PROCEEDINGS OF SCIENCE

Perturbative stability of the QCD predictions for the ratio $R = F_L/F_T$ and azimuthal asymmetry in heavy-quark leptoproduction

N. Ya. Ivanov*
Yerevan Physics Institute, Alikhanian Brs. 2, Yerevan 0036, Armenia
E-mail: nikiv@yerphi.am

We analyze the perturbative and parametric stability of the QCD predictions for the Callan-Gross ratio $R(x,Q^2) = F_L/F_T$ and azimuthal $\cos(2\varphi)$ asymmetry in heavy-quark leptoproduction. Our analysis shows that large radiative corrections to the structure functions cancel each other in their ratio $R(x,Q^2)$ and azimuthal asymmetry with good accuracy. As a result, the NLO contributions to the Callan-Gross ratio and $\cos(2\varphi)$ asymmetry are less than 10% in a wide region of the variables x and Q^2. We provide compact analytic predictions for $R(x,Q^2)$ and asymmetry in the case of low $x \ll 1$. Simple formulae connecting the high-energy behavior of the Callan-Gross ratio and azimuthal asymmetry with the low-x asymptotics of the gluon density in the target are derived. It is shown that the obtained hadron-level predictions for $R(x,Q^2)$ and azimuthal asymmetry are stable at $x \ll 1$ under the DGLAP evolution of the gluon distribution function.

Concerning the experimental aspects, we propose to exploit the observed perturbative stability of the Callan-Gross ratio and $\cos(2\varphi)$ asymmetry in the extraction of the structure functions from the corresponding reduced cross sections. In particular, our obtained analytic expressions simplify essentially the determination of $F_2^c(x,Q^2)$ and $F_2^b(x,Q^2)$ from available data of the H1 Collaboration. Our results will also be useful in extraction of the azimuthal asymmetries from the incoming and future data on heavy-quark leptoproduction.

XXI International Baldin Seminar on High Energy Physics Problems
September 10-15, 2012
JINR, Dubna, Russia

*Speaker.

1. Introduction

In the framework of perturbative quantum chromodynamics (QCD), the basic spin-averaged characteristics of heavy-flavor photo- [1, 2], electro- [3], and hadro-production [4 – 6] are known exactly up to the next-to-leading order (NLO).[1] Although these explicit results are widely used at present for a phenomenological description of available data (for a review, see Ref. [9]), the key question remains open: How to test the applicability of QCD at fixed order to heavy-quark production? The basic theoretical problem is that the NLO corrections are sizeable; they increase the leading-order (LO) predictions for both charm and bottom production cross sections by approximately a factor of two. Moreover, soft-gluon resummation of the threshold Sudakov logarithms indicates that higher-order contributions can also be substantial. (For reviews, see Refs. [10, 11].) On the other hand, perturbative instability leads to a high sensitivity of the theoretical calculations to standard uncertainties in the input QCD parameters. The total uncertainties associated with the unknown values of the heavy-quark mass, m, the factorization and renormalization scales, μ_F and μ_R, the asymptotic scale parameter Λ_{QCD} and the parton distribution functions (PDFs) are so large that one can only estimate the order of magnitude of the pQCD predictions for charm production cross sections in the entire energy range from the fixed-target experiments [12, 13] to the RHIC collider [9].

Since these production cross sections are not perturbatively stable, it is of special interest to study those observables that are well-defined in pQCD. Nontrivial examples of such observables were proposed in Refs. [14 – 22], where the azimuthal $\cos(2\varphi)$ asymmetry and Callan-Gross ratio $R(x,Q^2) = F_L/F_T$ in heavy-quark leptoproduction were analyzed.[2,3] In particular, the NLO soft-gluon corrections to the basic mechanism, photon-gluon fusion (GF), were calculated. It was shown that, contrary to the production cross sections, the azimuthal $\cos(2\varphi)$ asymmetry in heavy-flavor photo- and leptoproduction is quantitatively well defined in pQCD: the contribution of the dominant GF mechanism to the asymmetry is stable, both parametrically and perturbatively. Therefore, measurements of this asymmetry should provide a clean test of pQCD.

The perturbative and parametric stability of the GF predictions for the Callan-Gross ratio $R(x,Q^2) = F_L/F_T$ in heavy-quark leptoproduction was considered in Refs. [20 – 22]. It was shown that large radiative corrections to the structure functions $F_T(x,Q^2)$ and $F_L(x,Q^2)$ cancel each other in their ratio $R(x,Q^2)$ with good accuracy. As a result, the next-to-leading order (NLO) contributions of the dominant GF mechanism to the Callan-Gross ratio are less than 10% in a wide region of the variables x and Q^2.

In the present paper, we continue the studies of perturbatively stable observables in heavy-quark leptoproduction,

$$\ell(l) + N(p) \to \ell(l-q) + Q(p_Q) + X[\bar{Q}](p_X). \tag{1.1}$$

[1]Some recent results concerning the ongoing computations of the next-to-next-to-leading order (NNLO) corrections to the heavy-flavor hadroproduction are presented in Refs. [7, 8]

[2]Well-known examples include the shapes of differential cross sections of heavy flavor production, which are sufficiently stable under radiative corrections.

[3]Note also the paper [23], where the perturbative stability of the QCD predictions for the charge asymmetry in top-quark hadroproduction has been observed.

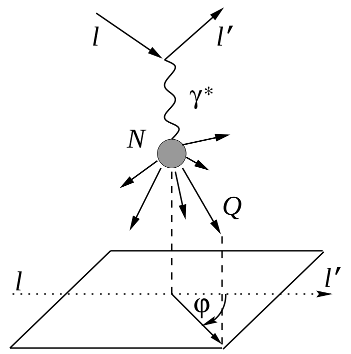

Figure 1: Definition of the azimuthal angle φ in the nucleon rest frame.

In the case of unpolarized initial states and neglecting the contribution of Z-boson exchange, the azimuth-dependent cross section of the reaction (1.1) can be written as

$$\frac{\mathrm{d}^3\sigma_{lN}}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}\varphi} = \frac{2\alpha_{\mathrm{em}}^2}{Q^4}\frac{y^2}{1-\varepsilon}\left[F_T(x,Q^2)+\varepsilon F_L(x,Q^2)+\varepsilon F_A(x,Q^2)\cos 2\varphi + 2\sqrt{\varepsilon(1+\varepsilon)}F_I(x,Q^2)\cos\varphi\right] \tag{1.2}$$

where α_{em} is Sommerfeld's fine-structure constant, $F_2(x,Q^2) = 2x(F_T+F_L)$, the quantity ε measures the degree of the longitudinal polarization of the virtual photon in the Breit frame [24], $\varepsilon = \frac{2(1-y)}{1+(1-y)^2}$, and the kinematic variables are defined by

$$\begin{aligned} \bar{S} &= (\ell+p)^2, \qquad & Q^2 &= -q^2, \qquad & x &= \frac{Q^2}{2p\cdot q}, \\ y &= \frac{p\cdot q}{p\cdot \ell}, \qquad & Q^2 &= xy\bar{S}, \qquad & \xi &= \frac{Q^2}{m^2}. \end{aligned} \tag{1.3}$$

In Eq. (1.2), $F_T\,(F_L)$ is the usual γ^*N structure function describing heavy-quark production by a transverse (longitudinal) virtual photon. The third structure function, F_A, comes about from interference between transverse states and is responsible for the $\cos 2\varphi$ asymmetry which occurs in real photoproduction using linearly polarized photons. The fourth structure function, F_I, originates from interference between longitudinal and transverse components [24]. In the nucleon rest frame, the azimuth φ is the angle between the lepton scattering plane and the heavy quark production plane, defined by the exchanged photon and the detected quark Q (see Fig. 1). The covariant definition of φ is

$$\cos\varphi = \frac{r\cdot n}{\sqrt{-r^2}\sqrt{-n^2}}, \qquad \sin\varphi = \frac{Q^2\sqrt{1/x^2+4m_N^2/Q^2}}{2\sqrt{-r^2}\sqrt{-n^2}}\,n\cdot\ell, \tag{1.4}$$

$$r^\mu = \varepsilon^{\mu\nu\alpha\beta}p_\nu q_\alpha \ell_\beta, \qquad n^\mu = \varepsilon^{\mu\nu\alpha\beta}q_\nu p_\alpha p_{Q\beta}. \tag{1.5}$$

In Eqs. (1.3) and (1.5), m and m_N are the masses of the heavy quark and the target, respectively.

In this talk, we review the perturbative and parametric stability of the Callan-Gross ratio, $R(x,Q^2)$, and azimuthal $\cos(2\varphi)$ asymmetry, $A(x,Q^2)$, defined as

$$R(x,Q^2) = \frac{F_L}{F_T}(x,Q^2), \qquad A(x,Q^2) = 2x\frac{F_A}{F_2}(x,Q^2). \tag{1.6}$$

First, we consider radiative corrections to the quantity $R(x,Q^2)$ using the explicit NLO results presented in [3, 25]. Our calculations show that complete $\mathcal{O}(\alpha_s^2)$ corrections to $R(x,Q^2)$ (including both the photon-gluon, $\gamma^* g \to Q\bar{Q}(g)$, and photon-(anti)quark, $\gamma^* q \to Q\bar{Q}q$, fusion components) do not exceed 10% in the energy range $x > 10^{-4}$.

Then, we analyze the perturbative stability of the azimuthal $\cos(2\varphi)$ asymmetry, $A(x,Q^2)$. Presently, the exact NLO predictions for the azimuth dependent structure function $F_A(x,Q^2)$ are not available. For this reason, we use the so-called soft-gluon approximation to estimate the radiative corrections to $F_A(x,Q^2)$. Our analysis shows that the NLO soft-gluon predictions for $A(x,Q^2)$ affect the LO results by less than a few percent at $Q^2 \lesssim m^2$ and $x \gtrsim 10^{-2}$.

In both cases, perturbative stability is mainly due to the cancellation of large radiative corrections to the structure functions F_L, F_T, F_A and F_2 in their ratios, $R(x,Q^2)$ and $A(x,Q^2)$, correspondingly. Note also that both the LO and NLO predictions for the Callan-Gross ratio and azimuthal asymmetry are sufficiently insensitive, to within ten percent, to standard uncertainties in the QCD input parameters μ_F, μ_R, $\Lambda_{\rm QCD}$ and PDFs.

We conclude that, in contrast to the production cross sections, the ratios $R(x,Q^2)$ and $A(x,Q^2)$ in heavy-quark leptoproduction are observables quantitatively well defined in pQCD. Measurements of these quantities in charm and bottom leptoproduction should provide a good test of the conventional parton model based on pQCD.

Since the ratios $R(x,Q^2)$ and $A(x,Q^2)$ are perturbatively stable, it makes sense to provide the LO hadron-level predictions for these quantities in analytic form that may be useful in some applications. For this reason, we derive compact hadron-level LO predictions for the the Callan-Gross ratio and azimuthal asymmetry in the limit of low $x \to 0$. Assuming the low-x asymptotic behavior of the gluon PDF to be of the type $g(x,Q^2) \propto 1/x^{1+\delta}$, we provide analytic result for the ratios $R(x \to 0,Q^2)$ and $A(x \to 0,Q^2)$ for arbitrary values of the parameter δ in terms of the Gauss hypergeometric function.[4]

In principle, the parameter δ is a function of Q^2 and this dependence is calculated using the DGLAP evolution equations [30 – 32]. However, our analysis shows that hadron-level predictions for $R(x \to 0,Q^2)$ and $A(x \to 0,Q^2)$ are practically independent of δ in the entire region of Q^2 for $\delta > 0.2$. We see that the hadron-level predictions for $R(x \to 0,Q^2)$ and $A(x \to 0,Q^2)$ are stable not only under the NLO corrections to the partonic cross sections, but also under the DGLAP evolution of the gluon PDF.

As to the experimental applications, we show that our compact LO formulae for $R(x \to 0,Q^2)$ conveniently reproduce the HERA results for $F_2^c(x,Q^2)$ and $F_2^b(x,Q^2)$ obtained by H1 Collaboration [33, 34] with the help of more cumbersome NLO estimations of $F_L(x,Q^2)$. Our analytic predictions will also be useful in extraction of the azimuthal asymmetries from the incoming COMPASS results as well as from future data on heavy-quark leptoproduction at the proposed EIC [35] and LHeC [36] colliders at BNL/JLab and CERN, correspondingly.

This paper is organized as follows. In Section 2, we analyze the exact NLO results for the Callan-Gross ratio. The soft-gluon contributions to $A(x,Q^2)$ are investigated in Section 3. The analytic LO results for the ratios $R(x,Q^2)$ and $A(x,Q^2)$ at low x are discussed in Section 4.

[4]The simplest case, $\delta = 0$, has been studied in Ref. [26]. The choice $\delta = 1/2$ historically originates from the BFKL resummation of the leading powers of $\ln(1/x)$ [27 – 29].

2. Exact NLO predictions for the Callan-Gross ratio $R(x,Q^2)$

At leading order, $\mathcal{O}(\alpha_{\rm em}\alpha_s)$, leptoproduction of heavy flavors proceeds through the photon-gluon fusion (GF) mechanism,

$$\gamma^*(q)+g(k_g)\to Q(p_Q)+\bar{Q}(p_{\bar{Q}}). \tag{2.1}$$

The relevant Feynman diagrams are depicted in Fig. 2. The corresponding $\gamma^* g$ cross sections, $\hat{\sigma}^{(0)}_{k,g}(z,\lambda)$ ($k=2,L,A,I$), have the form [37]:

$$\begin{aligned}
\hat{\sigma}^{(0)}_{2,g}(z,\lambda) &= \frac{\alpha_s}{2\pi}\hat{\sigma}_B(z)\Big\{\big[(1-z)^2+z^2+4\lambda z(1-3z)-8\lambda^2 z^2\big]\ln\frac{1+\beta_z}{1-\beta_z} \\
&\quad -[1+4z(1-z)(\lambda-2)]\,\beta_z\Big\}, \\
\hat{\sigma}^{(0)}_{L,g}(z,\lambda) &= \frac{2\alpha_s}{\pi}\hat{\sigma}_B(z)z\Big\{-2\lambda z\ln\frac{1+\beta_z}{1-\beta_z}+(1-z)\,\beta_z\Big\}, \\
\hat{\sigma}^{(0)}_{A,g}(z,\lambda) &= \frac{\alpha_s}{\pi}\hat{\sigma}_B(z)z\Big\{2\lambda\,[1-2z(1+\lambda)]\ln\frac{1+\beta_z}{1-\beta_z}+(1-2\lambda)(1-z)\beta_z\Big\}, \\
\hat{\sigma}^{(0)}_{I,g}(z,\lambda) &= 0,
\end{aligned} \tag{2.2}$$

with $\hat{\sigma}_B(z)=(2\pi)^2 e_Q^2\alpha_{\rm em}\,z/Q^2$, where e_Q is the electric charge of quark Q in units of the positron charge and $\alpha_s\equiv\alpha_s(\mu_R^2)$ is the strong-coupling constant. In Eqs. (2.2), we use the following definition of partonic kinematic variables:

$$z=\frac{Q^2}{2q\cdot k_g},\qquad \lambda=\frac{m^2}{Q^2},\qquad \beta_z=\sqrt{1-\frac{4\lambda z}{1-z}}. \tag{2.3}$$

The hadron-level cross sections, $\sigma_{k,GF}(x,Q^2)$ ($k=2,L,A,I$), corresponding to the GF subprocess, have the form

$$\sigma_{k,GF}(x,Q^2)=\int_{x(1+4\lambda)}^{1}{\rm d}z\,g(z,\mu_F)\hat{\sigma}_{k,g}\left(x/z,\lambda,\mu_F\right), \tag{2.4}$$

where $g(z,\mu_F)$ is the gluon PDF of the proton.

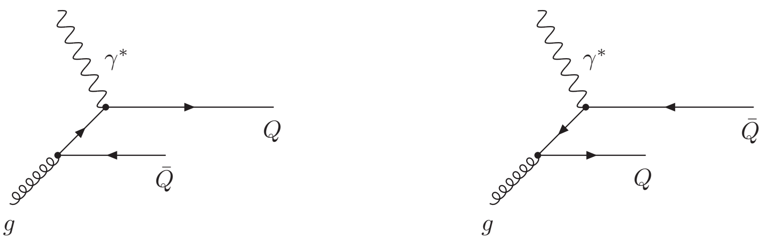

Figure 2: Feynman diagrams of photon-gluon fusion at LO.

The leptoproduction cross sections $\sigma_k(x,Q^2)$ are related to the structure functions $F_k(x,Q^2)$ as follows:

$$\begin{aligned} F_k(x,Q^2) &= \frac{Q^2}{8\pi^2\alpha_{\rm em}x}\sigma_k(x,Q^2) \qquad (k=T,L,A,I), \\ F_2(x,Q^2) &= \frac{Q^2}{4\pi^2\alpha_{\rm em}}\sigma_2(x,Q^2), \end{aligned} \tag{2.5}$$

where $\sigma_2(x,Q^2) = \sigma_T(x,Q^2) + \sigma_L(x,Q^2)$.

At NLO, $\mathscr{O}(\alpha_{\rm em}\alpha_s^2)$, the contributions of both the photon-gluon, $\gamma^* g \to Q\bar{Q}(g)$, and photon-(anti)quark, $\gamma^* q \to Q\bar{Q}q$, fusion components are usually presented in terms of the dimensionless coefficient functions $c_k^{(n,l)}(z,\lambda)$ as

$$\hat{\sigma}_k(z,\lambda,\mu^2) = \frac{e_Q^2\alpha_{\rm em}\alpha_s}{m^2}\left\{c_k^{(0,0)}(z,\lambda) + 4\pi\alpha_s\left[c_k^{(1,0)}(z,\lambda) + c_k^{(1,1)}(z,\lambda)\ln\frac{\mu^2}{m^2}\right] + \mathscr{O}(\alpha_s^2)\right\}, \tag{2.6}$$

where we identify $\mu = \mu_F = \mu_R$.

The coefficients $c_{k,g}^{(1,1)}(z,\lambda)$ and $c_{k,q}^{(1,1)}(z,\lambda)$ $(k=T,L,A,I)$ of the μ-dependent logarithms can be evaluated explicitly using renormalization group arguments [1, 3]. The results of direct calculations of the coefficient functions $c_{k,g}^{(1,0)}(z,\lambda)$ and $c_{k,q}^{(1,0)}(z,\lambda)$ $(k=T,L)$ are presented in Refs. [3, 25]. Using these NLO predictions, we analyze the Q^2 dependence of the ratio $R(x,Q^2) = F_L/F_T$ at fixed values of x.

The panels (a), (b) and (c) of Fig. 3 show the NLO predictions for Callan-Gross ratio $R(x,Q^2)$ in charm leptoproduction as a function of $\xi = Q^2/m^2$ at $x = 10^{-1}$, 10^{-2} and 10^{-3}, correspondingly. In our calculations, we use the CTEQ6M parametrization of the PDFs together with the values $m_c = 1.3$ GeV and $\Lambda = 326$ MeV [38].[5] Unless otherwise stated, we use $\mu = \sqrt{4m_c^2 + Q^2}$ throughout this paper.

For comparison, the panel (d) of Fig. 3 shows the Q^2 dependence of the QCD correction factor for the transverse structure function, $K(x,Q^2) = F_T^{\rm NLO}/F_T^{\rm LO}$. One can see that sizable radiative corrections to the structure functions $F_T(x,Q^2)$ and $F_L(x,Q^2)$ cancel each other in their ratio $R(x,Q^2) = F_L/F_T$ with good accuracy. As a result, the NLO contributions to the ratio $R(x,Q^2)$ are less than 10% for $x > 10^{-4}$.

Another remarkable property of the Callan-Gross ratio closely related to fast perturbative convergence is its parametric stability.[6] Our analysis shows that the fixed-order predictions for the ratio $R(x,Q^2)$ are less sensitive to standard uncertainties in the QCD input parameters than the corresponding ones for the production cross sections. For instance, sufficiently above the production threshold, changes of μ in the range $(1/2)\sqrt{4m_c^2+Q^2} < \mu < 2\sqrt{4m_c^2+Q^2}$ only lead to 10% variations of $R(x,Q^2)$ at NLO. For comparison, at $x = 0.1$ and $\xi = 4.4$, such changes of μ affect the NLO predictions for the quantities $F_T(x,Q^2)$ and $R(x,Q^2)$ in charm leptoproduction by more than 100% and less than 10%, respectively.

[5] Note that we convolve the NLO CTEQ6M distribution functions with both the LO and NLO partonic cross sections that makes it possible to estimate directly the degree of stability of the pQCD predictions under radiative corrections.

[6] Of course, parametric stability of the fixed-order results does not imply a fast convergence of the corresponding series. However, a fast convergent series must be parametrically stable. In particular, it must exhibit feeble μ_F and μ_R dependences.

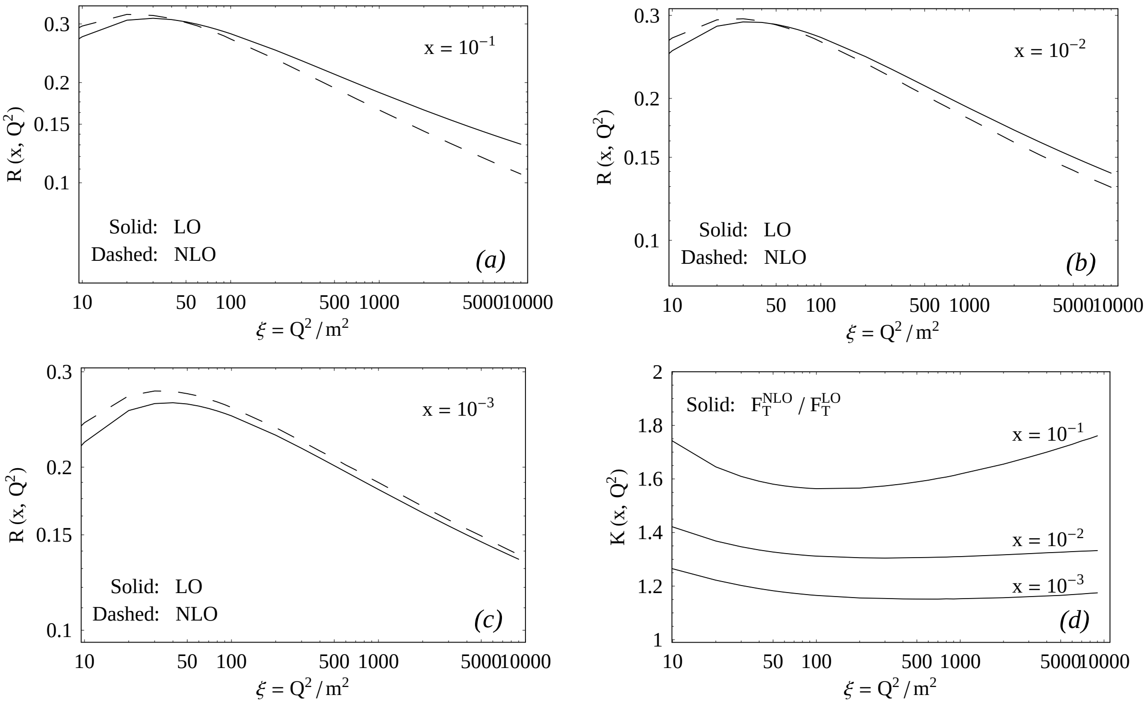

Figure 3: (a), (b) and (c) *panels:* Q^2 dependence of the LO (solid curves) and NLO (dashed curves) predictions for the Callan-Gross ratio, $R(x,Q^2) = F_L/F_T$, in charm leptoproduction at $x = 10^{-1}$, 10^{-2} and 10^{-3}. (d) *panel:* Q^2 dependence of the K factor for the transverse structure function, $K(x,Q^2) = F_T^{\mathrm{NLO}}/F_T^{\mathrm{LO}}$, at the same values of x.

Keeping the value of the variable Q^2 fixed, we analyze the dependence of the pQCD predictions on the uncertainties in the heavy-quark mass. We observe that changes of the charm-quark mass in the interval 1.3 GeV $< m_c <$ 1.7 GeV affect the Callan-Gross ratio by 2%–3% at $Q^2 = 10$ GeV^2 and $x < 10^{-1}$. The corresponding variations of the structure functions $F_T(x,Q^2)$ and $F_L(x,Q^2)$ are about 20%. We also verify that the recent CTEQ versions [38, 39] of the PDFs lead to NLO predictions for $R(x,Q^2)$ that coincide with each other with an accuracy of about 5% at $10^{-3} \leq x < 10^{-1}$.

3. Soft-gluon corrections to the azimuthal asymmetry $A(x,Q^2)$ at NLO

Presently, the exact NLO predictions for the azimuth dependent structure function $F_A(x,Q^2)$ are not available. For this reason, we consider the NLO predictions for the azimuthal $\cos(2\varphi)$ asymmetry within the soft-gluon approximation. For the reader's convenience, we collect the final results for the parton-level GF cross sections to the next-to-leading logarithmic (NLL) accuracy. More details may be found in Refs. [10, 15, 17, 20].

At NLO, photon-gluon fusion receives contributions from the virtual $\mathscr{O}(\alpha_{\mathrm{em}}\alpha_s^2)$ corrections to the Born process (2.1) and from real-gluon emission,

$$\gamma^*(q) + g(k_g) \to Q(p_Q) + \bar{Q}(p_{\bar{Q}}) + g(p_g). \tag{3.1}$$

The partonic invariants describing the single-particle inclusive (1PI) kinematics are

$$\begin{aligned} s' &= 2q\cdot k_g = s+Q^2 = \zeta S', & t_1 &= (k_g - p_Q)^2 - m^2 = \zeta T_1, \\ s_4 &= s' + t_1 + u_1, & u_1 &= (q-p_Q)^2 - m^2 = U_1, \end{aligned} \tag{3.2}$$

where ζ is defined through $\vec{k}_g = \zeta \vec{p}$ and s_4 measures the inelasticity of the reaction (3.1). The corresponding 1PI hadron-level variables describing the reaction (1.1) are

$$\begin{aligned} S' &= 2q\cdot p = S+Q^2, & T_1 &= (p-p_Q)^2 - m^2, \\ S_4 &= S' + T_1 + U_1, & U_1 &= (q-p_Q)^2 - m^2. \end{aligned} \tag{3.3}$$

The exact NLO calculations of unpolarized heavy-quark production [1−4] show that, near the partonic threshold, a strong logarithmic enhancement of the cross sections takes place in the collinear, $|\vec{p}_{g,T}| \to 0$, and soft, $|\vec{p}_g| \to 0$, limits. This threshold (or soft-gluon) enhancement is of universal nature in perturbation theory and originates from an incomplete cancellation of the soft and collinear singularities between the loop and the bremsstrahlung contributions. Large leading and next-to-leading threshold logarithms can be resummed to all orders of the perturbative expansion using the appropriate evolution equations [40]. The analytic results for the resummed cross sections are ill-defined due to the Landau pole in the coupling constant α_s. However, if one considers the obtained expressions as generating functionals and re-expands them at fixed order in α_s, no divergences associated with the Landau pole are encountered.

Soft-gluon resummation for the photon-gluon fusion was performed in Ref. [10] and confirmed in Refs. [15, 17]. To NLL accuracy, the perturbative expansion for the partonic cross sections, $\mathrm{d}^2\hat{\sigma}_k(s',t_1,u_1)/(\mathrm{d}t_1\,\mathrm{d}u_1)$ $(k = T,L,A,I)$, can be written in factorized form as

$$s'^2 \frac{\mathrm{d}^2\hat{\sigma}_k}{\mathrm{d}t_1 \mathrm{d}u_1}(s',t_1,u_1) = B_k^{\mathrm{Born}}(s',t_1,u_1)\left[\delta(s'+t_1+u_1) + \sum_{n=1}^{\infty}\left(\frac{\alpha_s C_A}{\pi}\right)^n K^{(n)}(s',t_1,u_1)\right]. \tag{3.4}$$

The functions $K^{(n)}(s',t_1,u_1)$ in Eq. (3.4) originate from the collinear and soft limits. Since the azimuthal angle φ is the same for both $\gamma^* g$ and $Q\bar{Q}$ center-of-mass systems in these limits, the functions $K^{(n)}(s',t_1,u_1)$ are also the same for all $\hat{\sigma}_k$, $k = T,L,A,I$. At NLO, the soft-gluon corrections to NLL accuracy in the $\overline{\mathrm{MS}}$ scheme read [10]

$$\begin{aligned} K^{(1)}(s',t_1,u_1) &= 2\left[\frac{\ln\left(s_4/m^2\right)}{s_4}\right]_+ - \left[\frac{1}{s_4}\right]_+ \left[1 + \ln\frac{u_1}{t_1} - \left(1 - \frac{2C_F}{C_A}\right)\left(1 + \mathrm{Re}L_\beta\right) + \ln\frac{\mu^2}{m^2}\right] \\ &\quad + \delta(s_4)\ln\frac{-u_1}{m^2}\ln\frac{\mu^2}{m^2}. \end{aligned} \tag{3.5}$$

In Eq. (3.5), $C_A = N_c$, $C_F = (N_c^2-1)/(2N_c)$, N_c is the number of quark colors, and $L_\beta = (1 - 2m^2/s)\{\ln[(1-\beta_z)/(1+\beta_z)] + \mathrm{i}\pi\}$ with $\beta_z = \sqrt{1-4m^2/s}$. The single-particle inclusive "plus" distributions are defined by

$$\left[\frac{\ln^l\left(s_4/m^2\right)}{s_4}\right]_+ = \lim_{\varepsilon\to 0}\left[\frac{\ln^l\left(s_4/m^2\right)}{s_4}\theta(s_4-\varepsilon) + \frac{1}{l+1}\ln^{l+1}\frac{\varepsilon}{m^2}\delta(s_4)\right]. \tag{3.6}$$

For any sufficiently regular test function $h(s_4)$, Eq. (3.6) implies that

$$\int_0^{s_4^{\max}} \mathrm{d}s_4\, h(s_4) \left[\frac{\ln^l\left(s_4/m^2\right)}{s_4}\right]_+ = \int_0^{s_4^{\max}} \mathrm{d}s_4 \left[h(s_4)-h(0)\right] \frac{\ln^l\left(s_4/m^2\right)}{s_4} + \frac{1}{l+1} h(0) \ln^{l+1}\frac{s_4^{\max}}{m^2}. \tag{3.7}$$

Standard NLL soft-gluon approximation allows us to determine unambiguously only the singular s_4 behavior of the cross sections defined by Eq. (3.6). To fix the s_4 dependence of the Born-level distributions $B_k^{\mathrm{Born}}(s',t_1,u_1)\big|_{u_1=s_4-s'-t_1}$ in Eq. (3.4), we use the method proposed in [20] and based on comparison of the soft-gluon predictions with the exact NLO results. According to [20],

$$B_k^{\mathrm{Born}}(s',t_1,u_1) \equiv s'^2 \frac{\mathrm{d}\hat{\sigma}_{k,g}^{(0)}}{\mathrm{d}t_1}(x_4 s', x_4 t_1), \qquad x_4 = -\frac{u_1}{s'+t_1} = 1 - \frac{s_4}{s'+t_1}, \tag{3.8}$$

where the LO GF differential distributions $s'^2 \mathrm{d}\hat{\sigma}_{k,g}^{(0)}(s',t_1)/\mathrm{d}t_1$ are

$$\begin{aligned}
s'^2 \frac{\mathrm{d}\hat{\sigma}_{T,g}^{(0)}}{\mathrm{d}t_1}(s',t_1) &= \pi e_Q^2 \alpha_{\mathrm{em}} \alpha_s \left\{ \frac{t_1}{u_1} + \frac{u_1}{t_1} + 4\left(\frac{s}{s'} - \frac{m^2 s'}{t_1 u_1}\right)\left[\frac{s'(m^2 - Q^2/2)}{t_1 u_1} + \frac{Q^2}{s'}\right]\right\}\Bigg|_{u_1=-s'-t_1} \\
s'^2 \frac{\mathrm{d}\hat{\sigma}_{L,g}^{(0)}}{\mathrm{d}t_1}(s',t_1) &= \pi e_Q^2 \alpha_{\mathrm{em}} \alpha_s \frac{8Q^2}{s'}\left(\frac{s}{s'} - \frac{m^2 s'}{t_1 u_1}\right)\Bigg|_{u_1=-s'-t_1} \\
s'^2 \frac{\mathrm{d}\hat{\sigma}_{A,g}^{(0)}}{\mathrm{d}t_1}(s',t_1) &= \pi e_Q^2 \alpha_{\mathrm{em}} \alpha_s\, 4\left(\frac{s}{s'} - \frac{m^2 s'}{t_1 u_1}\right)\left(\frac{m^2 s'}{t_1 u_1} + \frac{Q^2}{s'}\right)\Bigg|_{u_1=-s'-t_1} \\
s'^2 \frac{\mathrm{d}\hat{\sigma}_{I,g}^{(0)}}{\mathrm{d}t_1}(s',t_1) &= \pi e_Q^2 \alpha_{\mathrm{em}} \alpha_s\, 4\sqrt{Q^2}\left(\frac{t_1 u_1 s}{s'^2} - m^2\right)^{1/2} \frac{t_1 - u_1}{t_1 u_1}\left(1 - \frac{2Q^2}{s'} - \frac{2m^2 s'}{t_1 u_1}\right)\Bigg|_{u_1=-s'-t_1}
\end{aligned} \tag{3.9}$$

Comparison with the exact NLO results given by Eqs. (4.7) and (4.8) in Ref. [3] indicates that the usage of the distributions $B_k^{\mathrm{Born}}(s',t_1,u_1)$ defined by Eqs. (3.8) and (3.9) in present paper provides an accurate account of the logarithmic contributions originating from collinear gluon emission. Numerical analysis shows that Eqs. (3.8) and (3.9) render it possible to describe with good accuracy the exact NLO predictions for the functions $\hat{\sigma}_T^{(1)}(s')$ and $\hat{\sigma}_L^{(1)}(s')$ near the threshold at relatively low virtualities $Q^2 \sim m^2$ [20].[7]

Our results for the x distribution of the azimuthal $\cos(2\varphi)$ asymmetry, $A(x,Q^2) = 2xF_A/F_2$, in charm leptoproduction at fixed values of ξ are presented in the left panel of Fig. 4. For comparison, the K factor, $K(x,Q^2) = F_2^{\mathrm{NLO}}/F_2^{\mathrm{LO}}$, for the structure function F_2 at the same values of ξ is shown in the right panel of Fig. 4. One can see that the sizable soft-gluon corrections to the production cross sections affect the Born predictions for $A(x,Q^2)$ at NLO very little, by a few percent only.

4. Analytic LO results for $R(x,Q^2)$ and $A(x,Q^2)$ at low x

Since the ratios $R(x,Q^2)$ and $A(x,Q^2)$ are perturbatively stable, it makes sense to provide the LO hadron-level predictions for these quantities in analytic form. In this Section, we derive

[7] Note that soft-gluon approximation is unreliable for high $Q^2 \gg m^2$.

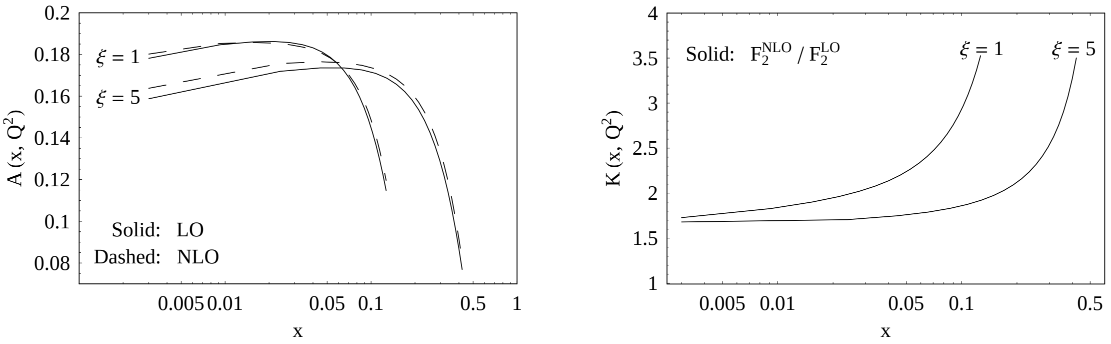

Figure 4: *Left panel:* LO (solid lines) and NLO (dashed lines) soft-gluon predictions for the x dependence of the azimuthal $\cos(2\varphi)$ asymmetry, $A(x,Q^2) = 2xF_A/F_2$, in charm leptoproduction at $\xi = 1$ and 5. *Right panel:* x dependence of the K factor, $K(x,Q^2) = F_2^{\rm NLO}/F_2^{\rm LO}$, at the same values of ξ.

compact low-x approximation formulae for the azimuthal $\cos(2\varphi)$ asymmetry and the quantity $R_2(x,Q^2)$ closely related to the Callan-Gross ratio $R(x,Q^2)$,

$$R_2(x,Q^2) = 2x\frac{F_L}{F_2}(x,Q^2) = \frac{R(x,Q^2)}{1+R(x,Q^2)}. \tag{4.1}$$

We will see below that our obtained results may be useful in the extraction of the structure functions F_k ($k = 2, L, A, I$) from experimentally measurable reduced cross sections.

To obtain the hadron-level predictions, we convolute the LO partonic cross sections given by Eqs. (2.2) with the low-x asymptotics of the gluon PDF:

$$g(x,Q^2) \xrightarrow{x\to 0} \frac{1}{x^{1+\delta}}. \tag{4.2}$$

The value of δ in Eq. (4.2) is a matter of discussion. The simplest choice, $\delta = 0$, leads to a non-singular behavior of the structure functions for $x \to 0$. Another extreme value, $\delta = 1/2$, historically originates from the BFKL resummation of the leading powers of $\ln(1/x)$ [27–29]. In reality, δ is a function of Q^2 (for an experimental review, see Refs. [41, 42]). Theoretically, the Q^2 dependence of δ is calculated using the DGLAP evolution equations [30–32].

First, we derive an analytic low-x formula for the ratio $R_2^{(\delta)}(Q^2) \equiv R_2^{(\delta)}(x \to 0, Q^2)$ with arbitrary values of δ in terms of the Gauss hypergeometric function. Our result has the following form:

$$R_2^{(\delta)}(Q^2) = 4\frac{\frac{2+\delta}{3+\delta}\Phi\left(1+\delta,\frac{1}{1+4\lambda}\right) - (1+4\lambda)\,\Phi\left(2+\delta,\frac{1}{1+4\lambda}\right)}{\left[1+\frac{\delta(1-\delta^2)}{(2+\delta)(3+\delta)}\right]\Phi\left(\delta,\frac{1}{1+4\lambda}\right) - (1+4\lambda)\left(4-\delta-\frac{10}{3+\delta}\right)\Phi\left(1+\delta,\frac{1}{1+4\lambda}\right)}, \tag{4.3}$$

where λ is defined in Eq. (2.3) and the function $\Phi(r,z)$ is

$$\Phi(r,z) = \frac{z^{1+r}}{1+r}\frac{\Gamma(1/2)\,\Gamma(1+r)}{\Gamma(3/2+r)}\,{}_2F_1\left(\frac{1}{2},1+r;\frac{3}{2}+r;z\right). \tag{4.4}$$

The hypergeometric function ${}_2F_1(a,b;c;z)$ has the following series expansion:

$${}_2F_1(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)\,\Gamma(b+n)}{\Gamma(c+n)}\frac{z^n}{n!}. \tag{4.5}$$

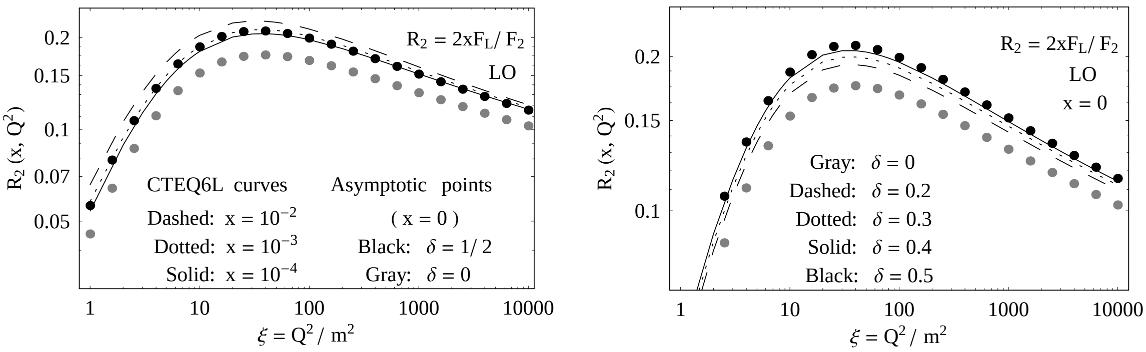

Figure 5: LO low-x predictions for the ratio $R_2(x,Q^2) = 2xF_L/F_2$ in charm leptoproduction. *Left panel:* Asymptotic ratios $R_2^{(0)}(Q^2)$ (gray points) and $R_2^{(1/2)}(Q^2)$ (black points), as well as CTEQ6L predictions for $R_2(x,Q^2)$ at $x = 10^{-2}$, 10^{-3} and 10^{-4}. *Right panel:* Asymptotic ratio $R_2^{(\delta)}(Q^2)$ at $\delta = 0$, 0.2, 0.3, 0.4 and 0.5.

In Fig. 5, we investigate the obtained result (4.3) for $R_2^{(\delta)}(Q^2)$. The left panel of Fig. 5 shows the ratio $R_2^{(\delta)}(Q^2)$ as functions of ξ for two extreme cases, $\delta = 0$ and $1/2$. One can see that the difference between these quantities varies slowly from 20% at low Q^2 to 10% at high Q^2. For comparison, also the LO results for $R_2(x,Q^2)$ calculated at several values of x using the CTEQ6L gluon PDF [38] are shown. We observe that, for $x \to 0$, the CTEQ6L predictions converge to the function $R_2^{(1/2)}(Q^2)$ practically in the entire region of Q^2. We have verified that the similar situation takes also place for other recent CTEQ PDF versions [38, 39]. In the right panel of Fig. 5, the δ dependence of the asymptotic ratio $R_2^{(\delta)}(Q^2)$ is investigated. One can see that the ratio $R_2^{(\delta)}(Q^2)$ rapidly converges to the function $R_2^{(1/2)}(Q^2)$ for $\delta > 0.2$. In particular, the relative difference between $R_2^{(0.5)}(Q^2)$ and $R_2^{(0.3)}(Q^2)$ varies slowly from 6% at low Q^2 to 2% at high Q^2.

As mentioned above, the Q^2 dependence of the parameter δ is determined with the help of the DGLAP evolution. However, our analysis shows that hadron-level predictions for $R_2^{(\delta)}(x \to 0,Q^2)$ depend weakly on δ practically in the entire region of Q^2 for $0.2 < \delta < 0.9$. For this reason, it makes sense to consider the ratio $R_2^{(\delta)}(Q^2)$ in particular case of $\delta = 1/2$. The result is:

$$R_2^{(1/2)}(Q^2) = \frac{8}{1+4\lambda}\,\frac{[3+4\lambda\,(13+32\lambda)]\,E(1/(1+4\lambda)) - 4\lambda\,(9+32\lambda)\,K(1/(1+4\lambda))}{(-37+72\lambda)\,E(1/(1+4\lambda)) + 2\,(23-36\lambda)\,K(1/(1+4\lambda))}, \tag{4.6}$$

where the functions $K(y)$ and $E(y)$ are the complete elliptic integrals of the first and second kinds defined as

$$K(y) = \int_0^1 \frac{\mathrm{d}t}{\sqrt{(1-t^2)(1-yt^2)}}, \qquad E(y) = \int_0^1 \mathrm{d}t\sqrt{\frac{1-yt^2}{1-t^2}}. \tag{4.7}$$

One can see from Fig. 5 that our simple formula (4.6) with $\delta = 1/2$ (i.e., without any evolution) describes with good accuracy the low-x CTEQ results for $R_2(x,Q^2)$. We conclude that the hadron-level predictions for $R_2(x \to 0,Q^2)$ are stable not only under the NLO corrections to the partonic cross sections, but also under the DGLAP evolution of the gluon PDF.

Then we calculate and investigate the LO hadron-level predictions for the azimuthal $\cos(2\varphi)$ asymmetry in the limit of $x \to 0$. Our result for the quantity $A^{(\delta)}(Q^2) \equiv A^{(\delta)}(x \to 0,Q^2)$ has the

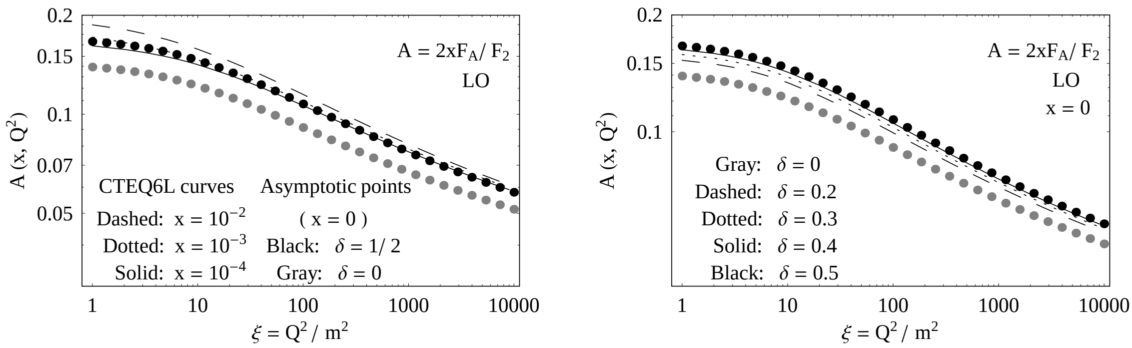

Figure 6: LO low-x predictions for the ratio $A(x,Q^2) = 2xF_A/F_2$ in charm leptoproduction. *Left panel:* Asymptotic ratios $A^{(0)}(Q^2)$ (gray points) and $A^{(1/2)}(Q^2)$ (black points), as well as CTEQ6L predictions for $A(x,Q^2)$ at $x = 10^{-2}$, 10^{-3} and 10^{-4}. *Right panel:* Asymptotic ratio $A^{(\delta)}(Q^2)$ at $\delta = 0$, 0.2, 0.3, 0.4 and 0.5.

following form:

$$A^{(\delta)}(Q^2) = 2\frac{\frac{2+\delta+2\lambda}{3+\delta}\Phi\left(1+\delta,\frac{1}{1+4\lambda}\right) - (1+4\lambda)\,\Phi\left(2+\delta,\frac{1}{1+4\lambda}\right)}{\left[1+\frac{\delta(1-\delta^2)}{(2+\delta)(3+\delta)}\right]\Phi\left(\delta,\frac{1}{1+4\lambda}\right) - (1+4\lambda)\left(4-\delta-\frac{10}{3+\delta}\right)\Phi\left(1+\delta,\frac{1}{1+4\lambda}\right)}. \tag{4.8}$$

Our analysis presented in Fig. 6 shows that the quantity $A^{(\delta)}(Q^2)$ defined by Eq. (4.8) has the properties very similar to the ones demonstrated by the ratio $R_2^{(\delta)}(Q^2)$. In particular, one can see from Fig. 6 that the hadron-level predictions for $A^{(\delta)}(Q^2)$ depend weakly on δ practically in the entire region of Q^2 for $\delta > 0.2$. So, the azimuthal $\cos(2\varphi)$ asymmetry $A(x \to 0, Q^2)$ is also stable under the DGLAP evolution of the gluon PDF.

Let us now discuss how the obtained analytic results may be used in the extraction of the structure functions F_k ($k = 2, L, A, I$) from experimentally measurable quantities. Usually, it is the so-called "reduced cross section", $\tilde{\sigma}(x,Q^2)$, that can directly be measured in DIS experiments:

$$\tilde{\sigma}(x,Q^2) = \frac{1}{1+(1-y)^2}\frac{xQ^4}{2\pi\alpha_{\mathrm{em}}^2}\frac{\mathrm{d}^2\sigma_{lN}}{\mathrm{d}x\mathrm{d}Q^2} = F_2(x,Q^2) - \frac{2xy^2}{1+(1-y)^2}F_L(x,Q^2) \tag{4.9}$$

$$= F_2(x,Q^2)\left[1-\frac{y^2}{1+(1-y)^2}R_2(x,Q^2)\right]. \tag{4.10}$$

In earlier HERA analyses of charm and bottom electroproduction, the corresponding longitudinal structure functions were taken to be zero for simplicity. In this case, $\tilde{\sigma}(x,Q^2) = F_2(x,Q^2)$. In recent papers [33, 34], the structure function $F_2(x,Q^2)$ is evaluated from the reduced cross section (4.9) where the longitudinal structure function $F_L(x,Q^2)$ is estimated from the NLO QCD expectations. Instead of this rather cumbersome procedure, we propose to use the expression (4.10) with the quantity $R_2(x,Q^2)$ defined by the analytic LO expressions (4.3) or (4.6). This simplifies the extraction of $F_2(x,Q^2)$ from measurements of $\tilde{\sigma}(x,Q^2)$ but does not affect the accuracy of the result in practice because of perturbative stability of the ratio $R_2(x,Q^2)$.

In Ref. [20], we used the analytic expressions (4.3) and (4.6) for the extraction of the structure functions $F_2^c(x,Q^2)$ and $F_2^b(x,Q^2)$ from the HERA measurements of the reduced cross sections $\tilde{\sigma}^c(x,Q^2)$ and $\tilde{\sigma}^b(x,Q^2)$, respectively. It was demonstrated that our LO formula (4.6) for $R_2(x,Q^2)$

with $\delta = 1/2$ usefully reproduces the results for $F_2^c(x,Q^2)$ and $F_2^b(x,Q^2)$ obtained by the H1 Collaboration [33, 34] with the help of the NLO evaluation of $F_L(x,Q^2)$. In particular, the results of our analysis of the HERA data on the charm electroproduction are collected in Table 1. In our calculations, the value $m_c = 1.3$ GeV for the charm quark mass is used. The LO predictions, F_2(LO), for the case of $\delta = 0.5$ are presented and compared with the NLO values, F_2(NLO), obtained in the H1 analysis [33, 34]. One can see that our LO predictions agree with the NLO results with an accuracy better than 1%.

Table 1: Values of $F_2^c(x,Q^2)$ extracted from the HERA measurements of $\tilde{\sigma}^c(x,Q^2)$ at low [34] and high [33] Q^2 (in GeV^2) for various values of x (in units of 10^{-3}). The NLO H1 results [33, 34] are compared with the LO predictions corresponding to the case of $\delta = 0.5$.

Q^2 (GeV^2)	x ($\times 10^{-3}$)	y	$\tilde{\sigma}^c$	Error (%)	F_2^c(NLO) H1	F_2^c(LO) $\delta = 0.5$
12	0.197	0.600	0.412	18	0.435 ± 0.078	0.435 ± 0.078
12	0.800	0.148	0.185	13	0.186 ± 0.024	0.185 ± 0.024
25	0.500	0.492	0.318	13	0.331 ± 0.043	0.331 ± 0.043
25	2.000	0.123	0.212	10	0.212 ± 0.021	0.212 ± 0.021
60	2.000	0.295	0.364	10	0.369 ± 0.040	0.369 ± 0.040
60	5.000	0.118	0.200	12	0.201 ± 0.024	0.200 ± 0.024
200	0.500	0.394	0.197	23	0.202 ± 0.046	0.202 ± 0.046
200	1.300	0.151	0.130	24	0.131 ± 0.032	0.130 ± 0.031
650	1.300	0.492	0.206	27	0.213 ± 0.057	0.213 ± 0.057
650	3.200	0.200	0.091	31	0.092 ± 0.028	0.091 ± 0.028

The structure functions F_A and F_I can be extracted from the φ-dependent DIS cross section,

$$\frac{\mathrm{d}^3\sigma_{lN}}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}\varphi} = \frac{2\alpha_{em}^2}{Q^4}\frac{y^2}{1-\varepsilon}\Big[\frac{1}{2x}F_2(x,Q^2) - (1-\varepsilon)F_L(x,Q^2) + \varepsilon F_A(x,Q^2)\cos 2\varphi + 2\sqrt{\varepsilon(1+\varepsilon)}F_I(x,Q^2)\cos\varphi\Big], \tag{4.11}$$

where $\varepsilon = \frac{2(1-y)}{1+(1-y)^2}$. For this purpose, one should measure the first moments of the $\cos(\varphi)$ and $\cos(2\varphi)$ distributions defined as

$$\langle\cos n\varphi\rangle(x,Q^2) = \frac{\int_0^{2\pi}\mathrm{d}\varphi\cos n\varphi\frac{\mathrm{d}^3\sigma_{lN}}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}\varphi}(x,Q^2,\varphi)}{\int_0^{2\pi}\mathrm{d}\varphi\frac{\mathrm{d}^3\sigma_{lN}}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}\varphi}(x,Q^2,\varphi)}. \tag{4.12}$$

Using Eq. (4.11), we obtain:

$$\langle\cos 2\varphi\rangle(x,Q^2) = \frac{1}{2}\frac{\varepsilon A(x,Q^2)}{1-(1-\varepsilon)R_2(x,Q^2)}, \qquad A(x,Q^2) = 2x\frac{F_A}{F_2}(x,Q^2), \tag{4.13}$$

$$\langle\cos\varphi\rangle(x,Q^2) = \frac{\sqrt{\varepsilon(1+\varepsilon)}A_I(x,Q^2)}{1-(1-\varepsilon)R_2(x,Q^2)}, \qquad A_I(x,Q^2) = 2x\frac{F_I}{F_2}(x,Q^2). \tag{4.14}$$

One can see from Eqs. (4.13) and (4.14) that, using the perturbatively stable predictions (4.3) for $R_2(x,Q^2)$, we will be able to determine the structure functions $F_A(x,Q^2)$ and $F_I(x,Q^2)$ from future data on the moments $\langle\cos 2\varphi\rangle$ and $\langle\cos\varphi\rangle$. On the other hand, according to Eq. (4.13), the analytic results (4.3) and (4.8) for the quantities $R_2(x,Q^2)$ and $A(x,Q^2)$ provide us with the perturbatively stable predictions for $\langle\cos 2\varphi\rangle$ which may be directly tested in experiment.

So, our obtained analytic and perturbatively stable predictions for the ratios $R_2(x,Q^2)$ and $A(x,Q^2)$ will simplify both the extraction of structure functions from the measurable φ-dependent cross section (4.11) and the test of self-consistency of the extraction procedure.

5. Conclusion

We conclude by summarizing our main observations. In the present paper, we studied the radiative corrections to the Callan-Gross ratio, $R(x,Q^2)$, and azimuthal $\cos(2\varphi)$ asymmetry, $A(x,Q^2)$, in heavy-quark leptoproduction. It turned out that large (especially, at non-small x) radiative corrections to the structure functions cancel each other in their ratios $R(x,Q^2) = F_L/F_T$ and $A(x,Q^2) = 2xF_A/F_A$ with good accuracy. As a result, the NLO contributions to the ratios $R(x,Q^2)$ and $A(x,Q^2)$ are less than 10% in a wide region of the variables x and Q^2. Our analysis shows that, sufficiently above the production threshold, the pQCD predictions for $R(x,Q^2)$ and $A(x,Q^2)$ are insensitive (to within ten percent) to standard uncertainties in the QCD input parameters and to the DGLAP evolution of PDFs. We conclude that, unlike the production cross sections, the Callan-Gross ratio and $\cos(2\varphi)$ asymmetry in heavy-quark leptoproduction are quantitatively well defined in pQCD. Measurements of the quantities $R(x,Q^2)$ and $A(x,Q^2)$ in charm and bottom leptoproduction would provide a good test of the conventional parton model based on pQCD.

As to the experimental aspects, we propose to exploit the observed perturbative stability of the Callan-Gross ratio and azimuthal asymmetry in the extraction of the structure functions from the experimentally measurable reduced cross sections. For this purpose, we provided compact LO hadron-level formulae for the ratios $R_2(x,Q^2) = 2xF_L/F_2 = R/(1+R)$ and $A(x,Q^2) = 2xF_A/F_2$ in the limit $x \to 0$. We demonstrated that these analytic expressions usefully reproduce the results for $F_2^c(x,Q^2)$ and $F_2^b(x,Q^2)$ obtained by the H1 Collaboration [33, 34] with the help of the more cumbersome NLO evaluation of $F_L(x,Q^2)$. Our obtained predictions will also be useful in extraction of the azimuthal asymmetries from the incoming COMPASS results as well as from future data on heavy-quark leptoproduction at the proposed EIC [35] and LHeC [36] colliders at BNL/JLab and CERN, correspondingly.

Acknowledgments

The author is thankful to Serge Bondarenko for invitation to XXI International Baldin Seminar on High Energy Physics Problems and help. We thank S. I. Alekhin and J. Blümlein for providing us with fast code [25] for numerical calculations of the NLO partonic cross sections. The author is also grateful to S. J. Brodsky, A. V. Efremov, A. V. Kotikov, A. B. Kniehl, E. Leader and C. Weiss for useful discussions. This work is supported in part by the State Committee of Science of RA, grant 11-1C015.

References

[1] R. K. Ellis and P. Nason, *QCD radiative corrections to the photoproduction of heavy quarks*, Nucl. Phys. B **312**, 551 (1989).

[2] J. Smith and W. L. van Neerven,*QCD corrections to heavy flavor photoproduction and electroproduction*, Nucl. Phys. B **374**, 36 (1992).

[3] E. Laenen, S. Riemersma, J. Smith, and W. L. van Neerven, *Complete $\mathscr{O}(\alpha_s)$ corrections to heavy flavor structure functions in electroproduction*, Nucl. Phys. B **392**, 162 (1993).

[4] P. Nason, S. Dawson, and R. K. Ellis, *The total cross-section for the production of heavy quarks in hadronic collisions*, Nucl. Phys. B **303**, 607 (1988).

[5] P. Nason, S. Dawson, and R. K. Ellis, *The one particle inclusive differential cross-section for heavy quark production in hadronic collisions*, Nucl. Phys. B **327**, 49 (1989); Erratum-ibid. B **335**, 260 (1990).

[6] W. Beenakker, H. Kuijf, W. L. van Neerven, and J. Smith, *QCD corrections to heavy quark production in $p\bar{p}$ collisions*, Phys. Rev. D **40**, 54 (1989).

[7] M. Czakon and A. Mitov, *NNLO corrections to top-pair production at hadron colliders: the all-fermionic scattering channels*, `arXiv:1207.0236 [hep-ph]`.

[8] M. Czakon and A. Mitov, *NNLO corrections to top pair production at hadron colliders: the quark-gluon reaction*, `arXiv:1210.6832 [hep-ph]`.

[9] R. Vogt, *The total charm cross-section*, Eur. Phys. J. ST **155**, 213 (2008) [`arXiv:0709.2531 [hep-ph]`].

[10] E. Laenen and S. -O. Moch, *Soft gluon resummation for heavy quark electroproduction* Phys. Rev. D **59**, 034027 (1999) [`hep-ph/9809550`].

[11] N. Kidonakis, *Next-to-next-to-next-to-leading-order soft-gluon corrections in hard-scattering processes near threshold*, Phys. Rev. D **73**, 034001 (2006) [`hep-ph/0509079`].

[12] M. L. Mangano, P. Nason, and G. Ridolfi, *Heavy quark correlations in hadron collisions at next-to-leading order*, Nucl. Phys. B **373**, 295 (1992).

[13] S. Frixione, M. L. Mangano, P. Nason, and G. Ridolfi, *Heavy quark correlations in photon-hadron collisions*, Nucl. Phys. B **412**, 225 (1994) [`hep-ph/9306337`].

[14] N. Ya. Ivanov, A. Capella, and A. B. Kaidalov, *Single spin asymmetry in heavy flavor photoproduction as a test of pQCD*, Nucl. Phys. B **586**, 382 (2000) [`hep-ph/9911471`].

[15] N. Ya. Ivanov, *Perturbative stability of the QCD predictions for single spin asymmetry in heavy quark photoproduction*, Nucl. Phys. B **615**, 266 (2001) [`hep-ph/0104301`].

[16] N. Ya. Ivanov, P. E. Bosted, K. Griffioen, and S. E. Rock, *Single spin asymmetry in open charm photoproduction and decay as a test of pQCD*, Nucl. Phys. B **650**, 271 (2003) [`hep-ph/0210298`].

[17] N. Ya. Ivanov, *Azimuthal asymmetries in heavy quark leptoproduction as a test of pQCD*, Nucl. Phys. B **666**, 88 (2003) [`hep-ph/0304191`].

[18] L. N. Ananikyan and N. Ya. Ivanov, *Azimuthal dependence of the heavy quark initiated contributions to DIS*, Phys. Rev. D **75**, 014010 (2007) [`hep-ph/0609074`].

[19] L. N. Ananikyan and N. Ya. Ivanov, *Azimuthal asymmetries in DIS as a probe of intrinsic charm content of the proton*, Nucl. Phys. B **762**, 256 (2007) [`hep-ph/0701076`].

[20] N. Ya. Ivanov and B. A. Kniehl, *On the perturbative stability of the QCD predictions for the ratio $R = F_L/F_T$ in heavy-quark leptoproduction*, Eur. Phys. J. C **59**, 647 (2009) [`arXiv:0806.4705 [hep-ph]`].

[21] N. Ya. Ivanov, *The ratio $R = F_L/F_T$ in DIS as a probe of the charm content of the proton*, Nucl. Phys. B **814**, 142 (2009) [`arXiv:0812.0722 [hep-ph]`].

[22] N. Ya. Ivanov, *How to measure the charm density in the proton at EIC*, in proceedings of 4th Workshop on *Exclusive Reactions at High Momentum Transfer*, ed. by A.Radyushkin, p. 433 (Hackensack, World Scientific, 2011) [`arXiv:1010.5424 [hep-ph]`].

[23] L. G. Almeida, G. Sterman, and W. Vogelsang, *Threshold resummation for the top quark charge asymmetry*, Phys. Rev. D **78**, 014008 (2008) [`arXiv:0805.1885 [hep-ph]`].

[24] N. Dombey, *Scattering of polarized leptons at high energy*, Rev. Mod. Phys. **41**, 236 (1969).

[25] S. I. Alekhin and J. Blumlein, *Mellin representation for the heavy flavor contributions to deep inelastic structure functions*, Phys. Lett. B **594**, 299 (2004) [`hep-ph/0404034`].

[26] A. Yu. Illarionov, B. A. Kniehl and A. V. Kotikov, *Heavy-quark contributions to the ratio F_L/F_2 at low x*, Phys. Lett. B **663**, 66 (2008) [`arXiv:0801.1502 [hep-ph]`].

[27] E. A. Kuraev, L. N. Lipatov and V. S. Fadin, *Multi - Reggeon processes in the Yang-Mills theory*, Sov. Phys. JETP **44**, 443 (1976) [Zh. Eksp. Teor. Fiz. **71**, 840 (1976)].

[28] E. A. Kuraev, L. N. Lipatov and V. S. Fadin, *The Pomeranchuk singularity in nonabelian gauge theories*, Sov. Phys. JETP **45**, 199 (1977) [Zh. Eksp. Teor. Fiz. **72**, 377 (1977)].

[29] I. I. Balitski and L. N. Lipatov, *The Pomeranchuk singularity in Quantum Chromodynamics*, Sov. J. Nucl. Phys. **28**, 822 (1978) [Yad. Fiz. **28**, 1597 (1978)].

[30] V. N. Gribov and L. N. Lipatov, *Deep inelastic ep scattering in perturbation theory*, Sov. J. Nucl. Phys. **15**, 438 (1972) [Yad. Fiz. **15**, 781 (1972)].

[31] Y. L. Dokshitzer, *Calculation of the structure functions for deep inelastic scattering and e^+e^- annihilation by perturbation theory in Quantum Chromodynamics*, Sov. Phys. JETP **46**, 641 (1977) [Zh. Eksp. Teor. Fiz. **73**, 1216 (1977)].

[32] G. Altarelli and G. Parisi, *Asymptotic freedom in parton language*, Nucl. Phys. B **126**, 298 (1977).

[33] H1 Collaboration, A. Aktas et al., *Measurement of $F_2^{c\bar{c}}$ and $F_2^{b\bar{b}}$ at high Q^2 using the H1 vertex detector at HERA*, Eur. Phys. J. C **40**, 349 (2005) [`hep-ex/0411046`].

[34] H1 Collaboration, A. Aktas et al., *Measurement of $F_2^{c\bar{c}}$ and $F_2^{b\bar{b}}$ at low Q^2 and x using the H1 vertex detector at HERA*, Eur. Phys. J. C **45**, 23 (2006) [`hep-ex/0507081`].

[35] D. Boer, M. Diehl, R. Milner et al., *Gluons and the quark sea at high energies: Distributions, polarization, tomography*, The EIC Science case: a report on the joint BNL/INT/JLab program, `arXiv:1108.1713 [nucl-th]`.

[36] J. B. Dainton, M. Klein, P. Newman, E. Perez, and F. Willeke, *Deep inelastic electron-nucleon scattering at the LHC*, J. Inst. **1**, P10001 (2006) [`hep-ex/0603016`].

[37] J. P. Leveille and T. Weiler, *Azimuthal dependence of diffractive Ψ and $D\bar{D}$ muoproduction and a test of gluon spin, parity, and $k_\perp$*, Phys. Rev. D **24**, 1789 (1981).

[38] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. Nadolsky, and W. K. Tung, *New generation of parton distributions with uncertainties from global QCD analysis*, JHEP **0207**, 012 (2002) [`hep-ph/0201195`].

[39] H.-L. Lai, M. Guzzi, J. Huston et al., *New parton distributions for collider physics*, Phys. Rev. D **82**, 074024 (2010) [arXiv:1007.2241[hep-ph]].

[40] H. Contopanagos, E. Laenen, and G. Sterman, *Sudakov factorization and resummation*, Nucl. Phys. B **484**, 303 (1997) [hep-ph/9604313].

[41] A. Vogt, *Parton Distributions: Progress and Challenges*, in Proceedings of the 15th International Workshop on *Deep-Inelastic Scattering and Related Subjects (DIS2007)*, edited by G. Grindhammer and K. Sachs, (DESY, Hamburg, 2007), p. 39 [arXiv:0707.4106 [hep-ph]].

[42] R. D. Ball, S. Carrazza, L. Del Debbio et al., *Parton Distribution Benchmarking with LHC Data*, arXiv:1211.5142 [hep-ph].