PROCEEDINGS OF SCIENCE

Differential cross sections and analyzing powers in the $dd \to {}^3He\ n$ reaction at intermediate energies

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The $dd \to {}^3He\ n$ reaction is considered at the energies between 200 MeV and 520 MeV. The calculations are performed within relativistic multiple scattering model based on the Alt-Grassberger-Sandhas equations. The angular dependences of the differential cross section, vector and tensor analyzing powers are given in comparison with the experimental data.

XXI International Baldin Seminar on High Energy Physics Problems
September 10-15, 2012
JINR, Dubna, Russia

[*]Speaker.
[†]supported by the Russian Foundation for Basic Research under grant No. 10-02-00087a.

PoS(Baldin ISHEPP XXI)098

1. Introduction

During several decades hadronic reactions with helium and tritium were extensively investigated at the energies of a few hundred MeV. A number of experiments were performed at TRIUMF, IUCF, RCNP, Saclay. The aim of these experiments was to study the helium internal structure. The simple relations between the helium wave function and differential cross section and polarization observables in the frame of the plane-wave-impulse-approximation (PWIA) give an opportunity to extract useful information about the ground state spin structure of helium.

Several years ago the experiment to study the $dd \to {}^3Hen$, $dd \to {}^3Hp$ reactions was carried out at RIKEN [1, 2]. The vector and tensor analyzing powers were obtained in a wide angular range at three deuteron kinetic energies: 140, 200, and 270 MeV. Previously the differential cross sections of the reactions ${}^2H(d,n){}^3He$ and ${}^2H(d,p){}^3H$ were measured for incident deuteron momenta between 1.1 GeV/c and 2.5 GeV/c [3].

The $dd \to {}^3Hen$ reaction was considered in the one-nucleon-exchange (ONE) framework in refs. [4, 5]. High sensitivity of some of the polarization observables to the spin structure of the ${}^3He$ was shown. However, the data obtained at RIKEN are in disagreement with ONE predictions. Only a small angular range, around $0^0 - 15^0$ and $165^0 - 180^0$, is reasonably described by ONE mechanism. This result stimulated further theoretical investigations of this reaction.

In the present paper the $dd \to {}^3Hen$ reaction is studied at the deuteron energies between 200 MeV and 520 MeV. We start our investigation from the Alt-Grassberger-Sandhas (AGS) equations for the four-body case [6] and then iterate them up to the first order terms over the nucleon-nucleon t-matrix. In such a way we include not only ONE mechanism into consideration but also the next term. It corresponds to the case when nucleons from different deuterons interact with each other and then form a three-nucleon bounded state and a free nucleon. The parameterization based on the modern phase-shift analysis data is applied to describe NN interaction. The partial wave decomposition is not used in this approach. It allows us to avoid the problem related with convergence which is important at the considered energies.

2. General formalism

Here we consider the reaction where four initial nucleons are bounded in pairs forming two deuterons, and three final nucleons are bounded to the helium or tritium and one nucleon is free. In other words, we have the reaction of the $(2)+(2) \to (3)+(1)$ type.

We write the transition operator $U(z)$ for our reaction as it was offered by Grassberger and Sandhas [6]:

$$U_{\beta\alpha}(z) = (1-\delta_{\beta\alpha})(z-H_\alpha) + \sum_{ik\not\subset\beta} T_{ik}(z)G_0(z)U_{ik,\alpha}(z) + \sum_{ik\not\subset\beta} V_\alpha \delta_{\alpha,ik}\,, \tag{2.1}$$

where α and β denote two-cluster partitions of the four-particles. Here these labels are referred to the initial and final states, respectively:

$$\begin{aligned} &1)\alpha = (ij)(kl);\;\; V_\alpha \equiv V_{(ij)(kl)} = V_{ij}+V_{kl};\quad \Phi_{(ij)(kl)} = |\vec{k}_{ij},\vec{k}_{kl}> |\psi_{ij}> |\psi_{kl}> \\ &2)\beta = (ijk);\quad\;\; V_\beta \equiv V_{(ijk)} = V_{ij}+V_{jk}+V_{ik};\quad \Phi_{(ijk)} = |\vec{k}_{ijk},\vec{k}_{l}> |\psi_{ijk}> . \end{aligned} \tag{2.2}$$

In accordance with the AGS-formalism the channel Hamiltonian is defined as a sum of the free particles Hamiltonian H_0 and the interaction potential:

$$H_{\alpha(\beta)} = H_0 + V_{\alpha(\beta)}. \tag{2.3}$$

The eigenfunctions of the channel Hamiltonian $|\Phi_\alpha>$ characterize possible initial and final configurations. These functions are products of plane waves and internal wave functions $|\psi_\alpha>$.

The operator $T_{ij}(z)$ in Eq.(2.1) is a two-body transition operator which satisfies the Lippmann-Schwinger equation:

$$T_{ij}(z) = V_{ij} + V_{ij} G_0(z) T_{ij} \ , \tag{2.4}$$

where G_0 is the resolvent of the four-nucleon kinetic energy operator $G_0(z) = (z - H_0)^{-1}$.

The operator $U_{ik,\alpha}$ in Eq.(2.1) corresponds to the case when the initial state α is determined as in Eq.(2.2) and the final state is a combination of two bounded nucleons (ik) and two free nucleons. This transition operator can be also defined from Eq.(2.1) if we put the final state $\beta = (ik)$. The notation $ik \not\subseteq \beta$ means that pair (ik) is not either equal to one cluster of β or contained in it.

We deal with four identical nucleons and two identical deuterons in the initial state. It means that symmetrized wave functions both for the initial and final states, should be built [7]. Using the properties of the symmetrized functions and permutation operators we get the following expression for the reaction amplitude:

$$\begin{aligned} < n^3He|U|dd >= \frac{1}{\sqrt{6}}[\ & < 4, \psi(123)_s|U|\psi(12)_s\psi(34)_s > - \\ & < 1, \psi(234)_s|U|\psi(12)_s\psi(34)_s >]. \end{aligned} \tag{2.5}$$

Here the wave functions of deuterons $\psi(ij)_s$ are antisymmetrized:

$$|\psi(ij)>_s = \frac{1}{\sqrt{2}}[|\psi(ij)> - |\psi(ji)>]. \tag{2.6}$$

Three-nucleon state $(ijk)_s$ is also presented by the antisymmetrized wave function

$$\begin{aligned} |\psi(123)>_s = \frac{1}{\sqrt{6}}[\ & |\psi(123)> - |\psi(213)> + |\psi(231)> - |\psi(321)> + \\ & |\psi(312)> - |\psi(132)>]. \end{aligned} \tag{2.7}$$

The same way it is necessary to find two matrix elements of the transition operator U. We start to consider the first of them. This term corresponds to the case of $\beta = (ijk) = (123)$, $\alpha = (12)(34)$. From Eq.(2.1) we get

$$\begin{aligned} U_{(123),(12)(34)}(z) \ = \ & (z - H_0) - V_{12} - V_{34} + T_{14}(z) G_0(z) U_{(14),(12)(34)}(z) + \\ & T_{24}(z) G_0(z) U_{(24),(12)(34)}(z) + T_{34}(z) G_0(z) U_{(34),(12)(34)}. \end{aligned} \tag{2.8}$$

The relation (2.8) contains transition operators for another reaction type. In the final state two particles are bounded and the other two are free, while the initial state is the same as before. In

order to derive expressions for these operators, it is convenient to rewrite Eq.(2.1) in the following form:

$$U_{\beta\alpha}(z) = (1-\delta_{\beta\alpha})(z-H_\beta) + \sum_{mn\not\subset\alpha} U_{\beta,mn}(z)G_0(z)T_{mn}(z) + \sum_{mn\not\subset\alpha} V_\beta \delta_{\beta,mn}. \quad (2.9)$$

Iterating equations (2.8) only up to the first order term over T-matrix and taking into account Eqs.(2.9), we get the following sequence for the $U_{(123),(12)(34)}$-operator:

$$U_{(123),(12)(34)} \approx (z-H_{12}) + T_{14}(z) + T_{24}(z). \quad (2.10)$$

Likewise we derive the expression for the other transition operator in Eq.(2.5):

$$U_{(234),(12)(34)} \approx (z-H_{34}) + T_{13}(z) + T_{14}(z). \quad (2.11)$$

Since the initial and final states are antisymmetrized, the contributions of the T_{24} and T_{14} matrix elements are equal to each other. In order to show it, we use the properties of the permutation operator: $P_{12}P_{12} = 1$, $P_{12}T_{24}P_{12} = T_{14}$.

$$< 4,\psi(123)_s|T_{24}|\psi(12)_s\psi(34)_s > = < 4,\psi(123)_s|P_{12}P_{12}T_{24}P_{12}P_{12}|\psi(12)_s\psi(34)_s > =$$
$$= < 4,\psi(213)_s|T_{12}|\psi(21)_s\psi(34)_s > = < 4,\psi(123)_s|T_{12}|\psi(12)_s\psi(34)_s > \quad (2.12)$$

It also concerns T_{13} and T_{14} matrix elements in the exchange contribution:

$$< 1,\psi(234)_s|T_{13}|\psi(12)_s\psi(34)_s > = < 1,\psi(234)_s|T_{14}|\psi(12)_s\psi(34)_s > . \quad (2.13)$$

In such a way Eq.(2.5) can be reduced to the following:

$$< n^3He|U|dd > = \frac{1}{\sqrt{6}}[< 4,\psi(123)_s|z-H_{12}|\psi(12)_s\psi(34)_s > - \quad (2.14)$$
$$< 1,\psi(234)_s|z-H_{34}|\psi(12)_s\psi(34)_s > + 2 < 4,\psi(123)_s|T_{14}^{sym}|\psi(12)_s\psi(34)_s >],$$

where antisymmetrized NN T-matrix is defined as $T_{14}^{sym} = (1-P_{14})T_{14}$.

Two first terms in Eq.(2.14) correspond to the one-nucleon-exchange (ONE) mechanism of the reaction. We call the first of them as "direct" and the second one as "exchange". Here one of the deuterons breaks in a neutron and proton. One of the nucleons becomes free, while the other interacts with the remained deuteron forming helium or tritium. Schematically it can be presented by diagrams in Figs.1a and 1b. The latter term corresponds to single scattering (SS) when two nucleons from different deuterons interact in the final state (Fig, 1c).

Taking the quantum numbers and momenta of all particles into account, we get the following expression for

ONE terms:

$$< n^3He|U|dd >_{ONE} = \frac{1}{\sqrt{6}}[< \vec{p}_n m_n \tau_n, \Psi^{123}(\vec{P}_h M_h \tau_h)|(2E_d - \hat{H}_{12})|\Psi^{12}(\vec{P}_1, M_1), \Psi^{34}(\vec{P}_2, M_2) > -$$
$$< \vec{p}_n m_n \tau_n, \Psi^{234}(\vec{P}_h M_h \tau_h)|(2E_d - \hat{H}_{34})|\Psi^{12}(\vec{P}_1, M_1), \Psi^{34}(\vec{P}_2, M_2) >]. \quad (2.15)$$

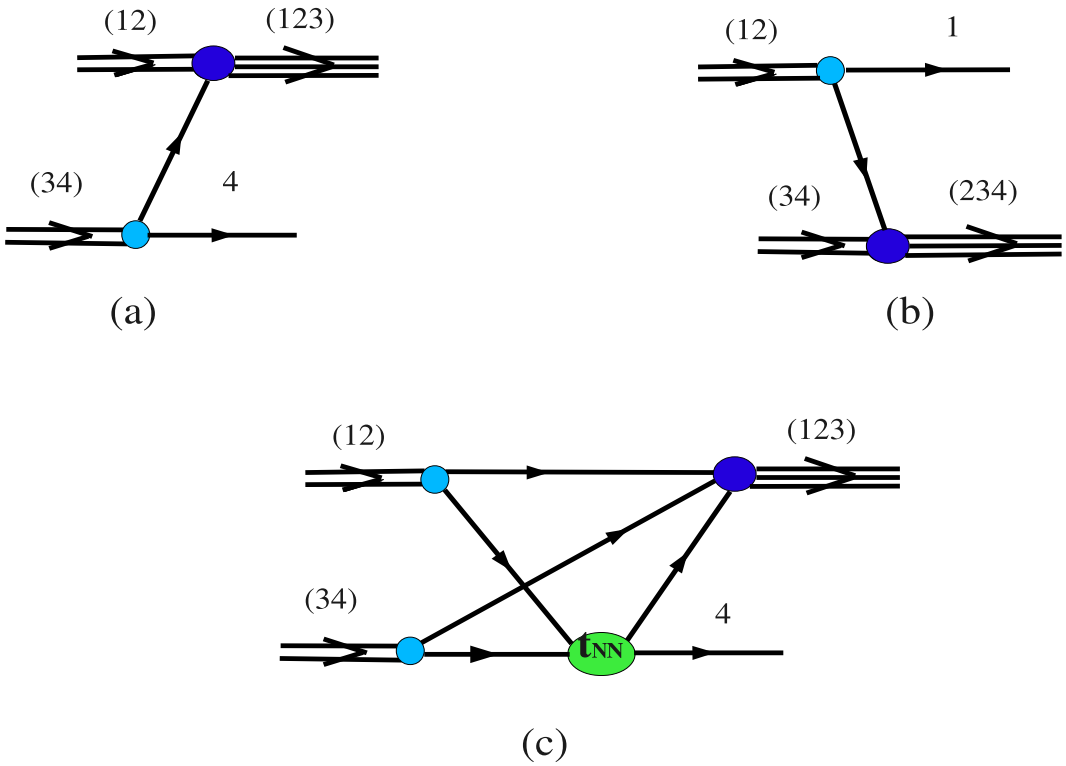

Figure 1: The diagrams taken into consideration: one-nucleon-exchange (a),(b), and single scattering (c) graphs.

and Single Scattering term:

$$< n^3He|U|dd >_{SS} = \frac{1}{\sqrt{6}} < \vec{p}_n m_n \tau_n, \Psi^{123}(\vec{P}_h M_h \tau_h)|T_{NN}^{sym}(E)|\Psi^{12}(\vec{P}_1, M_1), \Psi^{34}(\vec{P}_2, M_2) > \tag{2.16}$$

Here we introduce notation $\Psi^{ijk}(\vec{P}_h, M_h, \tau_h)$ for the ^3He wave function, where ^3He is formed by i, j, k nucleons and has momentum $\vec{P}_h$, spin projection M_h and isospin projection τ_h. Note in case $\tau = -1/2$ we deal with the reaction of $dd \to tp$. The $\Psi^{ij}(\vec{P}, M)$ denotes the wave function of the deuteron with momentum $\vec{P}$ and spin projection M. In our calculation we use the parameterized wave functions both for a three-nucleon system ref.[8] and for the deuteron [9, 10].

The nucleon-nucleon scattering is described by the T-matrix. We use the parameterization of this matrix offered by Love and Franey [11]. A new fit of the model parameters [12] was done in accordance with the phase-shift-analysis data SP07 [13].

3. Results and discussions.

The formalism presented above was applied to describe the experimental data obtained for $dd \to {}^3Hen$ and $dd \to tp$ reactions at the deuteron kinetic energies of a few hundred MeV. The calculations have been performed with CD-Bonn deuteron [9] and helium [8] wave functions.

In Figs.2-4 the results of the calculations of the differential cross sections are presented in comparison with the data. We consider three energies, 300 MeV, 457 MeV and 520 MeV, which

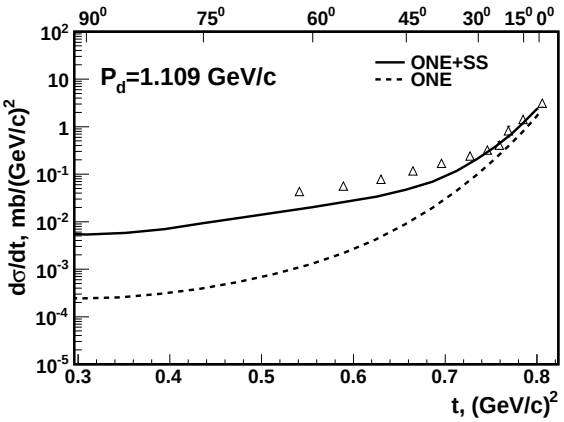

Figure 2: The differential cross section at the deuteron momentum of 1.109 GeV/c as a function of t. The data are taken from [3].

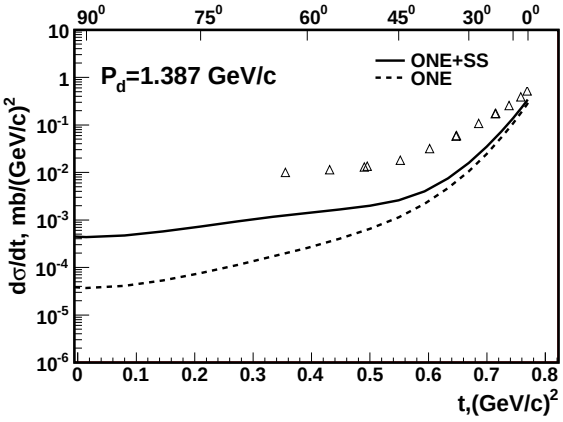

Figure 3: The differential cross section at the deuteron momentum of 1.387 GeV/c as a function of t. The data are taken from [3].

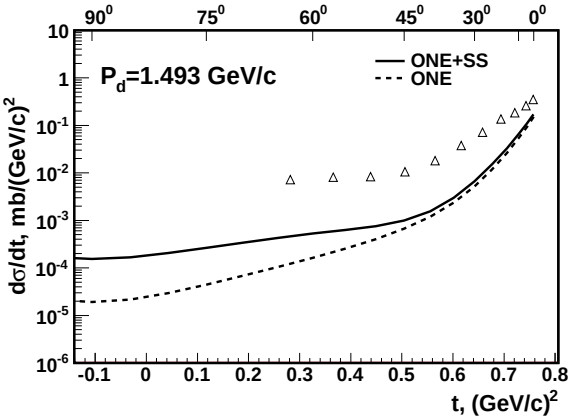

Figure 4: The differential cross section at the deuteron momentum of 1.493 GeV/c as a function of t. The data are taken from [3].

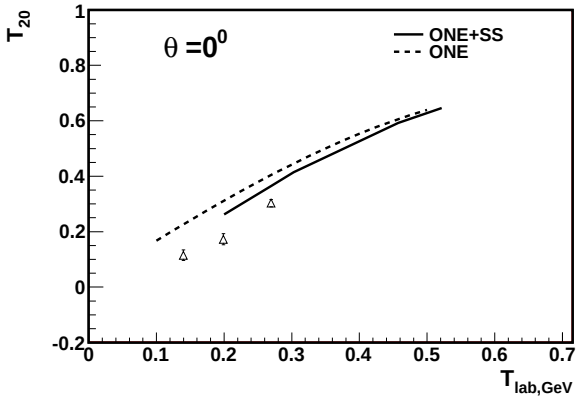

Figure 5: The energy dependence of tensor analyzing power T_{20} at the zero scattering angle. The data are taken from [1].

correspond to the laboratory momenta $P_{lab} = 1.109,\ 1.387$, and 1.493 GeV/c, respectively. In order to demonstrate the contribution of the single scattering term, we have considered two cases. One of them corresponds to the calculations including only ONE terms. The results of these calculations are given with the dashed curves. The other case corresponds to the calculations taking into account both ONE and single scattering contributions. These results are presented with the solid curves.

The contribution of the rescattering term is not large at small scattering angles ($t \sim 0.7 - 0.8(GeV/c)^2$). It is in agreement with the results obtained in ref.[2]. However, the difference between these two curves increases with the scattering angle increasing and reaches the maximal value at 90^0. Taking the single scattering diagram into consideration significantly improves the agreement between the experimental data and theoretical predictions. We have a good description of the data for $P_{lab} = 1.109\ GeV/c$ (Fig.2). Nevertheless, the underestimation of the differential cross sections is observed at the deuteron energies above 300 MeV (Figs.3,4).

The formalism presented here gives us an opportunity to calculate not only the differential

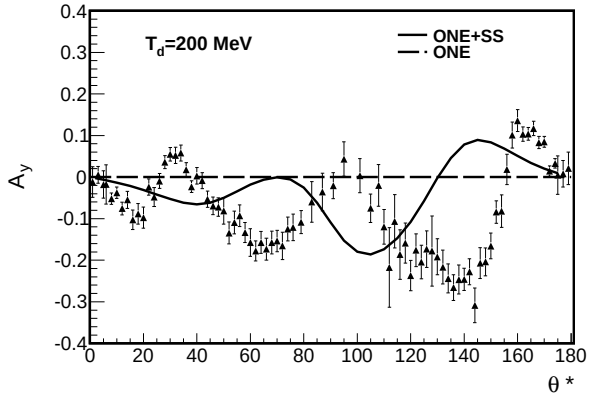

Figure 6: The angular dependence of vector analyzing power A_y at the deuteron energy of 200 MeV. The data are taken from [14].

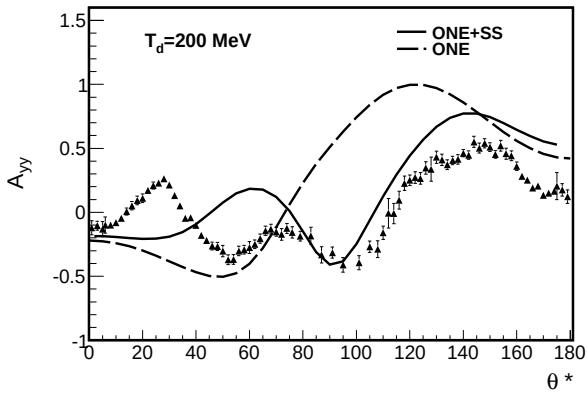

Figure 7: The angular dependence of tensor analyzing power A_{yy} at the deuteron energy of 200 MeV. The data are taken from [14].

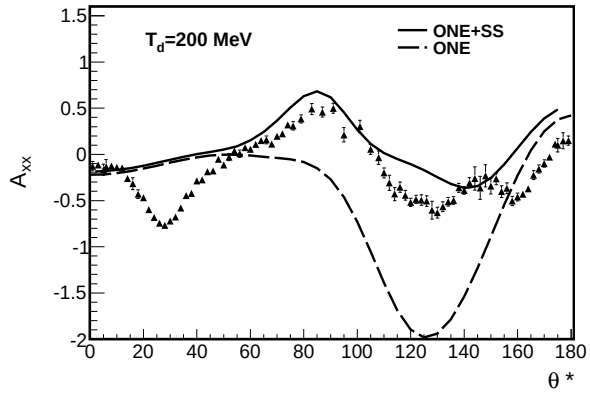

Figure 8: The angular dependence of tensor analyzing power A_{xx} at the deuteron energy of 200 MeV. The data are taken from [14].

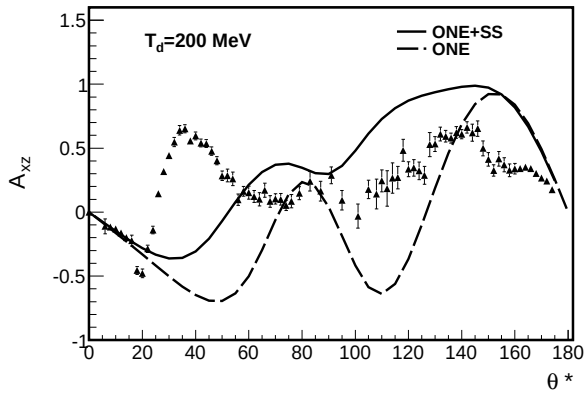

Figure 9: The angular dependence of tensor analyzing power A_{xz} at the deuteron energy of 200 MeV. The data are taken from [14].

cross sections but also polarization observables. In Fig.5 the energy dependence of tensor analyzing power T_{20} is presented at the scattering angle equal to zero. The experimental data were obtained at RIKEN [1]. As it is mentioned above, the contribution of the single scattering term is not large at small angles. Nevertheless, one can observe some improvement of the agreement between the data and theory predictions. Unfortunately, we do not have enough experimental data to confirm this tendency.

The angular dependence of vector analyzing power, A_y, has been considered at the deuteron energy equal to 200 MeV (Fig.6). As well known, the vector analyzing power is equal to zero in the ONE framework. However, the experimental data demonstrate significant deviation of the A_y from zero value. Taking the single-scattering diagram into consideration we derive the non zero value of the vector analyzing power. Unfortunately, the description of the experimental data is not

good even with SS contribution.

Tensor analyzing powers, A_{yy}, A_{xx}, and A_{xz} at the same deuteron energy are given in Figs.7-9. Here we have reasonable description of the data when the single-scattering is included into consideration, especially, at the scattering angle above 60^0. The difference between the results obtained with SS term and without it is significant. Inclusion of the single-scattering diagram improves the agreement between data and theory. The solid curves describe the experimental data quit good in the angular range between 60^0 and 150^0. However, it is necessary to note that the peaks at the scattering angle about $20-40^0$ are not described by adding SS term. Moreover, the curves with SS term and without it are practically undistinguished for A_{xx} in this angular range. It notes us, that additional reaction mechanisms must be included into considerations.

4. Conclusion.

The $dd \to {}^3He\ n$ reaction has been considered at the deuteron energies between 200 MeV and 520 MeV. The theoretical model for description of this process has been suggested. This model is based on the multiple scattering expansion formalism taking relativistic kinematics and relativistic spin theory into account.The one-nucleon-exchange and single-scattering reaction mechanisms have been included into consideration. It was shown that the inclusion of the single scattering term gives significant contribution into the reaction amplitude.A reasonable agreement between the theoretical predictions and experimental data was obtained for the differential cross section at the deuteron energy of 300 MeV. Also the qualitative description of the differential cross sections were obtained at the deuteron energies about of 500 MeV and 520 MeV. A reasonable description of the tensor analyzing powers A_{yy}, A_{xx}, A_{xz} were obtained at the scattering angles above 60^0. Unfortunately, the observed peaks in the tensor analyzing powers at the scattering angles between 20^0 and 40^0 are not explained in the present model. This fact notes us, that addition reaction mechanisms should be taken into account. Perhaps, this discrepancy can be reduced, if the Δ-excitation in the intermediate state is taken into account. This possibility is discussed in ref.[3], where the Δ-isobar is taken into consideration in the simplest phenomenological model.

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