  $z \simeq (0.3 - 70) \cdot 10^2$ . The scaling function changes about twelve orders of magnitude in this range. The CDF measurements at  $\sqrt{s} = 1960$  GeV give new confirmation of the properties of  $z$ -scaling (the angular and energy independence of  $\psi(z)$ ) found at lower energies [25]. The obtained results mean that the mechanism of jet production manifests the property of self-similarity. In contrast to the  $p_T$ -dependence of differential cross sections, this feature is distinctly visible in the  $z$ -representation of jet spectra.

Figure 5 shows the inclusive spectra of jet production [19] measured by the D $\emptyset$  Collaboration in  $p\bar{p}$  collisions at energy  $\sqrt{s} = 1960$  GeV over a transverse momentum  $p_T = 50 - 600$  GeV/c and pseudorapidity  $|\eta| < 2.4$  ranges. The experimental data correspond to the integrated luminosity of  $0.7 \text{ fb}^{-1}$ . As seen from Fig. 5, the spectra obtained in different pseudorapidity intervals demonstrate a strong angular dependence. The difference in the behavior of the cross sections is enhanced with the increasing  $p_T$ . The jet yields decrease in the measured momentum range more than nine orders of magnitude. Figure 6(a) demonstrates the angular independence of scaling function  $\psi(z)$  for the same data over a wide range of variable  $z$ . Figure 6(b) shows  $z$ -presentation of the jet spectra measured by the D $\emptyset$  Collaboration in the central region ( $\theta = 90^\circ$ ) at energies  $\sqrt{s} = 630, 1800$  and 1960 GeV [28, 29]. The data demonstrate the energy independence of  $z$ -scaling for jet production. The scaling function is described by a power law,  $\psi(z) \sim z^{-\beta}$ , with a constant slope  $\beta$ . The obtained value of parameter  $\beta$  is in agreement with  $\beta = 5.48 \pm 0.02$  found in [25].

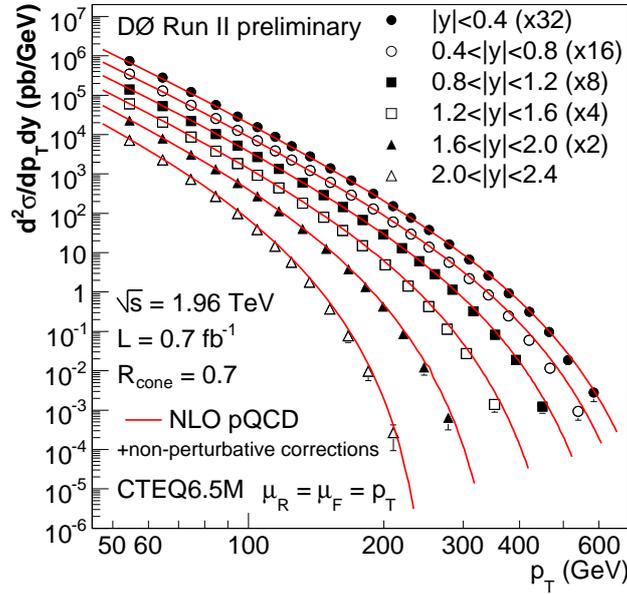
Based on the performed study we conclude that the Tevatron data on jet production measured by the CDF and D $\emptyset$  Collaborations in Run II confirm  $z$ -scaling. The results of the analysis mean that the interactions of hadron constituents, their substructure and mechanism of jet formation reveal properties of self-similarity over a wide scale range (up to  $10^{-4}$  Fm).



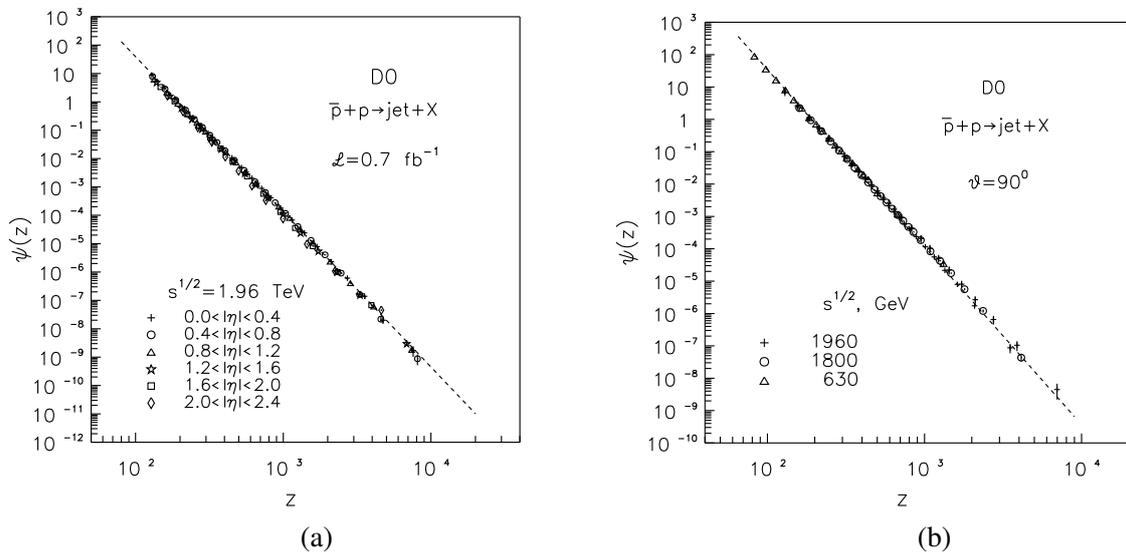
**Figure 3:** Inclusive spectra of jet production in  $p\bar{p}$  collisions at  $\sqrt{s} = 1960$  GeV and different pseudorapidity intervals. The experimental data obtained by the CDF Collaboration are taken from [18].



**Figure 4:** The  $z$ -presentation of inclusive spectra of jet production measured by the CDF Collaborations in  $p\bar{p}$  collisions (a) for different pseudorapidity intervals at  $\sqrt{s} = 1960$  GeV [18] and (b) for  $\theta \simeq 90^\circ$  at  $\sqrt{s} = 630, 1800$  and  $1960$  GeV [30, 31, 32].



**Figure 5:** Inclusive spectra of jet production in  $p\bar{p}$  collisions at  $\sqrt{s} = 1960$  GeV and different pseudorapidity intervals. The experimental data obtained by the DØ Collaboration are taken from [19].



**Figure 6:** The  $z$ -presentation of inclusive spectra of jet production measured by the DØ Collaborations in  $p\bar{p}$  collisions (a) for different pseudorapidity intervals at  $\sqrt{s} = 1960$  GeV [19] and (b) at  $\sqrt{s} = 630, 1800$  and  $1960$  GeV and  $\theta \simeq 90^\circ$  [28, 29].

### 3.3 Jets at LHC

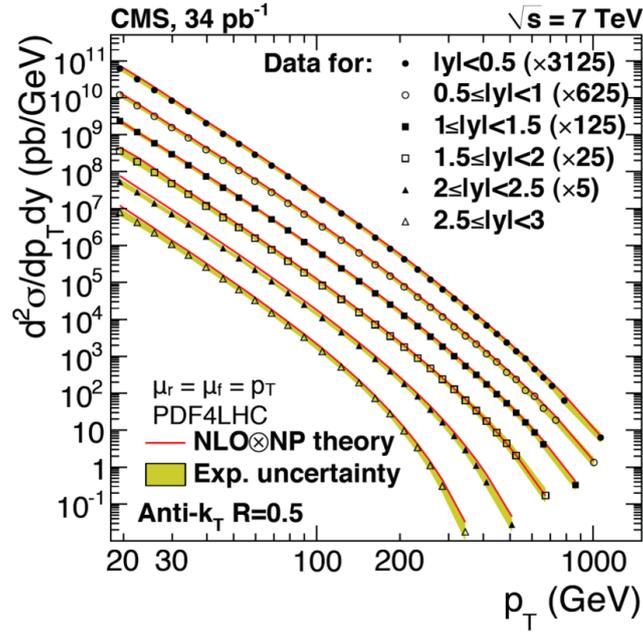
The inclusive production of jets was measured in proton-proton collisions at the LHC. The world's largest collider gives us possibility to investigate jet physics in the new energy domain [15, 16, 17]. It concerns a study of the interactions of proton constituents in processes with high transverse momenta up to  $p_T \simeq 5000$  GeV and search for new physics in the reactions where jets are used as triggers. We analyzed the data on inclusive cross sections of jet production in  $pp$  collisions at energies  $\sqrt{s} = 7000$  GeV [21, 22, 23] and  $\sqrt{s} = 2760$  GeV [24] in the framework of  $z$ -scaling and compared the results with the Tevatron data [28, 29, 30, 31, 32] at lower energies. The LHC and Tevatron data allow us to study the energy and angular independence of scaling function  $\psi(z)$  over a wide range of collision energy  $\sqrt{s} = 630 - 7000$  GeV and pseudorapidity  $|\eta| < 4.4$ .

Figure 7 shows the spectra of jets [21] measured by the CMS Collaboration in  $pp$  collisions at  $\sqrt{s} = 7000$  GeV and  $|\eta| < 3$ . The data sample corresponds to the integral luminosity of  $34 \text{ pb}^{-1}$ . The spectra in various pseudorapidity intervals are represented in terms of variable  $z$  in Fig. 8(a). A comparison of the  $z$ -presentation of the jet distribution in the most central pseudorapidity interval with jet spectra measured by the DØ Collaboration at the Tevatron is shown in Fig. 8(b). As follows from Fig.8, the experimental data manifest angular (a) and energy (b) independences of the scaling function. The corresponding values of  $\psi(z)$  change more than twelve orders of magnitude. A power behavior of the scaling function is observed over a wide range of  $z \simeq (0.1 - 7) \cdot 10^3$ . The dashed line indicates the asymptotic behavior of  $\psi(z)$  at high  $z$ . Some deviation of the data from the power asymptotics for  $z < 10^2$  is seen.

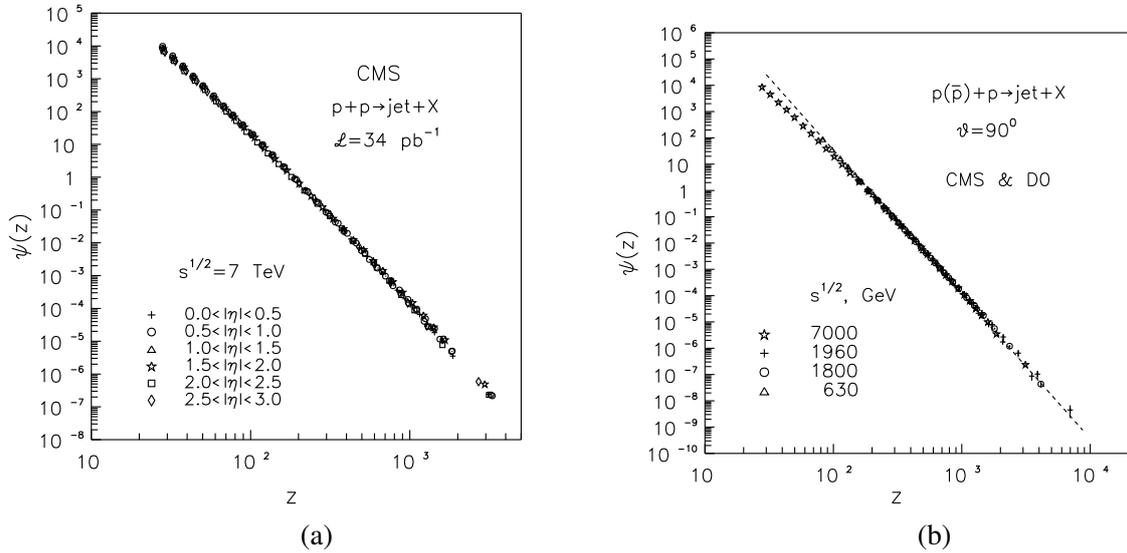
Figure 9 shows the inclusive spectra of jets measured by the ATLAS Collaboration [22] in  $pp$  collisions at energy  $\sqrt{s} = 7000$  GeV over the transverse momentum  $p_T = 20 - 1350$  GeV/c and pseudorapidity  $|\eta| < 4.4$  ranges. The  $z$ -presentation of the same data is demonstrated in Fig. 10(a). The points correspond to the mean values of the transverse momenta in the respective  $p_T$ -bins. The angular independence and power law of the scaling function is observed up to  $z \simeq 10^4$ . The energy independence of scaling function  $\psi(z)$  is demonstrated in Fig. 10(b) with the data measured at energies  $\sqrt{s} = 630, 1800, 1960$  and  $7000$  GeV. The dashed line corresponds to the asymptotic behavior of  $\psi(z)$ .

Figure 11 shows the new data on jet inclusive spectra [23] obtained by the CMS Collaboration in  $pp$  collisions at  $\sqrt{s} = 7000$  GeV for different angular intervals. The data sample corresponds to integral luminosity  $4.7 \text{ fb}^{-1}$ . Jets were reconstructed over the range  $p_T = 114 - 2000$  GeV and  $|y| < 2.5$ . The angular independence of the scaling function for these data is illustrated in Fig. 12(a). The new CMS measurements confirmed the energy independence of  $\psi(z)$  over a wide range of  $z$ . This is demonstrated in Fig. 12(b) where the data in the central region at  $\sqrt{s} = 7000$  GeV are compared with the jet spectra measured by the DØ Collaboration [28, 29] at lower energies.

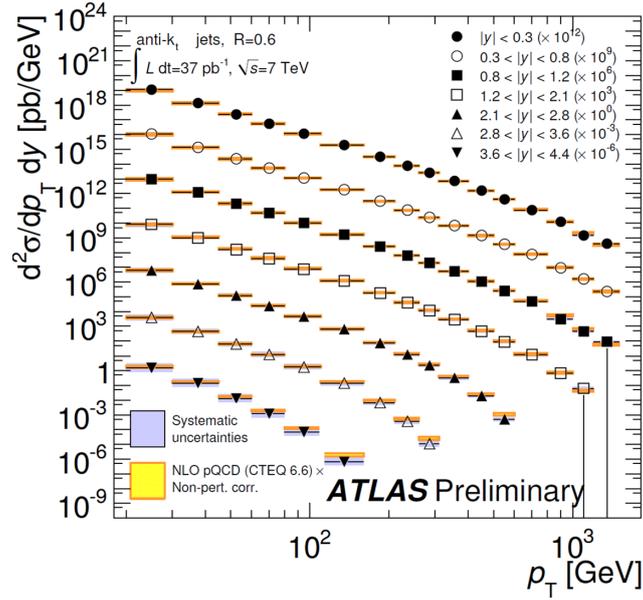
As one can see from Figs. 8(a), 10(a) and 12(a), the combined analysis of the CMS and ATLAS data manifests the angular independence of scaling function  $\psi(z)$ . A comparison of the Tevatron and LHC data shown in Figs. 8(b), 10(b) and 12(b) demonstrates the energy independence of  $\psi(z)$ . The scaling function is described by a power law  $\psi(z) \sim z^{-\beta}$  at high  $z$  with a constant value of slope parameter  $\beta$ . The value of  $\beta$  is found to be the same for  $p\bar{p}$  and  $pp$  collisions. The power behavior of  $\psi(z)$  is observed over a wide range of  $z$ . The obtained results give us a new confirmation of the self-similarity of jet production in  $pp$  collisions at the LHC.



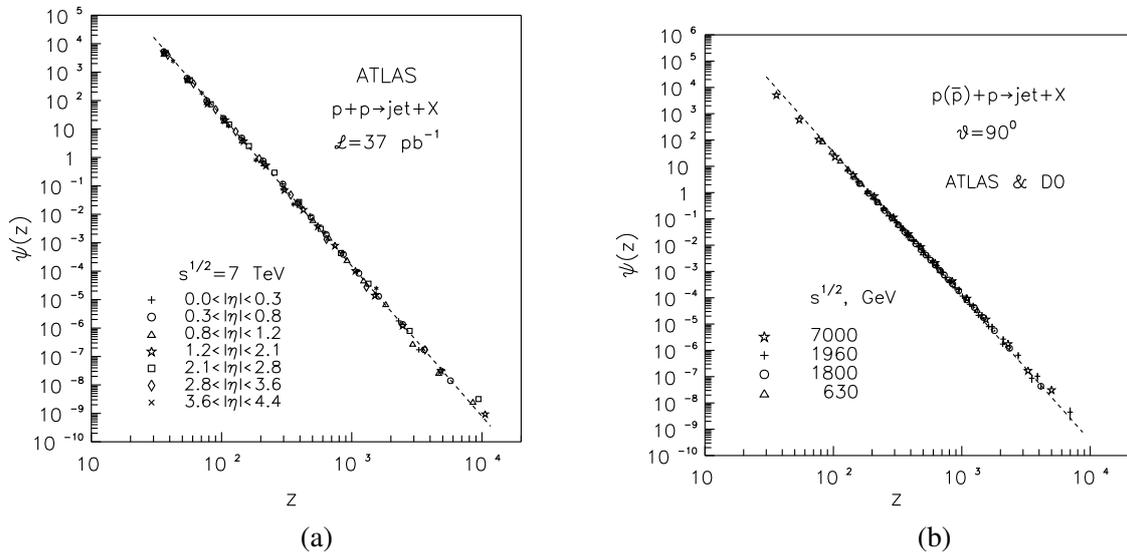
**Figure 7:** Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 7000$  GeV over a pseudorapidity range  $|\eta| < 3$  measured by the CMS Collaboration [21] at the LHC. The data correspond to integral luminosity  $L = 34 \text{ pb}^{-1}$



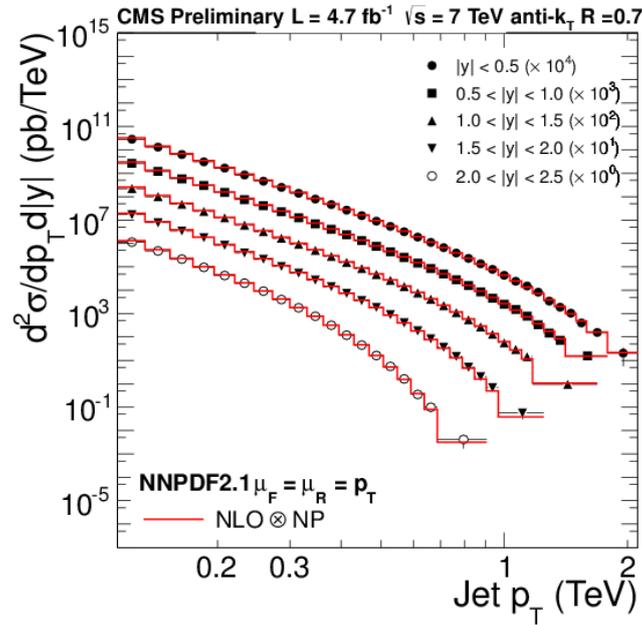
**Figure 8:** (a) Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 7000$  GeV over a pseudorapidity range  $|\eta| < 3$  measured by the CMS Collaboration [21] in  $z$ -presentation. (b) Inclusive spectra at different energies  $\sqrt{s} = 630, 1800, 1960$  and  $7000$  GeV in the central pseudorapidity region in  $z$ -presentation. The data are taken from [28, 29] and [21].



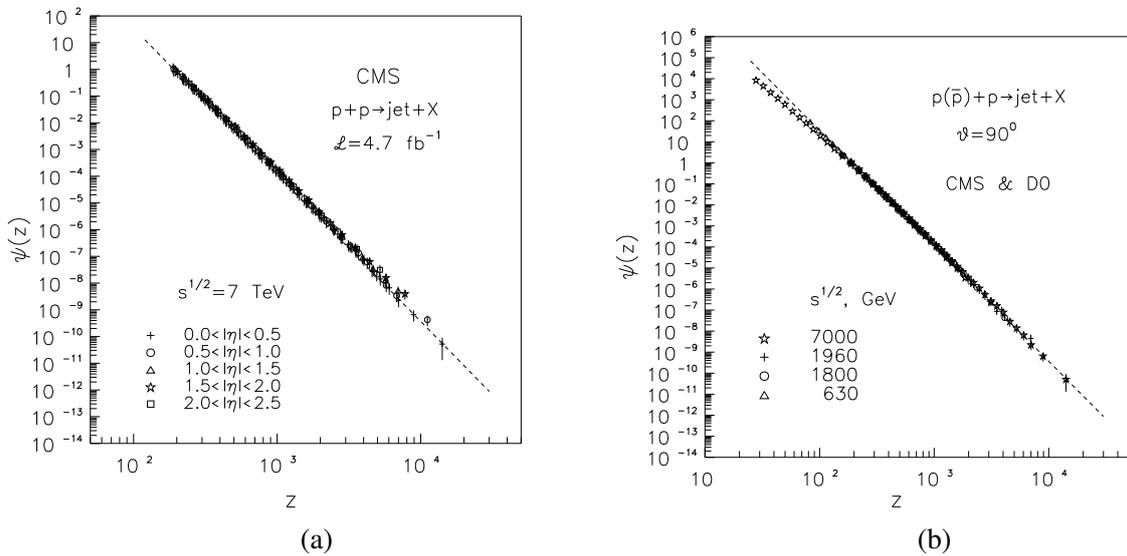
**Figure 9:** Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 7000$  GeV over a pseudorapidity range  $|\eta| < 4.4$  measured by the ATLAS Collaboration [22] at the LHC. The data correspond to integral luminosity  $L = 37 \text{ pb}^{-1}$



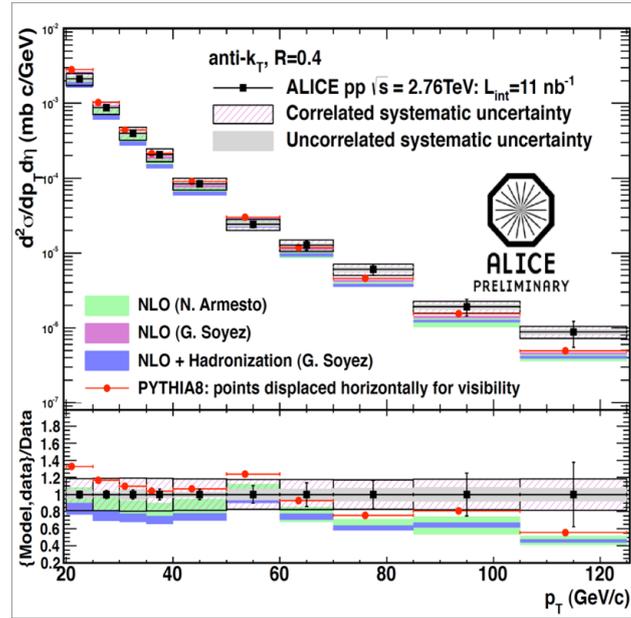
**Figure 10:** (a) Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 7$  TeV over a pseudorapidity range  $|\eta| < 4.4$  measured by the ATLAS Collaboration [22] in  $z$ -presentation. (b) The  $z$ -presentation of inclusive spectra at different energies  $\sqrt{s} = 630, 1800, 1960$  and  $7000$  GeV in the central pseudorapidity region. The data are taken from [28, 29] and [22].



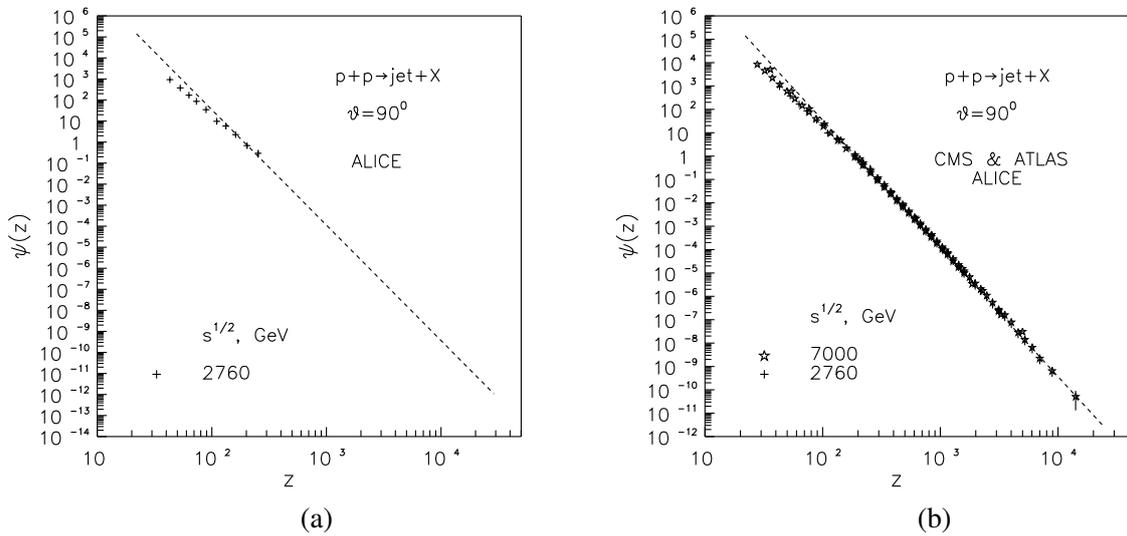
**Figure 11:** Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 7000$  GeV over a pseudorapidity range  $|\eta| < 2.5$  measured by the CMS Collaboration [23] at the LHC. The data correspond to integral luminosity  $L = 4.7 \text{ fb}^{-1}$ .



**Figure 12:** (a) Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 7$  TeV over a pseudorapidity range  $|\eta| < 2.5$  measured by the CMS Collaboration [23] in  $z$ -presentation. (b) The  $z$ -presentation of inclusive spectra at different energies  $\sqrt{s} = 630, 1800, 1960$  and  $7000$  GeV in the central pseudorapidity region. The data are taken from [28, 29] and [23].



**Figure 13:** Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 2760$  GeV over a pseudorapidity range  $|\eta| < 3$  measured by the ALICE Collaboration [24] at the LHC. The data correspond to integral luminosity  $L = 11 \text{ nb}^{-1}$ .



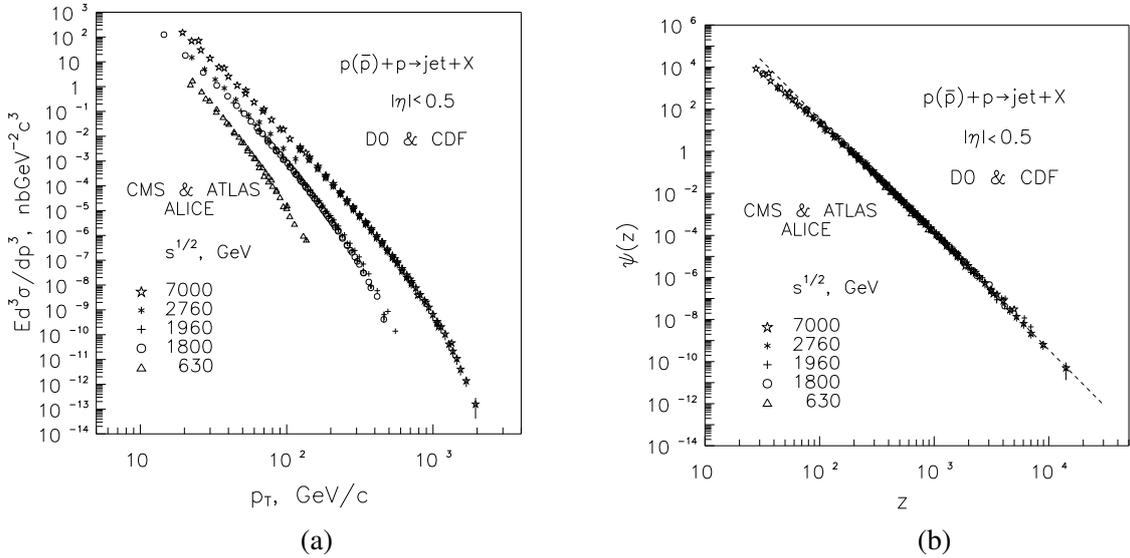
**Figure 14:** (a) Inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 2760$  GeV over a pseudorapidity range  $|\eta| < 3$  measured by the ALICE Collaboration [24] in  $z$ -presentation. The asymptotic curve of  $\psi(z)$  is shown by the dashed line. (b) The  $z$ -presentation of inclusive spectra at energies  $\sqrt{s} = 7000$  and  $2760$  GeV in the central pseudorapidity region. The data are taken from [22, 23] and [24].

The ALICE Collaboration presented the data [24] on jet  $p_T$  distribution in  $pp$  collisions at energy  $\sqrt{s} = 2760$  GeV. The spectrum measured in the momentum  $p_T = 20 - 130$  GeV/c and pseudorapidity  $|\eta| < 3$  range is depicted in Fig. 13. The data sample corresponds to integral luminosity  $L = 11 \text{ nb}^{-1}$ . Figure 14(a) shows the data in  $z$ -presentation. The dashed line represents the asymptotic curve of  $\psi(z)$  found from analysis of the DØ, CDF, CMS and ATLAS data. The ALICE data indicate on deviation from the asymptotic behavior of the scaling function at low  $p_T$ . Mutual comparison of the  $z$ -presentations of the ALICE data at  $\sqrt{s} = 2760$  GeV with the CMS and ATLAS data at  $\sqrt{s} = 7000$  GeV is shown in Fig. 14(b).

#### 4. Universality of jet production at Tevatron and LHC

A jet is usually considered as a direct evidence of hard interaction of hadron constituents (quarks and gluons). The spectra of jets can be connected with the constituent interactions in terms of scaling variable  $z$ . We have analysed the new LHC data [21, 22, 23] and [24] on inclusive cross sections of jet production in  $pp$  collisions at  $\sqrt{s} = 7000$  and 2760 GeV in the framework of  $z$ -scaling. The results were compared with the Tevatron data [28, 29, 30, 31, 32] at lower collision energies. The data sets allowed us to study the energy and angular independence of scaling function  $\psi(z)$  over a wide range of  $\sqrt{s} = 630 - 7000$  GeV. The analysis was performed with the same values of parameters  $\delta = 1$  and  $m_1 = m_2 = 0$  as used in [25].

Figure 15 shows comparative study of (a)  $p_T$ - and (b)  $z$ -presentation of the jet spectra measured in the central pseudorapidity region by the CMS, ATLAS and ALICE Collaborations at the LHC



**Figure 15:** The (a)  $p_T$ - and (b)  $z$ -presentation of inclusive spectra of jet production in  $pp$  collisions at  $\sqrt{s} = 2760, 7000$  GeV and in  $p\bar{p}$  collisions at  $\sqrt{s} = 630, 1800, 1960$  GeV measured in the central pseudorapidity region. The respective data were obtained by the CMS [21, 23], ATLAS [22] and ALICE [24] Collaborations at the LHC and by the DØ [28, 29] and CDF [31, 32] Collaborations at the Tevatron.

and by the DØ and CDF Collaborations at the Tevatron. The  $p_T$ -distributions show the strong dependence on collision energy  $\sqrt{s}$ . In contrast to this, all considered data confirm the energy independence of scaling function  $\psi(z)$ . The function is characterized by a power law  $\psi(z) \sim z^{-\beta}$  at high  $z$  with a constant value of slope parameter  $\beta$ . As seen from Fig. 15(b), the power behavior is observed over a wide range of  $z > 10^2$ . The maximal value of  $z \sim 10^4$  reached in central region  $|\eta| < 0.5$  corresponds to jet transverse momentum  $p_T = 2000$  GeV measured at LHC energy  $\sqrt{s} = 7000$  GeV. Moreover, the data on jet production in various pseudorapidity intervals obtained by the CMS and ATLAS Collaborations demonstrate the angular independence of  $\psi(z)$ . The result is in a good agreement with the angular independence of  $z$ -presentation of the jet spectra measured by the CDF and DØ Collaborations at lower energies [39]. Comparison of the LHC and Tevatron data indicates universality of the shape of the scaling function in  $pp$  and  $p\bar{p}$  collisions at high energies. It gives us new confirmation of the self-similarity of jet production in  $pp$  and  $p\bar{p}$  collisions at small scales.

## 5. QCD test of $z$ -scaling

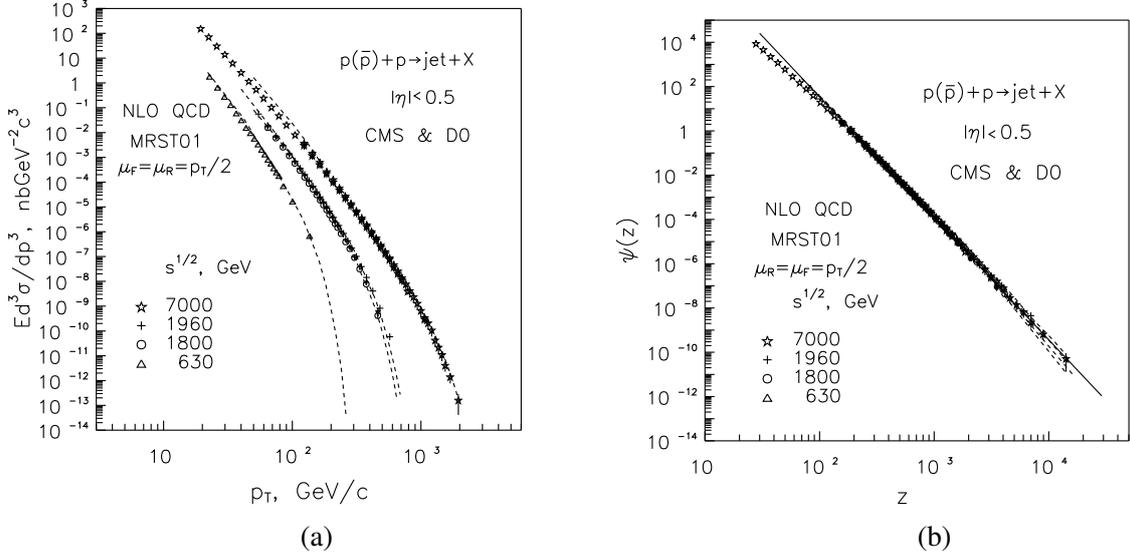
The  $z$ -scaling is a phenomenological method of analysis of spectra of inclusive particles produced in the collisions of extended objects like hadrons and nuclei. It is based on the principles of locality and self-similarity of interactions of hadron constituents at different scales. Validity of the scaling was obtained from many analyses of numerous experimental data on differential cross sections of inclusive reactions. In that view one of the interesting problems is QCD test of  $z$ -scaling. The theory of strong interactions, QCD, is based on the fundamental principles and was experimentally verified in many situations. Study of relation between QCD and  $z$ -scaling could give more deep understanding of physical phenomena especially those which require non-perturbative approach or phenomenological ingredients for their description.

For this purpose we performed calculations using the code from Ellis-Kunszt-Soper [40] for the jet transverse momentum spectra. The calculations by EKS group are based on the matrix elements published in [41, 42]. For the parton distribution functions (PDF) we used the CTEQ6M [43] and MRST01 [44] which incorporate a large amount of experimental information evaluated at the NLO level. The choice of the factorization ( $\mu_F$ ) and renormalization ( $\mu_R$ ) scales influences the NLO jet cross sections. The typical value of these parameters varies in the range  $(p_T/2, 2p_T)$  in standard calculations. In our calculations the renormalization and factorization scales are taken to be  $\mu_R = \mu_F = p_T/2$ . The quantity  $p_T$  refers to the transverse momentum of the jet in jet-level expressions. The NLO results depend on the jet-cone radius  $R = [(\Delta\eta)^2 + (\Delta\phi)^2]^{1/2}$  in the pseudorapidity and azimuth  $\{\eta, \phi\}$  space and the jet-separation parameter  $R_{sep}$ . Both parameters are connected by relation  $R_{sep} = 2R$  and the "optimal value" of the cone radius  $R = 0.7$  is taken.

The inclusive differential cross section for hadron production with transverse momentum  $p_T$  and pseudorapidity  $\eta$  is written as follows

$$E_C \frac{d^3\sigma}{dp_C^3} = \sum_{abc} \int dx_a dx_b \frac{dz_c}{z_c} f_{a/A}(x_a, \mu_F) f_{b/B}(x_b, \mu_F) \epsilon_c \frac{d^3\sigma}{dk_c^3}(p_c/z_c\sqrt{s}, \mu_R) D_{C/c}(z_c, \mu_H) \quad (5.1)$$

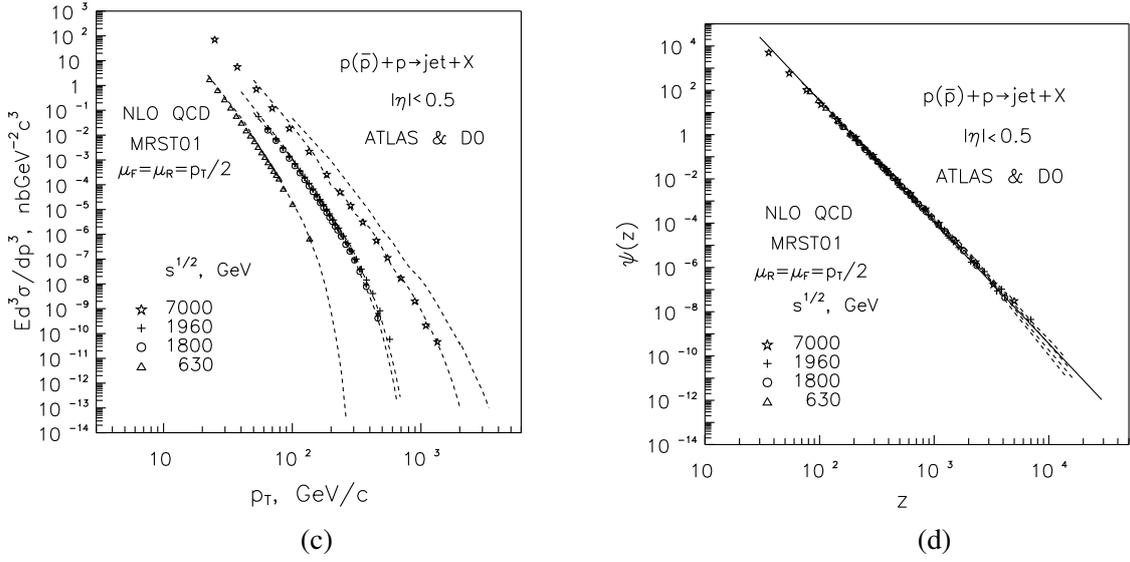
The sum is taken over the various flavors of partons that can participate in the hard scattering process. The functions  $f_{a/A}(x_a, \mu_F)$  and  $f_{b/B}(x_b, \mu_F)$  describe the distributions of a parton  $a$  and  $b$



**Figure 16:** Inclusive cross sections of jet production in  $p\bar{p}$  and  $pp$  collisions in (a)  $p_T$ - and (b)  $z$ -presentation. The dashed lines represent calculated results in NLO QCD with MRST01 [44] PDFs at  $\sqrt{s} = 630, 1800, 1960, 7000$  and  $14000$  GeV. The points ( $\Delta, \circ, +, \star$ ) correspond to the experimental data obtained by the DØ [28, 29] and CMS [21, 23] Collaborations at  $\sqrt{s} = 630, 1800, 1960$  and  $7000$  GeV, respectively.

in the hadron  $A$  and  $B$  on momentum fraction  $x_a$  and  $x_b$  at the factorization scale  $\mu_F$ , respectively. The fragmentation of parton  $c$  into hadron  $C$  is described by function  $D_c^C(z_c, \mu_H)$ . The parton momentum fraction carried by hadron  $C$  at the fragmentation scale  $\mu_H$  is equal to  $z_c$ . The cross section  $\epsilon_c d^3 \sigma / dk_c^3$  of the parton sub-process  $a + b \rightarrow c + X$  is calculated perturbatively in the NLO QCD. Here we have  $c \equiv \text{jet}$ . It is assumed that fragmentation does not destroy the transverse distribution of jet and  $z_c = 1$ . The  $\overline{\text{MS}}$  scheme is used to subtract final state collinear singularities. The strong coupling  $\alpha_S(\mu_R)$  is defined in the  $\overline{\text{MS}}$  renormalization scheme at scale  $\mu_R$ .

The invariant cross sections of jet production as a function of energy  $\sqrt{s}$  and transverse momentum  $p_T$  calculated in the NLO QCD are shown in Figs. 16(a) and 17(a). The dashed lines depict the calculations performed with the parton distribution functions MRST01 [44]. The renormalization, factorization and fragmentation scales were set to be equal each other  $\mu_R = \mu_F = \zeta p_T$ , where  $\zeta$  is a scale factor which varies usually in the range  $\zeta = 0.5 - 2$ . Here we used  $\zeta = 0.5$ . The spectra were calculated at  $\theta \simeq 90^\circ$  over a wide range of the transverse momentum  $p_T = 20 - 3000$   $\text{GeV}/c$  and the energy  $\sqrt{s} = 630 - 14000$  GeV. As seen from Figs. 16(a) and 17(a), the strong dependence of the spectra on the collision energy increases with  $p_T$ . The experimental data on cross sections obtained by the DØ Collaboration [28, 29] at energies  $\sqrt{s} = 630, 1800, 1960$  and by the CMS [21, 23] and ATLAS [22] Collaborations at  $\sqrt{s} = 7000$  GeV are shown by the symbols ( $\Delta, \circ, +$ ) and ( $\star$ ), respectively. The same data are plotted in dependence on variable  $z$  in Figs. 16(b) and 17(b). The solid line shows the asymptotic behavior of function  $\psi(z)$  obtained from the analysis of the experimental data at large  $z$ . The value of slope parameter  $\beta$  corresponds to  $5.48 \pm 0.02$ . The



**Figure 17:** Inclusive cross sections of jet production in  $p\bar{p}$  and  $pp$  collisions in (a)  $p_T$ - and (b)  $z$ -presentation. The dashed lines represent calculated results in NLO QCD with MRST01 [44] PDFs at  $\sqrt{s} = 630, 1800, 1960, 7000$  and  $14000$  GeV. The points ( $\Delta, \circ, +, \star$ ) correspond to the experimental data obtained by the D $\phi$  [28, 29] and ATLAS [22] Collaborations at  $\sqrt{s} = 630, 1800, 1960$  and  $7000$  GeV, respectively.

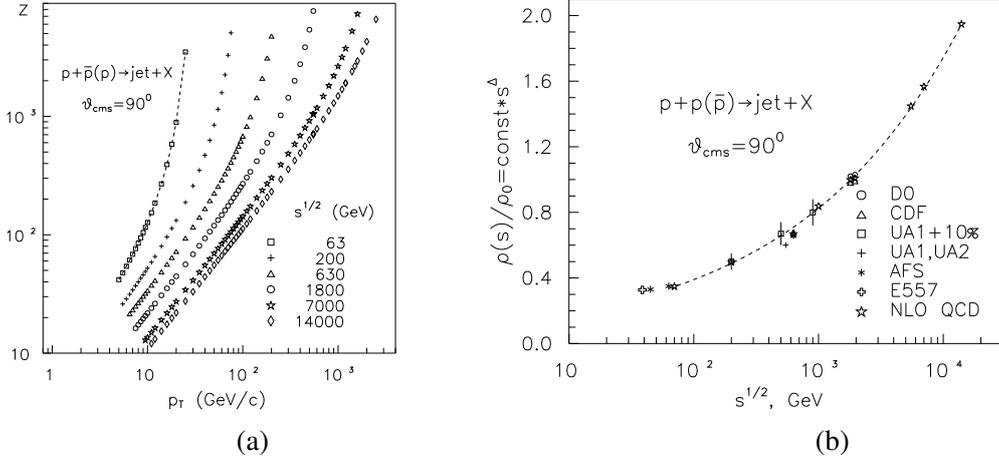
dashed lines depict  $z$ -presentation of the NLO QCD calculations shown in Figs. 16(a) and 17(a).

Note that a good agreement between the experimental data and the corresponding NLO QCD calculations is observed. One can see, however, that the NLO QCD predictions demonstrate some deviation from the asymptotic behavior of  $\psi(z)$  predicted by  $z$ -scaling as the collision energy and transverse momentum increase. The deviation is clearly visible in the region of high  $p_T$  where experimental measurements of inclusive jet transverse momentum spectra are not performed yet. Some factors can modify the asymptotic behavior of  $\psi(z)$ . They are quark and gluon distribution functions. We assume that especially large uncertainties of the gluon distribution function could be restricted by the power behavior of  $\psi(z)$  established over a wide range of  $\sqrt{s}$  and high  $p_T$ . The behavior of the scaling function can give a new additional constraint on PDFs which are phenomenological ingredients weakly controlled by the perturbative QCD.

Based on the obtained results we conclude that self-similar features of jet production dictated by  $z$ -scaling give strong restriction on the asymptotic behavior of scaling function  $\psi(z)$  for  $p\bar{p}$  and  $pp$  collisions. The behavior of  $\psi(z)$  is reasonably reproduced at Tevatron and LHC energies by the NLO QCD evolution of the cross sections with the phenomenological parton distribution functions used in the present analysis.

## 6. $z$ - $p_T$ plot and jet multiplicity density

The performed systematic analysis of experimental data on inclusive jet production in  $pp/\bar{p}p$  collisions measured at the RHIC, Tevatron and LHC showed that the jet cross sections can be well described by power function  $\psi(z) \sim z^{-\beta}$  over a wide kinematic range (up to the collision energy

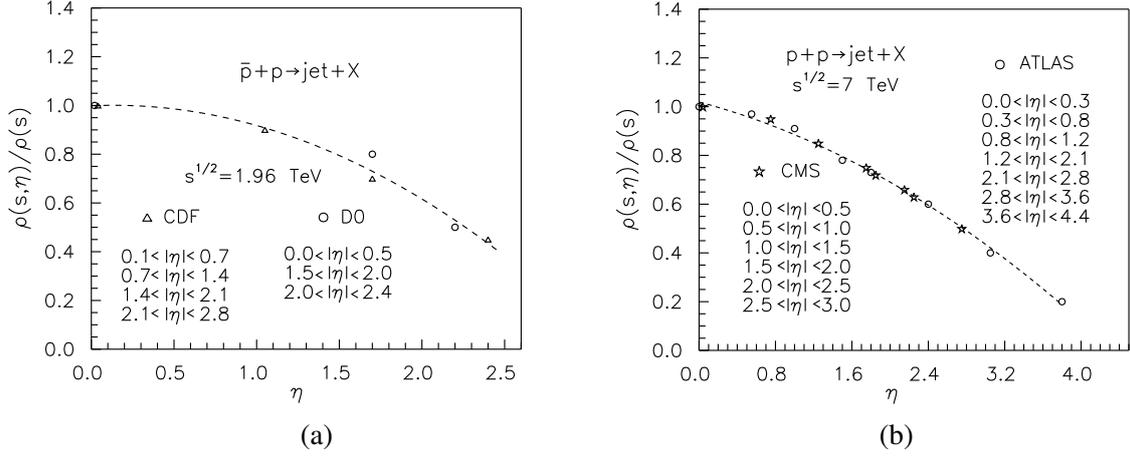


**Figure 18:** (a)  $z - p_T$  plot for jet production in  $pp$  and  $p\bar{p}$  collisions at  $\sqrt{s} = 63 - 14000$  GeV and angle  $\theta_{cms} = 90^\circ$ . (b) The energy dependence of the normalized jet multiplicity density  $\rho(s)/\rho_0$  used in our analysis of  $pp$  and  $p\bar{p}$  collisions.

$\sqrt{s} = 7000$  GeV and the transverse momentum  $p_T \simeq 2000$  GeV/c). We interpret this result as a strong confirmation that self-similarity is a general property of hadron structure, interactions of hadron constituents and mechanism of jet formation. More extensive analysis of the properties of  $z$ -scaling is desirable to study the self-similarity of jet production in physically interesting unexplored regions. Search for new phenomena in processes with jets requires knowledge of kinematic conditions which are preferable for such investigations. The dependence of variable  $z$  on the transverse momentum (the  $z - p_T$  plot) shown in Fig. 18(a) allows us to select the suitable regions. In this context, let us remind that  $z$ -presentation of data shown in Figs. 2(b), 4, 6, 8, 10, 12 and 15(b) is consistent with the power law of  $\psi(z)$  at large  $z$ . This property of  $z$ -scaling is valid up to  $z \simeq 1.4 \cdot 10^4$ . As seen from Fig. 18(a), verification of the observed regularity for even larger values of  $z$  would correspond to transverse momentum  $p_T > 60, 500, 2000$  and  $3000$  GeV/c at energies  $\sqrt{s} = 200, 1800, 7000$  and  $14000$  GeV, respectively. We consider that these are the kinematic regions where potentially new effects of jet production may be expected.

Figure 18(b) shows normalized jet multiplicity density  $\rho(s)/\rho_0$  at midrapidity  $\eta = 0$  as a function of collision energy  $\sqrt{s}$ . The multiplicity density is normalized at  $\sqrt{s} = 1800$  GeV and its energy dependence shown by the dashed line is taken in form  $\rho(s) \sim s^\Delta$  with  $\Delta = 0.163$ . The parametrization corresponds to the results [25] found in jet analysis at lower energies. The estimated values of  $\rho(s)/\rho_0$  at  $\sqrt{s} = 7000$  GeV and  $14000$  GeV are found to be  $1.56$  and  $1.95$ , respectively.

Figure 19 shows the  $\eta$ -dependences of ratio  $\rho(s, \eta)/\rho(s)$  of the jet multiplicity density related to its value at  $\eta = 0$ . The symbols represent the quantity obtained from the requirement of the same normalization of  $\psi(z)$  at different angles. One can see the mutual correspondence of the ratio extracted from the jet spectra measured by the CDF and DØ Collaborations at  $\sqrt{s} = 1960$  GeV, and by the CMS and ATLAS Collaborations at  $\sqrt{s} = 7000$  GeV. The dashed lines represent a parametrization normalized to unity at  $\eta = 0$ .



**Figure 19:** The pseudorapidity dependence of normalized jet multiplicity density  $\rho(s, \eta)/\rho(s)$  for (a)  $p\bar{p}$  collisions at  $\sqrt{s} = 1960$  GeV and (b)  $pp$  collisions at  $\sqrt{s} = 7000$  GeV. The symbols correspond to the values of multiplicity density used in  $z$ -presentation of the experimental data at different angles. The dashed lines show a parametrization normalized to unity at  $\eta = 0$ .

## 7. Conclusions

The experimental data on inclusive spectra of jet production in  $pp$  and  $p\bar{p}$  collisions obtained at the RHIC, Tevatron and LHC were analysed in the framework of  $z$ -scaling. We found a new confirmation of the energy and angular independence of the  $z$ -presentation of these data. The power behavior of the scaling function,  $\psi(z) \sim z^{-\beta}$ , is observed up to the highest jet transverse momentum  $p_T \simeq 2000$  GeV/c measured so far. The obtained results show that properties of jet production reflect the self-similarity, locality and fractality of hadron interactions at a constituent level.

A QCD test of  $z$ -scaling of jet production in  $p\bar{p}$  and  $pp$  collisions was performed in a wide region of  $z$ . The inclusive cross sections were calculated in the NLO QCD with different parton distribution functions and compared in the  $z$ -presentation with the scaling function obtained from analysis of experimental data. It was shown that self-similar features of particle production dictated by  $z$ -scaling give strong restriction on the asymptotic behavior of the scaling function and inclusive spectra in high- $p_T$  region. The result is important for further tests of the QCD and more precise specification of the phenomenological ingredients (such as PDFs and FFs) of the theory. It can be used as an additional constraint on the gluon distribution function in the global QCD analysis of experimental data.

More detailed study of  $z$ -scaling of jet production in  $pp$  and  $p\bar{p}$  collisions is desirable in physically interesting regions where experimental data are not obtained yet. A possible violation of the scaling is suggested to be considered as a signature of new phenomena which may be expected in the region  $z > 10^4$ .

## Acknowledgments

The investigations have been partially supported by RVO61389005 and by the Ministry of

Education, Youth and Sports of the Czech Republic, grant LA08002.

## References

- [1] R.P. Feynman, *Phys. Rev. Lett.* **23**, 1415 (1969).
- [2] J.D. Bjorken, *Phys. Rev.* **179**, 1547 (1969).  
J.D. Bjorken and E.A. Paschos, *Phys. Rev.* **185**, 1975 (1969).
- [3] P. Bosted *et al.*, *Phys. Rev. Lett.* **49**, 1380 (1972).
- [4] J. Benecke *et al.*, *Phys. Rev.* **188**, 2159 (1969).
- [5] A.M. Baldin, *Sov. J. Part. Nucl.* **8**, 429 (1977).
- [6] V.S. Stavinsky, *Sov. J. Part. Nucl.* **10**, 949 (1979).
- [7] G.A. Leksin, Report No. ITEF-147, 1976.  
G.A. Leksin, *Proceedings of the XVIII Int. Conf. on High Energy Physics*, Tbilisi, Georgia, 1976, edited by N.N. Bogolubov *et al.*, (JINR Report No. D1,2-10400, Tbilisi, 1977), p. A6-3.  
G.A. Leksin, *Phys. At. Nucl.* **65**, 1985 (2002).
- [8] A.M. Polyakov, *Zh. Eksp. Theor. Fiz.* **59**, 542 (1970).  
A.M. Polyakov, *Zh. Eksp. Theor. Fiz.* **60**, 1572 (1971).  
Z. Koba, H.B. Nielsen and P. Olesen, *Nucl. Phys. B* **40**, 317 (1972).
- [9] V.A. Matveev, R.M. Muradyan and A.N. Tavkhelidze, *Part. Nucl.* **2**, 7 (1971).  
V.A. Matveev, R.M. Muradyan and A.N. Tavkhelidze, *Lett. Nuovo Cim.* **5**, 907 (1972).  
V.A. Matveev, R.M. Muradyan and A.N. Tavkhelidze, *Lett. Nuovo Cim.* **7**, 719 (1973).
- [10] S. Brodsky and G. Farrar, *Phys. Rev. Lett.* **31**, 1153 (1973).  
S. Brodsky and G. Farrar, *Phys. Rev. D* **11**, 1309 (1975).
- [11] J.M. Campbell, J.W. Huston and W.J. Stirling, *Rep. Prog. Phys.* **70**, 89 (2007).
- [12] S. D. Ellis *et al.*, *Prog. Part. Nucl. Phys.* **60**, 484 (2008).
- [13] R.J. Teuscher, "High pT Jet Physics", XXXI Physics in Collision, Vancouver, Canada, August 28 - September 1, 2011; arXiv:1111.2484.
- [14] A. Bhatti and D. Lincoln, *Ann. Rev. Nucl. Part. Sci.* **60**, 267 (2010); arXiv:1002.1708.
- [15] ATLAS Collab. (F. Muller), "Jet Production Measurement with the ATLAS Detector" Physics at the LHC, Perugia, Italy, June 5-11, 2011; <http://www.pg.infn.it/plhc2011/>.
- [16] ATLAS Collab. (M. Plamondon), "Recent hard QCD results from ATLAS", Physics at the LHC, Perugia, Italy, June 5-11, 2011; <http://www.pg.infn.it/plhc2011/>.
- [17] K. Kousouris, "Hard QCD Results from CMS" Physics at the LHC, Perugia, Italy, June 5 - 11, 2011; <http://www.pg.infn.it/plhc2011/>.
- [18] CDF Collab. (A. Abulencia *et al.*), *Phys. Rev. D* **75**, 092006 (2007).
- [19] DØ Collab. (V.M. Abazov *et al.*), *Phys. Rev. Lett.* **101**, 062001 (2008).
- [20] STAR Collab. (B. Surrow), XVIII International Workshop on Deep-Inelastic Scattering and Related Subject (DIS 2010), Firenze, Italy, April 19-23, 2010; PoS (DIS 2010) 249.  
STAR Collab. (M. Calderon), Extreme QCD 2011, San Carlos, Mexico, July 18-20, 2011; <http://www.nucleares.unam.mx/XQCD11/>.

- [21] CMS Collab. (S. Chatrchyan *et al.*), *Phys. Rev. Lett.* **107**, 132001 (2011).
- [22] ATLAS Collab. (G. Aad *et al.*), *Phys. Rev. D* **86**, 014022 (2012).
- [23] CMS Collab. "Measurement of Differential Jet Cross Sections at  $\sqrt{s} = 7$  TeV with the CMS Detector", CMS-PAS-QCD-11-004.
- [24] ALICE Collab. (R.Ma), "Hard Probes 2012", Gagliari, Italy, May 27 - June 1, 2012; <http://www.ca.infn.it/hp12/>.
- [25] M.V. Tokarev and T.G. Dedovich, *Int. J. Mod. Phys. A* **15**, 3495 (2000).
- [26] M.V. Tokarev, T.G. Dedovich and I.Zborovský *Int. J. Mod. Phys. A* **27**, 1250115 (2012).
- [27] I. Zborovský and M. Tokarev, *Phys. Rev. D* **75**, 094008 (2007).
- [28] DØ Collab. (B. Abbott *et al.*), *Phys. Rev. Lett.* **82**, 2451 (1999).  
DØ Collab. (B. Abbott *et al.*), *Phys. Rev. D* **64**, 032003 (2001).  
D. Elvira, Ph.D Thesis Universidad de Buenos Aires, Argentina (1995).  
DØ Collab. (V.M. Abazov *et al.*), *Phys. Lett. B* **525**, 211 (2002).
- [29] DØ Collab. (M. Voutilainen), XIV International Workshop on Deep-Inelastic Scattering and Related Subjects (DIS2006), Tsukuba, Japan, April 20-24, 2006; <http://www-conf.kek.jp/dis06/>  
DØ Collab. (J. Cammin), XV International Workshop on Deep-Inelastic Scattering and Related Subjects (DIS2007), Munich, Germany, April 16-20, 2007; <http://www.mppmu.mpg.de/dis2007/>
- [30] CDF Collab. (A.A. Bhatti), Divisional Meeting of the Division of Particles and Fields of the APS (DPF96), August 10-15, 1996, Minneapolis, USA; preprint Fermilab-Conf-96/352-E.  
CDF Collab. (J. Lamoureux), *Proceedings of the 16th Int. Conf. on Physics in Collision (PIC96)*, Mexico City, Mexico, June 19-21, 1996; preprint Fermilab-Conf-97/017-E.
- [31] CDF Collab. (F. Abe *et al.*), *Phys. Rev. Lett.* **77**, 438 (1996).  
CDF Collab. (T. Affolder *et al.*), *Phys. Rev. D* **64**, 032001 (2001).
- [32] CDF Collab. (A. Abulencia *et al.*), *Phys. Rev. Lett.* **96**, 122001 (2006).
- [33] G. C. Blazey *et al.*, arXiv:hep-ex/0005012.  
CDF Collab. (T. Aaltonen *et al.*), *Phys. Rev. D* **78**, 052006 (2008).  
CDF Collab. (A. Abulencia *et al.*), *Phys. Rev. D* **74**, 071103 (2006).
- [34] G.P. Salam, *Acta Physica Polonica, Proc. Suppl.* **1**, 455 (2008).  
S.D. Ellis, LHC Physics Center at Fermilab, "Single jets and jet substructure", November 18, 2010; <http://lpc.fnal.gov/programs/topic/archive.shtml>  
G. Soyez, Initial Conditions in Heavy-Ion Collisions, "Jet finding at the LHC era", Goa, India, September 1-19, 2008; <http://theory.tifr.res.in/qcdinit/program.html>  
M. Cacciari, G.P. Salam and G. Soyez, *JHEP* **0804**, 063 (2008).  
G.P. Salam and G. Soyez, *JHEP* **0705**, 086 (2007).  
S. Catani *et al.*, *Nucl. Phys. B* **406**, 187 (1993).  
S. D. Ellis and D. E. Soper, *Phys. Rev. D* **48**, 3160 (1993).  
Y. L. Dokshitzer *et al.*, *JHEP* **9708**, 001 (1997).
- [35] E557 Collab. (C. Stewart *et al.*), *Phys. Rev. D* **42**, 1385 (1990).
- [36] AFS Collab. (T. Akesson *et al.*), *Phys. Lett. B* **118**, 185 (1982).  
AFS Collab. (T. Akesson *et al.*), *Phys. Lett. B* **118**, 193 (1982).  
AFS Collab. (T. Akesson *et al.*), *Phys. Lett. B* **123**, 133 (1983).

- [37] UA1 Collab. (G. Arnison *et al.*), *Phys. Lett. B* **172**, 461 (1986).  
UA1 Collab. (C. Albajar *et al.*), *Nucl. Phys. B* **309**, 405 (1988).
- [38] G. C. Blazey and B. L. Flaugher, *Ann. Rev. Nucl. Part. Sci.* **49**, 633 (1999).  
DØ Collab. (V. Abazov *et al.*), *Phys. Rev. D* **85**, 052006 (2012).
- [39] M.V. Tokarev and T.G. Dedovich, *Proceedings of the XIX International Baldin Seminar on High Energy Physics Problems "Relativistic Nuclear Physics and Quantum Chromodynamics"*, Dubna, Russia, September 29 - October 4, 2008, edited by A.N.Sissakian, V.V.Burov, A.I.Malakhov, S.G.Bondarenko and E.B.Plekhanov, JINR, Vol.2, pp.187-197, 2008.
- [40] S.D. Ellis, Z. Kunszt and D. Soper, *Phys. Rev. Lett.* **69**, 1496 (1992);  
<http://zebu.uoregon.edu/~soper/soper.html>
- [41] S.D. Ellis, Z. Kunszt and D. Soper, *Phys. Rev. D* **40**, 2188 (1989).
- [42] Z. Kunszt and D. Soper, *Phys. Rev. D* **46**, 192 (1992).
- [43] J. Pumplin *et al.*, *JHEP* **0207**, 012 (2002); hep-ph/0201195;  
<http://www.phys.psu.edu/~cteq>
- [44] A.D. Martin *et al.*, *Eur. Phys. J. C* **23**, 73 (2002); hep-ph/0110215;  
<http://www.durpdg.dur.ac.uk/hepdata/mrs>