PROCEEDINGS OF SCIENCE

Self-similarity of high-$\mathrm{p_T}$ hadron production in pA collisions

A. Aparin*
Joint Institute for Nuclear Research, Dubna, Russia
E-mail: aparin@jinr.ru

M. Tokarev
Joint Institute for Nuclear Research, Dubna, Russia
E-mail: tokarev@jinr.ru

New data on inclusive transverse momentum spectra of charged hadrons produced in pA collisions at the U70 are analyzed in the framework of z-scaling approach. Scaling function $\psi(z)$ is expressed via invariant cross section $Ed^3\sigma/dp^3$ and average multiplicity density $dN_{ch}/d\eta(\sqrt{s},\eta)$ of charged particles. Results of the analysis are compared with the data obtained by J. Cronin, R. Sulyaev, G. Leksin and D. Jaffe groups. The concept of z-scaling is discussed. The A-dependence of scaling function $\psi(z)$ at high z is verified. Self-similarity of the hadron production in pA collisions over a wide kinematical range is confirmed.

XXI International Baldin Seminar on High Energy Physics Problems,
September 10-15, 2012
JINR, Dubna, Russia

*Speaker.

PoS(Baldin ISHEPP XXI)067

1. Introduction

Search for clear signatures of the phase transtion of the nuclear matter in collisions of hadrons and nuclei is the main goal of the heavy ion experimental programms at the Relativistic Heavy Ion Collider at BNL [1], Super Proton Synchrotron [2], and Large Hadron Collider at CERN [3]. The hypothesis of self-similarity of the hadron production is an important concept for data analysis in searching for new physics. The hypothesis is related with established scaling laws such as the Bjorken scaling - in the deep inelatic scattering, the Feynman scaling - in the inclusive hadron production, the P-KNO scaling - in the multiparticle production, and others. Among the others there are the quark counting rules describing the power asymptotics of the electromagnetic form-factors of hadrons and cross sections of exclusive processes. [4]-[13]. The phase transitions in the nuclear matter produced in the heavy ion collisions at the high energy density and temperature are new phenomena related to collective interactions of quarks and gluons near the phase boundaries and the critical point.

It is well known that the general concepts in the critical phenomena are related with the notions of "scaling" and "universality" [14]. Scaling means that the system near the critical point exhibiting self-similar properties is invariant under transformation of the scale. According to universality, quite different systems behave in a remarkably similar way near the respective critical point.

It is assumed that transition of the nuclear matter from hadron to quark and gluon degrees of freedom near the critical point should reveal large fluctuations, correlations and discontinuty of some experimental quantities characterizing the system.

In the present paper we use the established features of z-scaling [15, 16, 17] to analise the data on inclusive spectra of hadrons produced in pA collisions in the kinematical region known as the cumulative one [8, 9, 10] (see, also [18] and references therein) in revealing signatures of new physics. The cumulative region is defined as a region forbidden for kinematics for free nucleons. It can be reached both in the fixed target and collider experiments but not at very high collision energy. We assumed (see also [23, 24]) that selection of events on centrality in the cumulative region could help to localize a position of the critical point. Changes of the parameters of the theory of z-scaling near the critical point considered as a signature of new physics, were also discussed in [25]. Relevance of the power asymptotics of $\psi(z)$ at high z was noted to anisotropy of momentum space due to spontaneous symmetry breaking, violation of the Lorenz and discrete (C,P,T) symmetries [23].

The validity of z-scaling was confirmed in the region far from the boundary of phase transitions or the region where the critical point (CP) can be located (see, for example, [27, 26, 28]. Nevertheless, it is assumed [23, 24] that the z-scaling approach can be also a suitable tool to search for phase transitions and the critical point in the hadron and nuclear matter. Although the scaling does not give us direct information on existence of the phase transition or the critical point, discontinuty of its parameters could indicate the vicinity of critical phenomena. Therefore violation of z-scaling is assumed to be a signature of new phenomena beyond the scaling theory known up to now.

In the report the main ideas and properties of z-scaling are briefly reminded in Section 1. We emphasize that these properties are treated as manifestation of the self-similarity of the structure of colliding objects (hadrons, nuclei), interactions of their constituents, and the hadronization process.

Some examples of self-similarity of the hadron production in pp and pD collisions at high p_T in the central range $\theta_{cms} \simeq 90^0$ are given in Section 2. Section 3 compares these data with inclusive spectra of the hadrons produced in pA collisions in the cumulative and low-p_T region. Results of our analysis of the new data on charged hadrons produced in pA collisions in the cumulative high-p_T region are discussed in Section 4. The main conclusions are summarized in Section 5.

2. z-Scaling

In this section, we would like to remind the basic ideas of z-scaling dealing with the investigation of the inclusive process. We follow the approach developed in [17]. The main idea of z-scaling is based on the assumptions that gross feature of the inclusive particle distribution of process $P_1 + P_2 \to p + X$ at high energies can be described in terms of the corresponding kinematic characteristics of the constituent subprocess written in the symbolic form:

$$(x_1M_1) + (x_2M_2) \to m_1 + (x_1M_1 + x_2M_2 + m_2) \tag{2.1}$$

satisfying the following condition:

$$(x_1P_1 + x_2P_2 - p)^2 = (x_1M_1 + x_2M_2 + m_2)^2. \tag{2.2}$$

The equation is the expression of locality of the hadron interaction at a constituent level. The x_1 and x_2 are the fractions of the incoming momenta P_1 and P_2 of the colliding objects with the masses M_1 and M_2. They determine the minimum energy necessary to produce the secondary particle with the mass m_1 and the four-momentum p. The parameter m_2 is introduced to satisfy the internal conservation laws (for charge, baryon, isospin, strangeness numbers, and so on).

Equation (2.2) reflects the minimum recoil mass hypothesis in the elementary subprocess. To connect kinematic and structural characteristics of the interaction, quantity Ω is introduced. It is chosen in the following form:

$$\Omega(x_1,x_2) = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}. \tag{2.3}$$

Here δ_1 and δ_2 are the fractal dimensions of the colliding objects. The fractions x_1 and x_2 are determined to maximize the value of $\Omega(x_1,x_2)$, simultaneously fulfilling the condition (2.2):

$$d\Omega(x_1,x_2)/dx_1|_{x_2=x_2(x_1)} = 0. \tag{2.4}$$

The fractions x_1 and x_2 are equal to unity along the phase space limit and cover the full phase space accessible at any energy.

Self-similarity is a scale-invariant property related with dropping of certain dimensional quantities out of the physical picture of the interactions. It means that dimensionless quantities to describe the physical processes are used. The scaling function $\psi(z)$ depends in a self-similar manner on the single dimensionless variable z. The function is expressed via the measurable quantities and written in the following form:

$$\psi(z) = -\frac{\pi s}{(dN/d\eta)\sigma_{in}} J^{-1} E \frac{d^3\sigma}{dp^3}. \tag{2.5}$$

Here, $Ed^3\sigma/dp^3$ is the invariant cross section, $dN/d\eta(s,\eta)$ is the multiplicity density as a function of the center-of-mass collision energy squared s and pseudorapidity η, σ_{in} is the inelastic cross section, J is the corresponding Jacobian. Factor J is the known function of the kinematic variables, the momenta and masses of the colliding and produced particles.

The function $\psi(z)$ is normalized as follows:

$$\int_0^\infty \psi(z)dz = 1. \tag{2.6}$$

The relation allows us to interpret the function as a probability density to produce a particle with the corresponding value of variable z. We note that the existence of the function $\psi(z)$ depending on a single dimensionless variable z and revealing scaling properties (independence of $\psi(z)$ on collision energy $\sqrt{s}$, an angle of produced particle) is not evident in advance. The validity of the scaling is confirmed a posteriori.

Self-similarity of an object revealing over a wide scale range is the general property of fractality. It means that the measure corresponding to the object diverges in terms of the resolution. In our case this measure is variable z which has the following form

$$z = z_0\Omega^{-1}. \tag{2.7}$$

Here, $z_0 = \sqrt{\hat{s}_\perp}/m(dN/d\eta)$ is the finite part of z. It is expressed via the ratio of the transverse energy $\sqrt{\hat{s}_\perp}$ released in the binary collision of constituents and the average multiplicity density $dN/d\eta$ at $\eta = 0$ and the nucleon mass m. The divergent part Ω^{-1} describes the resolution at which the collision of the constituents can be singled out of this process. The $\Omega(x_1,x_2)$ represents a relative number of all initial configurations containing the constituents which carry fractions x_1 and x_2 of the incoming momenta. The δ_1 and δ_2 are fractal dimensions of the colliding objects. The momentum fractions x_1 and x_2 are determined to minimize the resolution $\Omega^{-1}(x_1,x_2)$ of the measure z with respect to all possible sub-processes (2.1) under the condition (2.2).

Note, that ψ and z are the scale dependent quantities. The both ones depend on the dimensional variables $\sqrt{s}$ and p_T. We assume that the hadron and nucleus interactions at high energies and transverse momenta are the interactions of fractals. In this region the internal structure of hadrons, interactions of their constituents and mechanism of hadronization manifest self-similarity.

3. Self-similarity in pp and pA collisions

The measurements of high-p_T identified hadron spectra in pA collisions have been performed at the FNAL (Batavia) [30, 31] and IHEP (Protvino) [32]. The invariant inclusive cross sections for the production of $\pi^\pm, K^\pm, \bar{p}$ hadrons in the collisions of the 200, 300 and 400 GeV protons with the H, D, Be, Ti and W targets, cover the transverse momentum range of $0.77 < p_T < 6.91$ GeV/c. The measurements have been performed at the laboratory angle of 77 mrad, which corresponds to angles near 90^0 in the center-of-mass of the incident proton and a single nucleon at rest. The data presented in [31] correspond to the cross sections for $\pi^\pm, K^\pm, p^\pm$ hadron production in pp and pD collisions at $p_{lab} = 400$ and 800 GeV/c. The produced particles were registered over the transverse momentum range of $p_T = 4.0 - 10.0$ GeV/c and at $\theta_{cms} \simeq 90^0$. The data [32] were obtained at the accelerator U70 using the proton beam with an incident momentum of 70 GeV/c

PoS(Baldin ISHEPP XXI)067

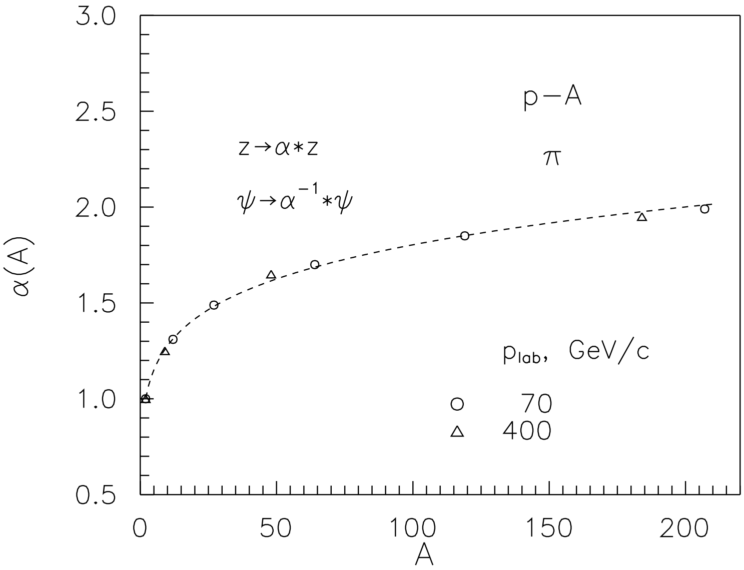


Figure 1: *A*-dependence of α - the scale transformation parameter.

and the nuclear targets of D,C,Al, Cu,Sn, and Pb. The produced hadrons were registered in the transverse momentum range $p_T = 0.99-4.65$ GeV/c and at angle θ_{cms} of 90^0.

Below we describe the procedure of data analysis used in [17]. The function ψ was calculated for every nucleus using Eq.(2.5) with normalization factor $\sigma^{pA}_{inel}/\sigma^{pp}_{inel}$ instead of σ_{inel}. The factor σ^{pA}_{in} is the total inelastic cross section for pA interactions [19]. To construct the function $\psi(z)$ for the particle production in pA interactions, it is necessary to know the values of the average multiplicity density $\rho_A(s,\eta) \equiv dN/d\eta(s,\eta)$ of secondaries produced in pA collisions. The relevant multiplicity densities of charged particles were obtained by the Monte Carlo generator HIJING [20, 21] for different nuclei ($A = 27-197$) and parametrized by the following formula

$$\rho_A(s) \simeq 0.67 \cdot A^{0.18} \cdot s^{0.105}, \quad A \geq 2. \tag{3.1}$$

The scaling functions for different nuclei obtained in this way reveal the energy and angular independence. The symmetry transformation

$$z \to \alpha(A) \cdot z, \quad \psi \to \alpha^{-1}(A) \cdot \psi \tag{3.2}$$

was used to compare the functions ψ found for different nuclei. The A-dependence of the scale parameter α is shown in Fig. 1. Symbols ($\circ, \triangle$) correspond to the values of α for which different functions coincide with each other. This is a calibration curve.

3.1 Hadron production in pp & pD collisions at $\theta_{cms} \simeq 90^0$

Figures 2(a) and 3(a) show the dependence of inclusive invariant cross sections of π^+ mesons produced in pp and pD collsions on the transverse momentum p_T at $p_{lab} = 70, 200, 300$, 400, 800 GeV/c and angle θ_{cms} near to 90^0 [30, 31, 32]. Note that the data cover a wide transverse momentum range, $p_T = 1-10$ GeV/c. As seen from Figs. 2(a) and 3(a) the hadron spectra demonstrate the power behavior and strong energy dependence which increases with the transverse momentum. Figures 2(b) and 3(b) show z-presentation of the same data sets. Taking into account the experimental errors we see that the scaling function $\psi(z)$ demonstrates the energy independence over a wide energy and transverse momentum range at $\theta_{cms} \simeq 90^0$.

3.2 Hadron production in pA collisions at $\theta_{cms} \simeq 90^0$

Figure 4(a) shows transverse momentum spectra for π^+ mesons produced in pA collisions at the incident proton momentum p_{lab} of 70, 400 GeV/c and the angle θ_{cms} of 90^0. The data [31] extend the transverse momentum range up to $p_T = 8.5$ GeV/c. A good compatibility of the [30] and [31] data sets in the overlapping region was observed. The values of the cross section of the same nucleus at $p_{lab} = 70$ and 400 GeV/c differ more than by two orders at $p_T > 4$ GeV/c. The solid and dashed lines are shown for visibility. They have been obtained by fitting the data for W, Pb and D nuclei, respectively. They demonstrate the strong dependence of p_T-spectra on collision energy $\sqrt{s}$. The same data in z-presentation are shown in Fig. 4(b). The obtained results is the evidence of z-scaling of high-p_T hadron production in pA collisions. We would like to emphasize that universality of the shape of the scaling function for different nuclei means that the mechanism of particle formation in the nuclear matter reveals a property of self-similarity, i.e. an elementary sub-process of constituent interactions is modified by nuclear matter in a self-similar manner.

Note that the results presented in Fig. 4(b) have confirmed the validity of z-scaling for the lightest (D) and heaviest (Pb) nuclei. The property of the function $\psi(z)$ allows us to conclude that the mechanism of hadron formation demonstrates features depending on global properties of the nuclear matter. The A-dependence of z-scaling is described by function $\alpha = \alpha(A)$. This function is found to be independent of kinematical variables p_{lab}, p_T, and $\sqrt{s}$.

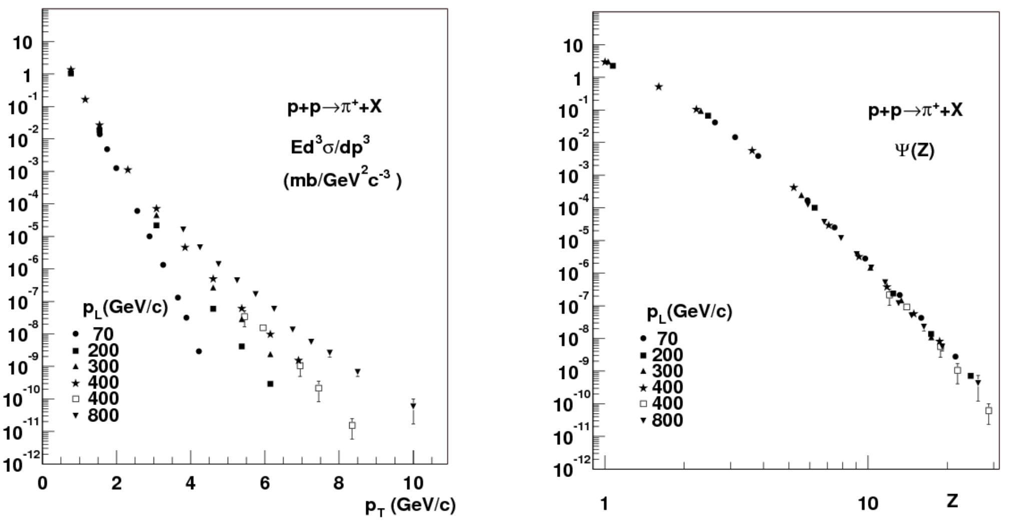
3.3 Cumulative hadron production in pA collisions at FNAL

Cumulative particles are the particles produced in the kinematical region forbidden for free nucleon-nucleon interactions [8, 9, 10] (see, also [18]). Such particles can be only produced in the processes with participation of nuclei. Production of such particles is not in disagreement with various conservation laws.

The data on inclusive invariant cross sections for $\pi^{\pm}, K^{\pm}, p^{\pm}$ hadrons produced in a backward hemisphere in pA collisions at $p_{lab} = 400$ GeV/c and at the angle θ_{lab} of $70^0, 90^0, 118^0$ and 160^0 were presented in [33]. The measurements were performed over the momentum range $0.2 < p < 1.25$ GeV/c. Nuclear targets, Li, Be, C, Al, Cu and Ta were used. These data cover, in particular, the kinematical range forbidden for particle production in nucleon-nucleon collisions.

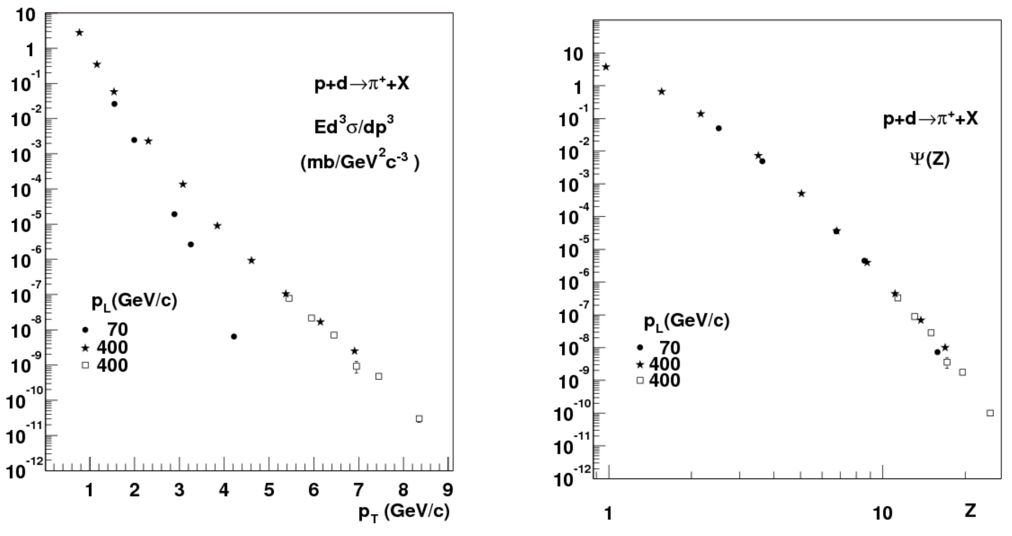
Figures 5(a), 6(a) and 7(a) show inclusive cross sections for the π^+ mesons produced in a backward hemisphere in pA collisions at $p_{lab} = 400$ GeV/c and the angle θ_{lab} of $70^0 - 160^0$. As it is seen from Figures 5(a), 6(a) and 7(a) the strong dependence of the cross section on angle θ_{lab} is observed for all nuclear targets, Li, Be, C, Al, Cu, Ta. The difference of cross sections at $\theta_{lab} = 90^0$ and 160^0 at momentum $p = 1 - 1.2$ GeV/c reaches 2-3 orders of the magnitude.

We assume that the shape of the scaling curve will be the same as for data points corresponding to the high-p_T region and $\theta_{cms} \simeq 90^o$. This hypothesis corresponds to validity of self-similarity of the hadron production over a wider kinematical region. There are no experimental data on the



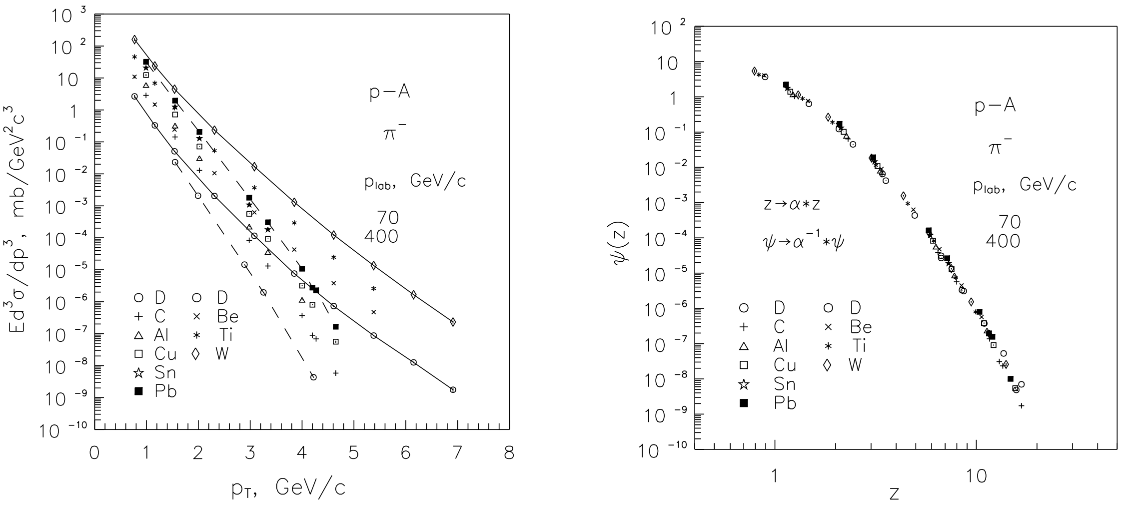
(a) (b)

Figure 2: (a) The inclusive differential cross section for π^+ mesons produced in pp collisions at $p_{lab} = 70, 200, 300, 400,$ 800 GeV/c and $\theta_{cm} \simeq 90^0$ as a function of transverse momentum p_T. The experimental data are taken from [30, 31, 32]. (b) The corresponding scaling function $\psi(z)$.



(a) (b)

Figure 3: (a) The inclusive differential cross section for π^+ mesons produced in pD collisions at $p_{lab} = 70$, 400 GeV/c and $\theta_{cm} \simeq 90^0$ as a function of transverse momentum p_T. The experimental data are taken from [30, 31, 32]. (b) The corresponding scaling function $\psi(z)$.



(a) (b)

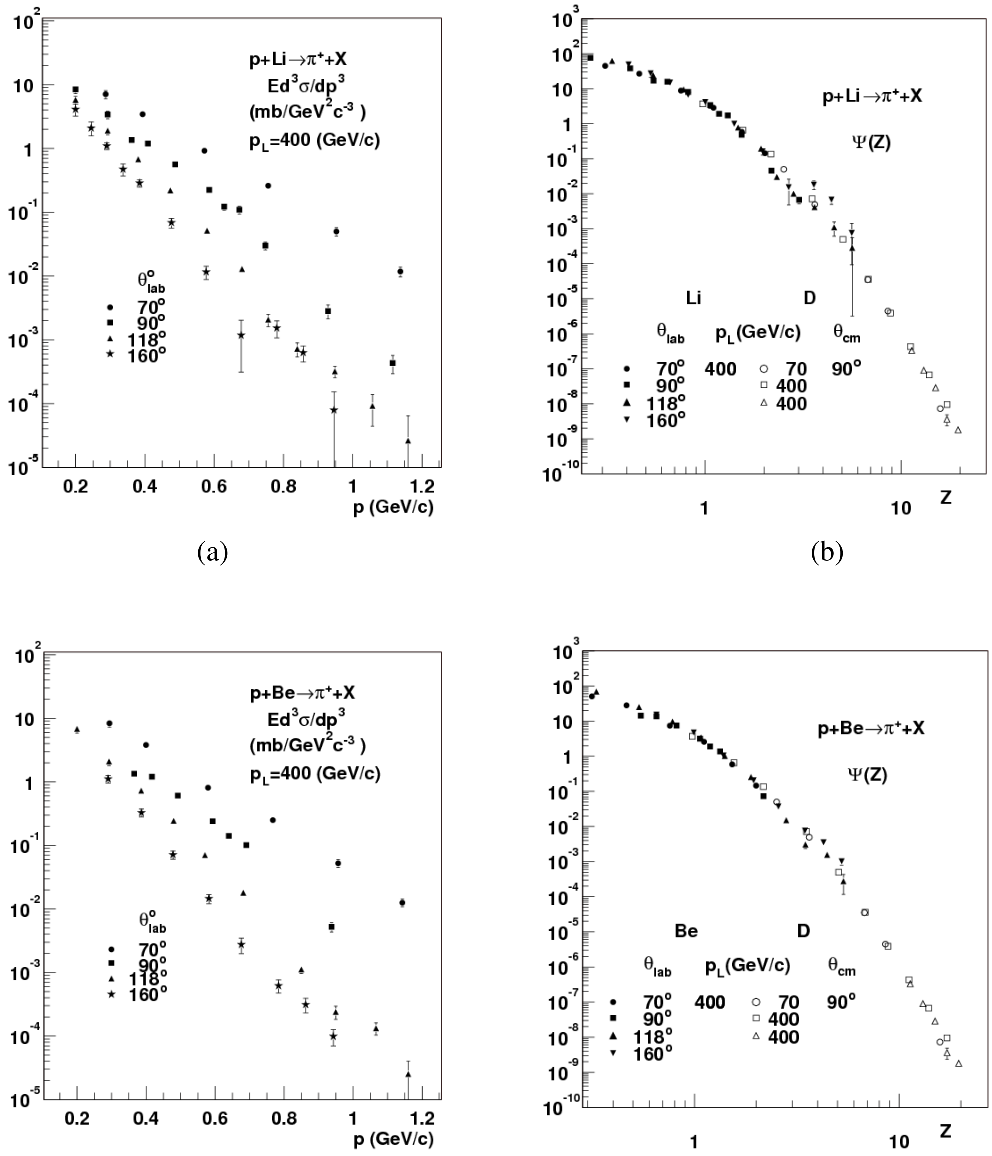
Figure 4: (a) The inclusive differential cross section for π^- mesons produced in pA collisions at $p_{lab} = 70$, 400 GeV/c and $\theta_{cms} \simeq 90^0$ as a function of transverse momentum p_T. The solid and dashed lines are obtained by fitting of the data for W, Pb and D, respectively. The experimental data are taken from [30, 31, 32]. (b) The corresponding scaling function $\psi(z)$.

angular dependence of $\rho(s,\eta,A)$ for particles produced in a backward hemisphere in order to obtain a normalization factor for the scaling function. Therefore we verify the possibility to restore the shape of the $\psi(z)$ found from the analysis of high-p_T data sets [32, 30, 31] using low-p_T data points [33]. The function $\rho(s,\eta,A)$ is parameterized in the form of $\rho(s,\eta,A) = \rho(s,A)|_{\eta=0} \cdot \chi(\theta,A)$, where the angular dependence is described by $\chi(\theta,A)$. It was found that $\chi_{Ta} = 0.75$ and 0.3 at $\theta_{lab} = 70^0$ and 160^0, respectively. The ratio χ_{Ta}/χ_{Li} decreases from 3.5 to 1.5 as the angle θ_{lab} increases from 70^0 to 160^0. The A-dependence of the ratio demonstrates the saturation reached for the backward particle production.

Figures 5(b), 6(b) and 7(b) show z-presentation of the same data [33]. The data [30, 31] and [32] for the deuteron target are given for comparison. One can see that curves found for all nuclei ($Li - Ta$) are in agreement with data z-presentation for D nucleus at $p_{lab} = 70,400$ GeV and $\theta_{cms} = 90^0$. The angular dependence of $\chi(\theta,A)$ describes both non-cumulative and cumulative data points. Taking into account experimental errors we can conclude that the shape of $\psi(z)$ can be restored using the angular dependence of the multiplicity density. We would like to note that all the data points [33] are out of the asymptotic region of $\psi(z)$. Nevertheless the low and high-p_T data overlap each other. They hsve demonstrated deviation from the power law $\psi(z) \sim z^\beta$ at $z < 4$. Verification of the law in cumulative processes at $z > 4$ is of interest to search for the signatures of phase transition (see [23] and references therein).

3.4 High-p_T cumulative hadron production in pA collisions at the U70

It is expected that the hadron production in cumulative region could give more clear information of properties on the compressed nuclear matter than in the processes without such selection of events. Note that it is practically impossible to reach this region in processes at high collision



(a) (b) (c) (d)

Figure 5: (a) The inclusive differential cross section for π^+-mesons produced in pLi and pBe collisions at $p_{lab} = 400$ GeV/c and $\theta_{lab} = 70^0, 90^0, 118^0$ and 160^0 as a function of the transverse momentum p_T. (b) The corresponding scaling function $\psi(z)$. The experimental data are taken from [30, 31, 32] and [33].

energy due to small cross sections near a free nucleon-nucleon kinematical boundary. Out of the region cross sections are much smaller. Therefore to investigate the nuclear matter in a cumulative region is more preferable by using light nuclear targets (He, Li, Be, C).

The cross sections of high-p_T cumulative charged hadrons produced in pA collisions at $p_{lab} =$ 50 GeV/c and $\theta_{lab} = 35^0$ were measured at the U70 [34]. The kinematical boundaries for π^- mesons produced in pA collisions on $\{p_T, \eta\}$ plane are shown in Fig. 8 by solid lines. Note that maximal values of the pion momentum reached in pp collisions at $\theta_{lab} = 9.2^0$ and 35^0 (the dashed

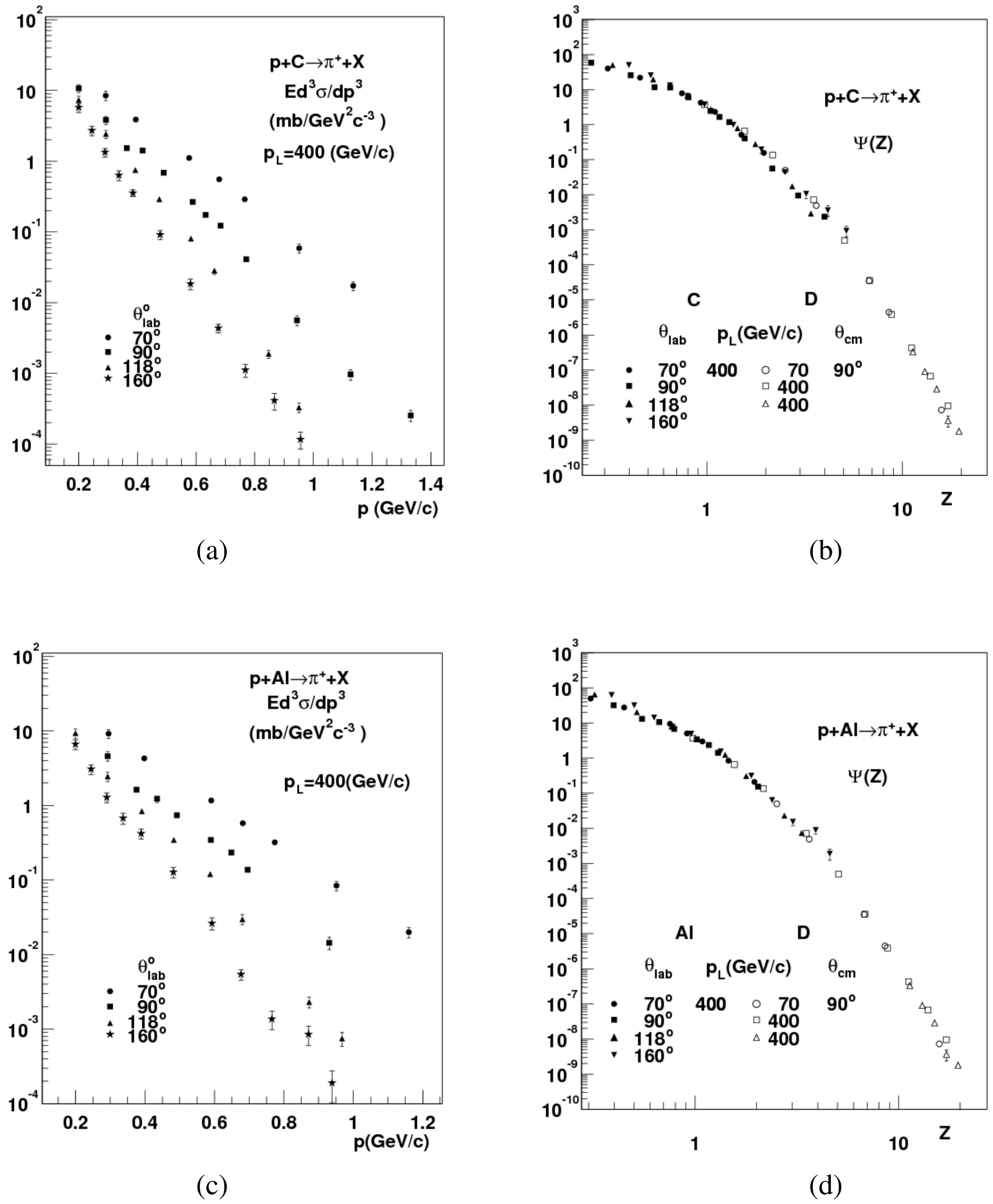
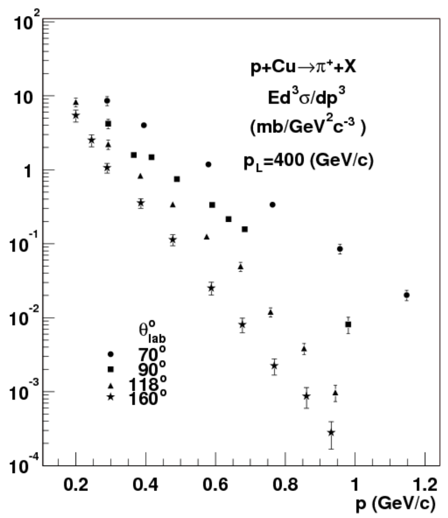


Figure 6: (a) The inclusive differential cross section for π^+-mesons produced in pC and pAl collisions at $p_{lab} = 400$ GeV/c and $\theta_{lab} = 70^0, 90^0, 118^0$ and 160^0 as a function of the transverse momentum p_T. (b) The corresponding scaling function $\psi(z)$. The experimental data are taken from [30, 31, 32] and [33].

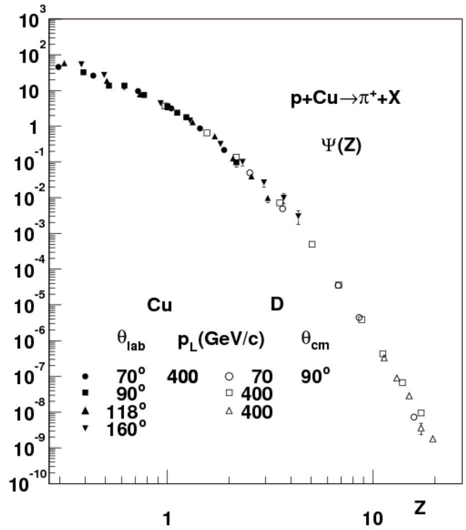
lines) correspond to 5.7 and 2.5 GeV/c, respectively.

In the framework of a microscopic scenario of constituent interactions developed in the theory of z-scaling the cumulative production of particles in nuclear collisions is determined by the conditions $x_1 < 1$ and $x_2 A_2 > 1$. The momentum fractions are the solution of the system of equations (2.2) and (2.4).

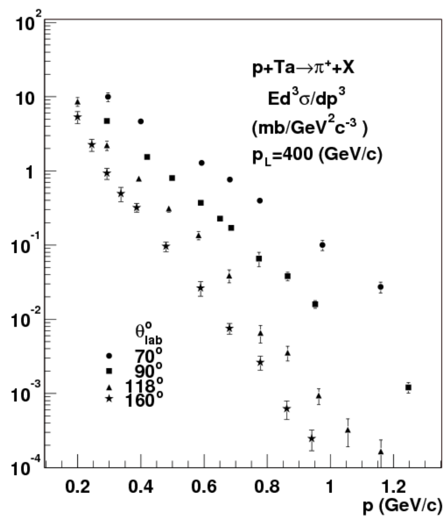
Figure 9 demonstrates the inclusive spectra of negative hadrons produced in pA collisions on nucleus targets (C, Al, Cu, W) [34] together with the data [32] for π^- mesons. The cross sections



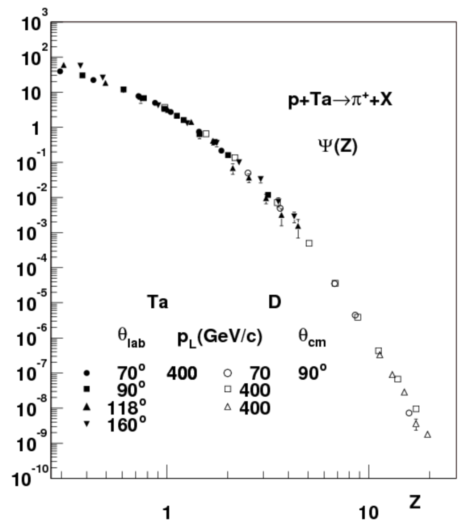
(a)



(b)



(c)



(d)

Figure 7: (a) The inclusive differential cross section for π^+-mesons produced in pCu and pTa collisions at $p_{lab} = 400$ GeV/c and $\theta_{lab} = 70^0, 90^0, 118^0$ and 160^0 as a function of the transverse momentum p_T. (b) The corresponding scaling function $\psi(z)$. The experimental data are taken from [30, 31, 32] and [33].

are measured over the transverse momentum range $p_T \simeq 0.5 - 3.7$ GeV/c. One can see from Fig. 9 that the spectra decrease by 9-10 orders of the magnitude. Note that the difference between the values of cross sections of the data [32] and [34] increases with p_T and reaches more than one order for $p_T > 3$ GeV/c.

Figure 10 shows z-presentation of the inclusive differential cross sections for h^- hadrons and π^- mesons produced in pA collisions at $p_{lab} = 50$ and 70 GeV/c and $\theta_{lab} \simeq 35^0$ and 9.2^0. One can see that the shape of $\psi(z)$ for the data [34] coincides with the other one for the data [32].

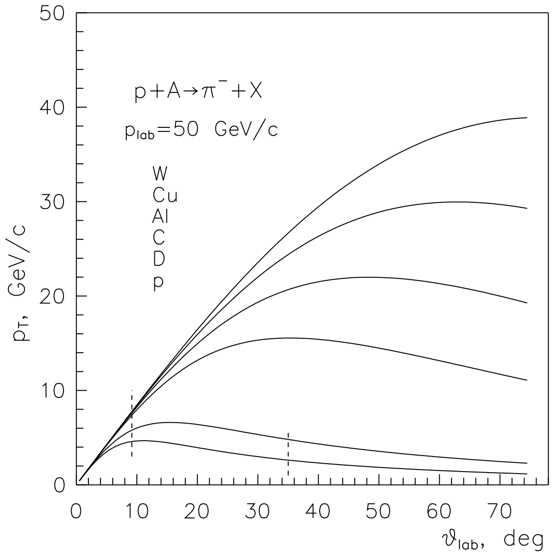


Figure 8: Kinematical boundary of π^- mesons produced in *pA* collisions at incident proton momentum $p_{lab} = 50$ GeV/c on $\{p_T, \theta_{lab}\}$ plane. The dashed lines correspond to the angles $\theta_{lab} = 9.2^0$ and 35^0.

The values of the fractal dimensions in the present analysis were used to be $\delta_1 = \delta_N = 0.5$ and $\delta_2 = A\delta_N$ as in [17]. New data reach z of 10. They clearly demonstrate a smooth transition to power behavior of $\psi(z)$ for all nuclei. Near the kinematical boundary for nucleon-nucleon interactions the discontinuity of $\psi(z)$ is not found. This is an unexpected result. It will be discussed below.

4. Discussion

Here we would like to discuss the obtained results. The interesting and unexpected observation is a smooth transition of $\psi(z)$ from the non-cumalative region to the cumulative one. It was expected to find a discontinuity of $\psi(z)$ near the point corresponding to the kinematical boundary for free nucleon-nucleon interactions. As it was noted in [23] the *A*-dependence of the fractal dimension of nucleus δ_A can significantly change in the cumulative region and violate the simple law $\delta_A = A\delta_N$. It did not happen. Let us look at the procedure of the analysis in detail. First of all, it should be noted that there is no absolute normalization of function $\psi(z)$ due to the absence of the angular dependence of the multiplicity density. However it is evident that this normalization factor can not distort the shape of $\psi(z)$ over a wide range of z. Therefore a measurement of multiplicity density $dN/d\eta$ as a function of η is important to develop the theory. Another important task is to measure the charged hadron spectra in *pA* collisions at higher p_T. Such data could allow us to verify the universality of the power law, $\psi(z) \sim z^{-\beta}$, at higher z for different nuclei and to determine the value of the slope parameter β. The *A*-dependence of the scaling function found in [17] means self-similarity of the hadron production in nuclear medium produced in *pA* collisions, i.e. elementary sub-process is modified by nuclear medium in a self-similar manner. It is reasonably to assume (see, also [23]) that signatures of the phase transition of nuclear matter and the critical point can be also manifested in cumulative processes. Moreover, the nuclear matter produced in

PoS(Baldin ISHEPP XXI)067

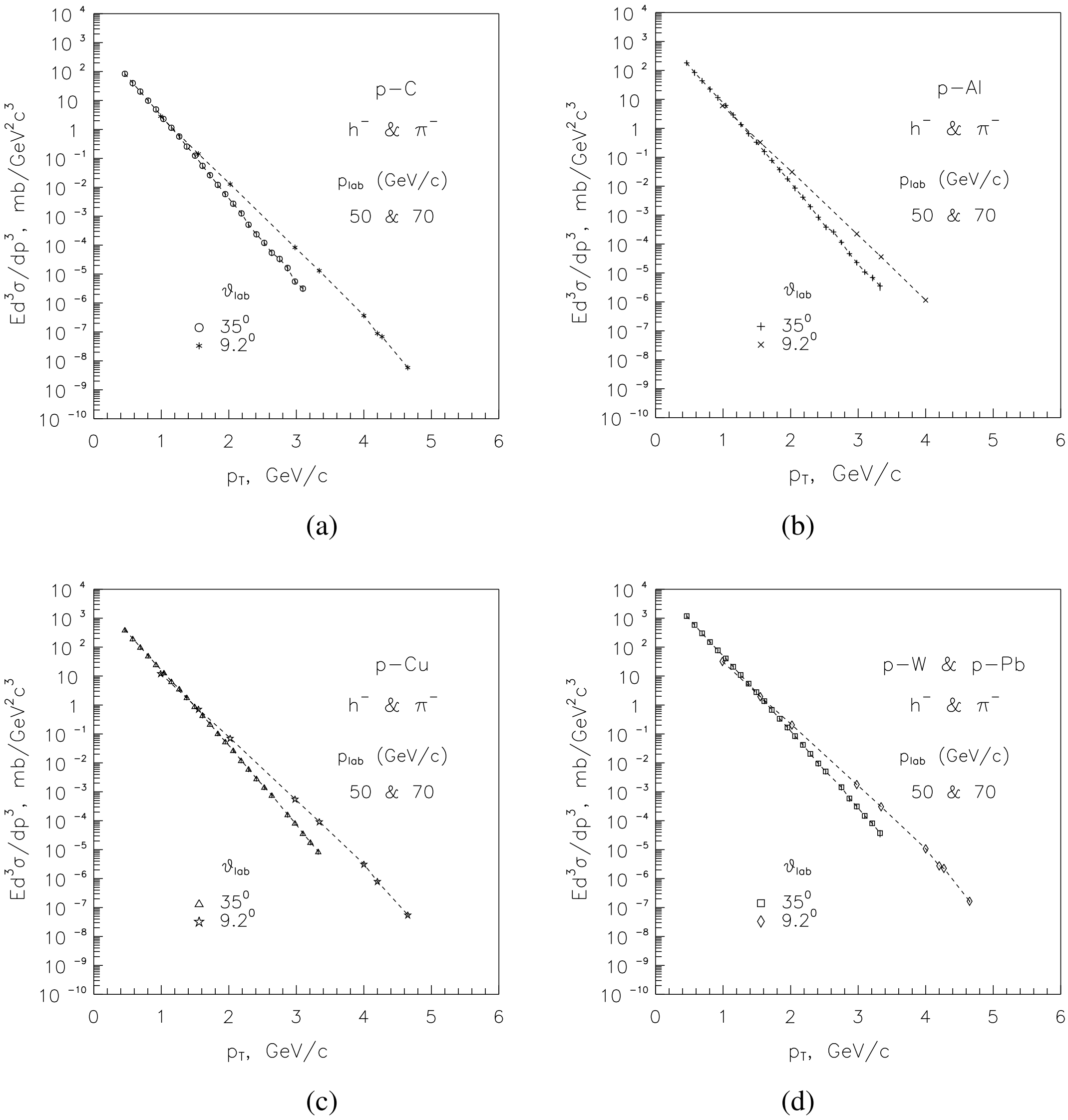


Figure 9: The inclusive differential cross sections for h^- hadrons and π^- mesons produced in pC(a), pAl (b), pCu (c) and $pW(Pb)$ (d) collisions as a function of the transverse momentum p_T at $p_{lab} = 50$ and 70 GeV/c and $\theta_{lab} \simeq 35^0$ and 9.2^0, respectively. The dashed lines are shown for visibility. The experimental data are taken from [32, 34].

the cumulative region can be much higher compressed than in the non-cumulative one. It is noted in [23] that localization of the critical point would be much more effective in the processes with smaller energy losses. The large energy losses can cause "smearing out" of dicontinuty of the parameters (fractal dimensions, specific heat, ...) characterizing the system. The high-p_T cumulative production is characterized by relatively small energy losses and therefore it is suitable for this type of investigations. In this region one can investigate the structure of the fluctons (particle-like fluctuations of the nuclear matter) and fragmentation processes induced by their collisions. Besides cumulativity, selection of the processes with large multiplicity is considered to be also preferable

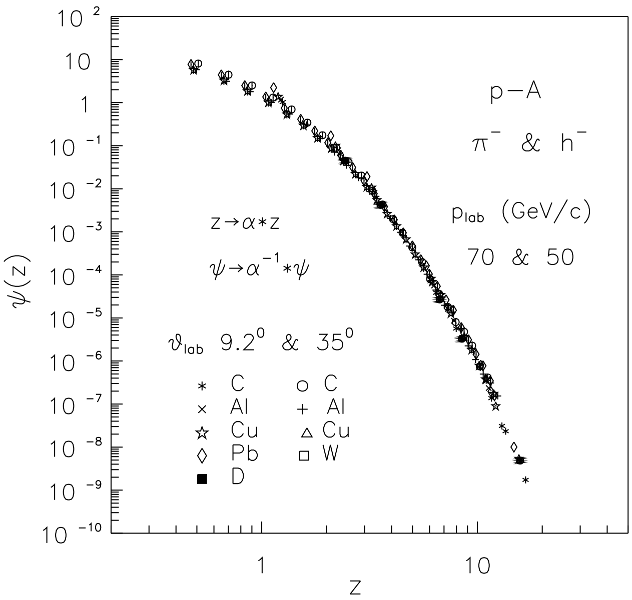


Figure 10: The z-presentation of inclusive spectra of h^- hadron and π^- meson production in pA collisions. The experimental data are taken from [32, 34].

to search for the critical point. The multiplicity density is one of the experimentally measurable quantities. It can be used to control the properties of the nuclear medium where the flucton interactions take place. The production of the compressed matter in pA collisions assunes using the special kinematic conditions ($x_2A_2 > 1$). Note that the data [34] have been obtained for the minimum bias trigger and therefore do not allow us to study the multiplicity dependence of transverse momentum spectra. Since the fractal dimension δ is sensitive to the behavior of spectra at high p_T, the $\delta - multiplicity$ correlation is expected to be stronger in the cumulative region. Study of these correlations could shed some light on the location of the critical point.

5. Conclusions

The new data on the inclusive transverse momentum spectra of the charged hadrons produced in pA collisions at the U70 have been analyzed in the framework of z-scaling approach. The theory of z-scaling for the high-p_T cumulative hadron production has been developed. The general ideas of the approach have been reviewed. It is shown that the scaling function $\psi(z)$ is expressed via the experimental observables - the invariant cross section, the average multiplicity density and total inelastic cross section. The scaling function $\psi(z)$ is interpreted as a probability density to produce a particle with a given value of z. The quantity z has a property of the fractal measure and δ_1, δ_2 are the fractal dimensions of colliding particles. It is argued that z-scaling reflects the fundamental symmetries such as locality, self-similarity, and fractality of hadron interactions at a constituent level.

The results of analysis of the U70 data [34] were compared with z-presentation of inclusive spectra for high-p_T hadron production in pA collisions at $\theta_{cms} \simeq 90^0$, and low-p_T cumulative hadrons produced in the backward hemisphere. The shape of function $\psi(z)$ for these processes has been found to be the same over a wide kinematical region. The smooth behavior both in the non-cumulative and cumulative regions of $\psi(z)$ has been observed. The discontinuity of $\psi(z)$ under transition of free nucleon-nucleon kinematical boundary has not been found. The indication of the power law of $\psi(z)$ for $z > 4$ has been seen. The search for signatures of the phase transition and critical point in the high-p_T cumulative hadron production has been suggested to be preferable to the other ones.

Acknowledgments

Some results were obtained in collaboration with Denis Toivonen and Mikhail Tokarev, one of the authors of this paper, is especially grateful to him for the joint work.

References

[1] BRAHMS Collab., I. Arsene *et al.*, *Nucl. Phys.* **A757**, 1 (2005).
PHOBOS Collab., B. B. Back *et al.*, *Nucl. Phys.* **A757**, 28 (2005).
STAR Collab., J. Adams *et al.*, *Nucl. Phys.* **A757**, 102 (2005).
PHENIX Collab., K. Adcox *et al.*, *Nucl. Phys.* **A757**, 184 (2005).
STAR Collaboration Decadal Plan, The STAR Collaboration, December, 2010; http://www.bnl.gov/npp/.
The PHENIX Experiment at RHIC, Decadal Plan 2011Ű2020, Brookhaven National Laboratory, Relativistic Heavy Ion Collider, October, 2010; http://www.bnl.gov/npp/.

[2] NA49 and NA61/SHINE Collab., M. Gazdzicki et al., *J. Phys. G: Nucl. Part. Phys.* **38** (2011) 124024.
NA61/SHINE Collab., M. Szuba, "The Old New Frontier: Studying the CERN-SPS Energy Range with NA61/SHINE". International Conference on New Frontiers in Physics, 10-16 June, 2012, Kolymbari, Crete, Greece; arXiv:1209.5936.

[3] ALICE Collab., P. Braun-Munzinger, "Status of ALICE", LHC Days in Split, October 1-6, 2012, Split, Croatia http://lhc2012.fesb.hr/
ALICE Collab., J. Stachel, "Overview of ALICE results", LHC Days in Split, LHC Days in Split, October 1-6, 2012, Split, Croatia http://lhc2012.fesb.hr/
ATLAS Collab., B.A. Cole, "High pT measurements by ATLAS in Pb+Pb Collisions at the LHC", 8th International Workshop on High pT Physics at LHC, 21-24 October 2012,Wuhan, China http://conf.ccnu.edu.cn/ hpt2012/
ATLAS Collab., A. Milov, "Heavy ion results from the ATLAS experiment", Xth Quark Confinement and the Hadron Spectrum, Munchen, Germany, 8Ű12 October 2012; http://www.confx.de/
CMS Collab., R.G. De Cassagnac, "Overview of CMS heavy ions results", LHC Days in Split, October 1-6, 2012, Split, Croatia http://lhc2012.fesb.hr/

[4] R.P. Feynman, *Phys. Rev. Lett.* **23**, 1415 (1969).

[5] J.D. Bjorken, *Phys. Rev.* **179**, 1547 (1969).
J.D. Bjorken and E.A. Paschos, *Phys. Rev.* **185**, 1975 (1969).

[6] J. Benecke *et al.*, *Phys. Rev.* **188**, 2159 (1969).

[7] P. Bosted P. *et al.*, *Phys. Rev. Lett.* **49**, 1380 (1972).

[8] A.M. Baldin, *Sov. J. Part. Nucl.* **8**, 429 (1977).

[9] V.S. Stavinsky, *Sov. J. Part. Nucl.* **10**, 949 (1979).

[10] G.A. Leksin, Report No. ITEF-147, 1976.
G.A. Leksin, *Proceedings of the XVIII Int. Conf. on High Energy Physics*, Tbilisi, Georgia, 1976, edited by N.N. Bogolubov *et al.*, (JINR Report No. D1,2-10400, Tbilisi, 1977),p. A6-3.
G.A. Leksin, *Phys. At. Nucl.* **65**, 1985 (2002).

[11] A.M. Polyakov, *Zh. Eksp. Theor. Fiz.* **59**, 542 (1970).
A.M. Polyakov, *Zh. Eksp. Theor. Fiz.* **60**, 1572 (1971).
Z. Koba, H.B. Nielsen and P. Olesen, *Nucl. Phys. B* **40**, 317 (1972).

[12] V.A. Matveev, R.M. Muradyan and A.N. Tavkhelidze, *Part. Nucl.* **2**, 7 (1971).
V.A. Matveev, R.M. Muradyan and A.N. Tavkhelidze, *Lett. Nuovo Cim.* **5**, 907 (1972).
V.A. Matveev, R.M. Muradyan and A.N. Tavkhelidze, *Lett. Nuovo Cim.* **7**, 719 (1973).

[13] S. Brodsky and G. Farrar, *Phys. Rev. Lett.* **31**, 1153 (1973).
S. Brodsky and G. Farrar, *Phys. Rev. D* **11**, 1309 (1975).

[14] H. Stanley, Introduction to Phase Transitions and Critical Phenomena, (Oxford University Press, London, 1971).

[15] I.Zborovský, Yu.A.Panebratsev, M.V.Tokarev, G.P.Škoro, *Phys. Rev.* **D54**, 5548 (1996).

[16] I. Zborovský, M.V.Tokarev, Yu.A.Panebratsev, G.P.Škoro, *Phys. Rev.* **C59**, 2227 (1999).

[17] M.V. Tokarev, I. Zborovský, Yu.A. Panebratsev, G.P. Škoro, JINR Preprint E2-99-113, Dubna, 1999; *Int. J. Mod. Phys.* **A16**, 1281 (2001).

[18] L.L. Frankfurt, M.I. Strikman, *Phys. Rep.* **160**, 235 (1988).
S.V. Boyarinov *et al.*, *Sov. J. Nucl. Phys.* **57**, 1452 (1994); ibid **46**, 1473 (1987); ibid **50**, 1605 (1989); ibid **54**, 119 (1991); ibid **56**, 125 (1993).
O.P. Gavrishchuk *et al.*, *Nucl. Phys.* **A523**, 589 (1991).
I.M. Beliaev *et al.*, *Yad. Fiz.* **56**, 135 (1993).
V.K. Bondarev, *Phys. Elem. Part. At. Nucl.* **28**, 13 (1997).
A.A. Baldin, A.M. Baldin, *Phys. Elem. Part. At. Nucl.* **29**, 577 (1998).

[19] J. Carroll *et al.*, Phys. Lett. B **80**, 319 (1979).

[20] X.N. Wang and M. Gyulassy, Phys. Rev. D **44**, 3501 (1991).

[21] X.N. Wang and M. Gyulassy, Phys. Rev. D **45**, 844 (1992).

[22] I.Zborovský, M.V.Tokarev, Yu.A.Panebratsev, G.P.Skoro, JINR Preprint E2-97-24, Dubna, 1997.

[23] M.V. Tokarev, I. Zborovský, *Phys. Part. Nucl. Lett.* **7**, 160 (2010).

[24] M.V. Tokarev, I.Zborovský, A. Kechechyan, A. Alakhverdyants, *Phys. Part. Nucl. Lett.* **8**, 533 (2011).

[25] M.V. Tokarev, I. Zborovský, *Phys. At. Nucl.* **75**, 700 (2012).

[26] M.V. Tokarev, *Phys. Part. Nucl. Lett.* **4**, 676 (2007).

[27] M.V. Tokarev, *Phys. Part. Nucl. Lett.* **3**, 7 (2006).

[28] M.V. Tokarev, O.V. Rogachevski, T.G. Dedovich, *J. Phys. G: Nucl. Part. Phys.* **26**, 1671 (2000).

[29] M.V. Tokarev, O.V. Rogachevski, T.G. Dedovich, JINR Preprint E2-2000-90, Dubna, 2000.

[30] J.W. Cronin *et al.*, *Phys. Rev.* **D11**, 3105 (1975).
D. Antreasyan *et al.*, *Phys. Rev.* **D19**, 764 (1979).

[31] D. Jaffe *et al.*, *Phys. Rev.* **D40**, 2777 (1989).

[32] V.V. Abramov *et al.*, *Pizma v ZEFT* **33**, 304 (1981).
V.V. Abramov *et al.*, *Sov. J. Nucl. Phys.* **41**, 357 (1985).

[33] N.A. Nikiforov *et al.*, *Phys. Rev.* **C22**, 700 (1980).

[34] N.N. Antonov et al., "Physics of Fundamental Interactions", Russian Academy of Science, ITEP, Moscow, Russia, November 21 - 25, 2011.