Studies Related to the CKM angles $\phi_2$ and $\phi_3$ at Belle

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We present a summary of recent measurements of the CKM angles $\phi_2$ and $\phi_3$, performed by the Belle experiment which collects $B\bar{B}$ pairs at the $\Upsilon(4S)$ resonance produced in asymmetric-energy $e^+e^-$ collisions. This includes the first evidence of CP violation in $B^0 \rightarrow a_1^{\pm} \pi$. We also discuss measurements of GLW and ADS observables as well as the first model-independent determination of $\phi_3$ in the GGSZ method.

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1. Introduction

The main goal of the Belle experiment at KEK is to constrain the unitarity triangle for $B$ decays. This allows us to test the Cabibbo-Kobayashi-Maskawa (CKM) mechanism for violation of the combined charge-parity ($CP$) symmetry [1, 2], as well as search for new physics effects beyond the Standard Model (SM). These proceedings give a summary of the experimental status of measurements of the CKM phases $\phi_2$ and $\phi_3$, defined from CKM matrix elements as $\phi_2 \equiv \arg(-V_{td}V_{tb}^*)/(V_{ud}V_{ub}^*)$ and $\phi_3 \equiv \arg(-V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)$.

First-order weak processes (tree) proceeding by $b \to u\bar{u}d$ quark transitions such as $B^0 \to \pi\pi$, $\rho\pi$, $\rho\rho$ and $a_1^\pm\pi$, are directly sensitive to $\phi_2$. In the quasi-two-body approach, CKM angles can be determined by measuring the time-dependent asymmetry between $B^0$ and $\bar{B}^0$ decays [3]. For the decay sequence $Y(4S) \to B_{CP}B_{\text{tag}} \to f_CPf_{\text{tag}}$, where one of the $B$ mesons decays at time $t_{CP}$, to a $CP$ eigenstate $f_CP$, and the other decays at time $t_{\text{tag}}$, to a flavour specific final state $f_{\text{tag}}$, with $q = +1(-1)$ for $B_{\text{tag}} = B^0(\bar{B}^0)$, the decay rate has a time-dependence given by

$$P(\Delta t, q) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_B}\left[1 + q(\mathcal{A}_{CP}\cos\Delta m_d\Delta t + \mathcal{S}_{CP}\sin\Delta m_d\Delta t)\right],$$

(1.1)

where $\Delta t \equiv t_{CP} - t_{\text{tag}}$ and $\Delta m_d$ is the mass difference between the $B_H$ and $B_L$ mass eigenstates. The parameters, $\mathcal{A}_{CP}$ and $\mathcal{S}_{CP}$, describe direct and mixing-induced $CP$ violation, respectively.

If a single first-order weak amplitude dominates the decay, then we expect $\mathcal{A}_{CP} = 0$ and $\mathcal{S}_{CP} = \sin 2\phi_2$. On the other hand, if second-order loop processes (penguins) are present, then direct $CP$ violation is possible, $\mathcal{A}_{CP} \neq 0$. Additionally, as these loop processes are not directly proportional to $V_{ub}$, our measurement of $\mathcal{S}_{CP}$ does not directly determine $\phi_2$, rather $\mathcal{S}_{CP} = \sqrt{1 - \mathcal{A}_{CP}^2 \sin(2\phi_2 - 2\Delta\phi_2)}$, where $\Delta\phi_2$ is the shift caused by the second order contributions.

A theoretically clean way of accessing $\phi_2$ is through $B^- \to DK^-$ decays where $D$ represents an admixture of $D^0$ and $\bar{D}^0$ states. This is possible through an interference if the $D$ decays to a common final state $|D\rangle = |D^0\rangle + r_B e^{i\theta}|\bar{D}^0\rangle$, where $\theta \equiv \delta_0 \pm \phi_1$ is the relative phase difference between the two processes for $B^+$ and $B^-$ in which $\delta_0$ is the relative strong phase difference in $B$ decays. The quantity $r_B$, is the amplitude ratio $A(B^- \to \bar{D}^0K^-)/A(B^- \to D^0K^-)$, and should be around the order of colour suppression as the two processes are of similar strength in the Cabibbo angle $\lambda$.

2. $B^0 \to a_1^\pm\pi^\mp$

Decays proceeding by first-order $b \to u\bar{u}d$ quark transitions such as $B^0 \to a_1^\pm\pi^\mp$, are sensitive to $\mathcal{S}_{CP} = \sin 2\phi_2$. On the other hand, if second-order loop processes are present, then direct $CP$ violation is possible and the measurement of $\phi_2$ is shifted by an amount $\Delta\phi_2$. Despite this, it is possible to recover $\phi_2$ with an $SU(2)$ isospin analysis [4], though many of the branching fractions required to constrain $\phi_2$ are at present unknown. A more practical method at this time is to use the $SU(3)$ related channels $B \to a_1K$ and $B \to K_{L3}\pi$, which are known, to constrain $|\Delta\phi_2|$ [5]. The decay $B^0 \to a_1^\mp\pi^\pm$, is a flavour non-specific final state, so 4 flavour-charge configurations $(q,c)$...
need to be considered. The time-dependence is governed by,
\[
\mathcal{P}(\Delta t, q, c) = (1 + c q \mathcal{S}_\text{CP}) e^{-|\Delta t|/\tau_{_{B}}^{0}} \left\{ 1 + q \left[ (\mathcal{S}_{\text{CP}} + c \Delta \mathcal{S}) \sin \Delta m_{d} \Delta t - (\mathcal{S}_{\text{CP}} + c \Delta \mathcal{S}) \cos \Delta m_{d} \Delta t \right] \right\},
\]
where \(c\) is the \(a_{1}\) charge. The parameter \(\mathcal{S}_{\text{CP}}\) measures mixing-induced CP violation, and \(\mathcal{S}_{\text{CP}}\) measures flavor-dependent direct CP violation. The quantity \(\Delta \mathcal{S}\) measures the rate asymmetry between the flavor-charge configurations where the \(a_{1}\) does not contain the spectator quark \((\Gamma[B^{0} \rightarrow a_{1} \pi^{-}] + \Gamma[\bar{B}^{0} \rightarrow a_{1} \pi^{+}]\), and where it does contain the spectator quark \((\Gamma[B^{0} \rightarrow a_{1} \pi^{-}] + \Gamma[\bar{B}^{0} \rightarrow a_{1} \pi^{+}]\), while \(\Delta \mathcal{S}\) is related to the strong phase difference between these two processes.

Belle has recently released its final CP violation measurement in this channels with 772 million \(B\bar{B}\) pairs [6], shown in Fig. 1. They obtain \(\mathcal{S}_{\text{CP}} = -0.51 \pm 0.14\) (stat) \(\pm 0.08\) (syst) which is first evidence of CP violation with a 3.1\(\sigma\) significance.

![Figure 1: Fit results for \(B^{0} \rightarrow a_{1}^{+} \pi^{+}\). The left plot shows the 3-pion mass highlighting the \(a_{1}\) (or2) contribution in blue (red). The middle and right plots show the \(\Delta t\) distributions for each flavour tag and its asymmetry, respectively.](image)

3. \(\phi_{3}\) with GLW

In the so-called GLW method [7], a theoretically clean measurement of the angle \(\phi_{3}\) can be obtained from the rate and asymmetry measurements of \(B^{-} \rightarrow D_{CP}^{(*)}K^{-}\) decays, where the \(D^{(*)}\) meson decays to CP-even \((D_{CP}^{+})\) and CP-odd \((D_{CP}^{-})\) eigenstates. The method benefits from the interference between the dominant \(b \rightarrow c \bar{u} \bar{s}\) transitions with the corresponding doubly CKM-suppressed \(b \rightarrow u \bar{c} \bar{s}\) transition.

The GLW variables are defined as,
\[
R_{CP^{\pm}} = \frac{\mathcal{B}(B^{-} \rightarrow D_{CP^{\pm}}K^{-}) + \mathcal{B}(B^{+} \rightarrow D_{CP^{\pm}}K^{+})}{\mathcal{B}(B^{-} \rightarrow D^{0}K^{-}) + \mathcal{B}(B^{+} \rightarrow D^{0}K^{+})},
\]
\[
A_{CP^{\pm}} = \frac{\mathcal{B}(B^{-} \rightarrow D_{CP^{\pm}}K^{-}) - \mathcal{B}(B^{+} \rightarrow D_{CP^{\pm}}K^{+})}{\mathcal{B}(B^{-} \rightarrow D_{CP^{\pm}}K^{-}) + \mathcal{B}(B^{+} \rightarrow D_{CP^{\pm}}K^{+})},
\]
which are related to \(\phi_{3}\) as
\[
CP\text{-even }D_{CP^{+}}\text{ decays}
R_{CP^{+}} = 1 + \frac{r_{B}^{2}}{1 + 2r_{B} \cos \delta_{B} \cos \phi_{3}},
A_{CP^{+}} = \frac{1 + 2r_{B} \sin \delta_{B} \sin \phi_{3}}{1 + 2r_{B} \cos \delta_{B} \cos \phi_{3}},
\]
\[
CP\text{-odd }D_{CP^{-}}\text{ decays}
R_{CP^{-}} = 1 + \frac{r_{B}^{2}}{1 + 2r_{B} \cos \delta_{B} \cos \phi_{3}},
A_{CP^{-}} = \frac{2r_{B} \sin \delta_{B} \sin \phi_{3}}{1 + 2r_{B} \cos \delta_{B} \cos \phi_{3}},
\]
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This analysis has been performed with the final Belle data set containing 772 million $B\bar{B}$ pairs using the $CP$-even channels $D_{CP}^+ \to \pi^+\pi^-$, $K^+K^-$, and the $CP$-odd channels $D_{CP}^- \to K^0_S\pi^0$, $K^0_S\eta$, shown in Fig. 2. From this measurement, the GLW observables were found to be

$$R_{CP^+} = (7.56 \pm 0.51)\%, \quad R_{CP^-} = (8.29 \pm 0.63)\%,$$
$$A_{CP^+} = (+28.7 \pm 6.0)\%, \quad A_{CP^-} = (-12.4 \pm 6.4)\%. \quad (3.3)$$

4. $\phi_3$ with ADS

In the so-called ADS method [8], $B^- \to DK^-$ with $D \to K^+\pi^-$ and the charge conjugate decays are used. Here, the favoured $B$ decay ($b \to c$) followed by the doubly CKM-suppressed $D$ decay interferes with the suppressed $B$ decay ($b \to u$) followed by the CKM-favoured $D$ decay. The relative similarity of the combined decay amplitudes enhances the possible $CP$ asymmetry.

The ADS variables are defined as,

$$R_{DK} = \frac{\mathcal{B}([K^+\pi^-]K^-) + \mathcal{B}([K^-\pi^+]K^+)}{\mathcal{B}([K^-\pi^+]K^-) + \mathcal{B}([K^+\pi^-]K^+)},$$
$$A_{DK} = \frac{\mathcal{B}([K^+\pi^-]K^-) - \mathcal{B}([K^-\pi^+]K^+)}{\mathcal{B}([K^+\pi^-]K^-) + \mathcal{B}([K^-\pi^+]K^+)}, \quad (4.1)$$

which are related to $\phi_3$ as

$$R_{DK} = r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos \phi_3,$$
$$A_{DK} = \frac{2r_B r_D \sin(\delta_B + \delta_D) \sin \phi_3}{R_{DK}}, \quad (4.2)$$

where the amplitude ratio $r_D = A(D^0 \to K^+\pi^-)/A(\bar{D}^0 \to K^+\pi^-)$, and $\delta_D$ is the strong phase difference between the two $D$ amplitudes.

This analysis has been performed with the final Belle data set containing 772 million $B\bar{B}$ pairs using the decays $D \to K^+\pi^-$ [9], $D^{*-0} \to D[K^+\pi^-]\pi^0$, and $D^{*-0} \to D[K^+\pi^-]\gamma$. From this
measurement, the ADS observables were found to be
\[
\mathcal{R}_{DK} = (1.63^{+0.44}_{-0.41} \pm 0.07) \times 10^{-2}, \quad \mathcal{R}_{D^0} = (1.0^{+0.8}_{-0.7} \pm 0.1) \times 10^{-2}, \quad \mathcal{R}_{D^+} = (3.6^{+1.4}_{-1.3} \pm 0.2) \times 10^{-2}, \quad \mathcal{R}_{D^0} = + 0.4^{+1.1}_{-0.7} \pm 0.2, \quad \mathcal{R}_{D^+} = - 0.51^{+0.33}_{-0.29} \pm 0.08.
\]
(4.3)

where the first uncertainty is statistical and the second is systematic.

5. \( B^- \to DK^-, D \to K^0_S \pi^+ \pi^- \)

In the so-called GGSZ method [10], \( B^- \to D^{(*)} K^- \) decays where the D decays to the CP eigenstate \( D \to K^0_S \pi^+ \pi^- \), are used. One can fit the \( D \to K^0_S \pi^+ \pi^- \) Dalitz plot with the matrix element \(|M_{\pm}(m_1^2, m_2^2)|^2 = |f_D(m_1^2, m_2^2) + r_B e^{i(\delta_B \pm \phi)} f_D(m_2^2, m_1^2)|^2\), thereby determining \( \phi_3 \) directly in the fit. The amplitude \( f_D \), which depends on the invariant squared masses \( m_{\pm}(K^0_S \pi^\pm) \) is typically parameterised as the coherent sum of 2-body decays via intermediate resonances and also measured in the fit.

This measurement has been performed previously at Belle using 657 million \( B \bar{B} \) pairs [11]. By combining the results of \( B^- \to DK^- \) and \( D^* \) \( K^- \), where \( D^* \to D \pi^0 \) and \( D^* \), \( \phi_3 = (78^{+12}_{-11} \pm 4 \text{ (stat)} \pm 9 \text{ (syst)} \pm 9 \text{ (model)})^\circ \) was obtained. Note that the dominant systematic uncertainty arises from model dependence in the parameterisation of \( f_D \) which would eventually dominate the total uncertainty at LHCb and the next generation \( B \) factories.

A new method removing the model uncertainty has recently been developed [12] which involves binning the Dalitz plot and working with the measured number of signal events in each bin instead. This can be compared in a \( \chi^2 \) fit with the expected number of events in each bin \( i \),
\[
N_{i} = h_B[K_i + r^2_{B}K_{i-1} + 2\sqrt{K_{i}K_{i-1}}(x_{\pm}c_i + y_{\pm}s_i)],
\]
(5.1)

where \( x_{\pm} = r_B \cos(\delta_B \pm \phi_3) \) and \( y_{\pm} = r_B \sin(\delta_B \pm \phi_3) \) are free parameters in the fit, constraining the phase \( \phi_3 \), and \( h_B \) is a normalisation constant. Here, \( K_i \) is the number of events in bin \( i \) determined from a flavour-tagged sample \( D^{(*)} \to D \pi^\pm \), while \( c_i = (\cos \Delta \delta_B) \) and \( s_i = (\sin \Delta \delta_B) \) are related to average strong phase difference in bin \( i \) and are measured by CLEO [13], but can also be measured at BES-III in the future.
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Figure 4: The left figure shows the optimised binning where the colours represent different bins. The right figure shows the fitted $B^- \to DK^-$ yields determined in each Dalitz plot bin as the data points, while the solid curve shows the expected yield in each bin.

Compared to measuring $|f_D|^2$, a binned analysis reduces the statistical precision of $\phi_3$, but this can be optimised. The advantage of this method is that the optimal binning depends on the model, however $\phi_3$ does not. Studies show that the precision depends strongly on the amplitude behaviour across the bins. Better precision can be achieved when the phase difference between the $D^0$ and $\bar{D}^0$ amplitudes varies as little as possible. The optimised binning was found using the amplitude measured by BaBar [14] and is shown in Fig. 4.

This analysis has been performed with the final Belle data set of 772 million $B\bar{B}$ pairs which obtained a total signal yield of $1176 \pm 43$ events. Following this, the signal yield in the optimised Dalitz plot bins is determined then compared in a $\chi^2$ fit with the expected signal yield given in Eq. 5.1. A significant $CP$ asymmetry can be seen in Fig. 4 which has a 0.4% probability of being a statistical fluctuation.

The parameters $x_\pm$ and $y_\pm$ are determined in the fit, constraining $\phi_3$, $r_B$ and $\delta_B$,

$$\phi_3 = (77.3^{+15.1}_{-14.9} \pm 4.2 \pm 4.3)\degree,$$

$$r_B = 0.145 \pm 0.030 \pm 0.011 \pm 0.011,$$

$$\delta_B = (129.9 \pm 15.0 \pm 3.9 \pm 4.7)\degree,$$

(5.2)

where the first error is statistical, the second systematic and the third is the precision on $c_i$ and $s_i$ from CLEO. This is a promising proof of concept as the precision on $\phi_3$ is comparable to the previous measurement with $B^- \to DK^-$ only.

References

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