

# Studies Related to the CKM angles $\phi_2$ and $\phi_3$ at Belle

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We present a summary of recent measurements of the CKM angles  $\phi_2$  and  $\phi_3$ , performed by the Belle experiment which collects  $B\bar{B}$  pairs at the  $\Upsilon(4S)$  resonance produced in asymmetric-energy  $e^+e^-$  collisions. This includes the first evidence of CP violation in  $B^0\to a_1^\pm\pi$ . We also discuss measurements of GLW and ADS observables as well as the first model-independent determination of  $\phi_3$  in the GGSZ method.

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#### 1. Introduction

The main goal of the Belle experiment at KEK is to constrain the unitarity triangle for B decays. This allows us to test the Cabibbo-Kobayashi-Maskawa (CKM) mechanism for violation of the combined charge-parity (CP) symmetry [1, 2], as well as search for new physics effects beyond the Standard Model (SM). These proceedings give a summary of the experimental status of measurements of the CKM phases  $\phi_2$  and  $\phi_3$ , defined from CKM matrix elements as  $\phi_2 \equiv \arg(-V_{td}V_{tb}^*)/(V_{ud}V_{ub}^*)$  and  $\phi_3 \equiv \arg(-V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)$ .

First-order weak processes (tree) proceeding by  $b \to u\bar{u}d$  quark transitions such as  $B^0 \to \pi\pi$ ,  $\rho\pi$ ,  $\rho\rho$  and  $a_1^\pm\pi$ , are directly sensitive to  $\phi_2$ . In the quasi-two-body approach, CKM angles can be determined by measuring the time-dependent asymmetry between  $B^0$  and  $\bar{B}^0$  decays [3]. For the decay sequence  $\Upsilon(4S) \to B_{CP}B_{Tag} \to f_{CP}f_{Tag}$ , where one of the B mesons decays at time  $t_{CP}$ , to a CP eigenstate  $f_{CP}$ , and the other decays at time  $t_{Tag}$ , to a flavour specific final state  $f_{Tag}$ , with q = +1(-1) for  $B_{Tag} = B^0(\bar{B}^0)$ , the decay rate has a time-dependence given by

$$P(\Delta t, q) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \left[ 1 + q(\mathscr{A}_{CP}\cos\Delta m_d \Delta t + \mathscr{S}_{CP}\sin\Delta m_d \Delta t) \right], \tag{1.1}$$

where  $\Delta t \equiv t_{CP} - t_{\text{Tag}}$  and  $\Delta m_d$  is the mass difference between the  $B_H$  and  $B_L$  mass eigenstates. The parameters,  $\mathcal{A}_{CP}$  and  $\mathcal{S}_{CP}$ , describe direct and mixing-induced CP violation, respectively.

If a single first-order weak amplitude dominates the decay, then we expect  $\mathcal{A}_{CP}=0$  and  $\mathcal{S}_{CP}=\sin 2\phi_2$ . On the other hand, if second-order loop processes (penguins) are present, then direct CP violation is possible,  $\mathcal{A}_{CP}\neq 0$ . Additionally, as these loop processes are not directly proportional to  $V_{ub}$ , our measurement of  $\mathcal{S}_{CP}$  does not directly determine  $\phi_2$ , rather  $\mathcal{S}_{CP}=\sqrt{1-\mathcal{S}_{CP}^2}\sin(2\phi_2-2\Delta\phi_2)$ , where  $\Delta\phi_2$  is the shift caused by the second order contributions.

A theoretically clean way of accessing  $\phi_3$  is through  $B^- \to DK^-$  decays where D represents an admixture of  $D^0$  and  $\bar{D}^0$  states. This is possible through an interference if the D decays to a common final state  $|D\rangle = |D^0\rangle + r_B e^{i\theta} |\bar{D}^0\rangle$ , where  $\theta \equiv \delta_B \pm \phi_3$  is the relative phase difference between the two processes for  $B^+$  and  $B^-$  in which  $\delta_B$  is the relative strong phase difference in B decays. The quantity  $r_B$ , is the amplitude ratio  $A(B^- \to \bar{D}^0K^-)/A(B^- \to D^0K^-)$ , and should be around the order of colour suppression as the two processes are of similar strength in the Cabibbo angle  $\lambda$ .

**2.** 
$$B^0 o a_1^{\pm} \pi^{\mp}$$

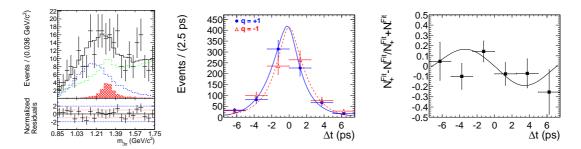
Decays proceeding by first-order  $b \to u\bar{u}d$  quark transitions such as  $B^0 \to a_1^\pm \pi^\mp$ , are sensitive to  $\mathscr{S}_{CP} = \sin 2\phi_2$ . On the other hand, if second-order loop processes are present, then direct CP violation is possible and the measurement of  $\phi_2$  is shifted by an amount  $\Delta\phi_2$ . Despite this, it is possible to recover  $\phi_2$  with an SU(2) isospin analysis [4], though many of the branching fractions required to constrain  $\phi_2$  are at present unknown. A more practical method at this time is to use the SU(3) related channels  $B \to a_1 K$  and  $B \to K_{1A} \pi$ , which are known, to constrain  $|\Delta\phi_2|$  [5]. The decay  $B^0 \to a_1^\pm \pi^\mp$ , is a flavour non-specific final state, so 4 flavour-charge configurations (q,c)

need to be considered. The time-dependence is governed by,

$$\mathscr{P}(\Delta t, q, c) = (1 + c\mathscr{A}_{CP}) \frac{e^{-|\Delta t|/\tau_{B^0}}}{8\tau_{B^0}} \bigg\{ 1 + q \bigg[ (\mathscr{S}_{CP} + c\Delta\mathscr{S}) \sin \Delta m_d \Delta t - (\mathscr{C}_{CP} + c\Delta\mathscr{C}) \cos \Delta m_d \Delta t \bigg] \bigg\}, \tag{2.1}$$

where c is the  $a_1$  charge. The parameter  $\mathscr{S}_{CP}$  measures mixing-induced CP violation, and  $\mathscr{C}_{CP}$  measures flavor-dependent direct CP violation. The quantity  $\Delta\mathscr{C}$  measures the rate asymmetry between the flavor-charge configurations where the  $a_1$  does not contain the spectator quark ( $\Gamma[B^0 \to a_1^+ \pi^-] + \Gamma[\bar{B}^0 \to a_1^- \pi^+]$ ), and where it does contain the spectator quark ( $\Gamma[B^0 \to a_1^- \pi^+] + \Gamma[\bar{B}^0 \to a_1^+ \pi^-]$ ), while  $\Delta\mathscr{S}$  is related to the strong phase difference between these two processes.

Belle has recently released its final CP violation measurement in this channels with 772 million  $B\bar{B}$  pairs [6], shown in Fig. 1. They obtain  $\mathcal{S}_{CP} = -0.51 \pm 0.14$  (stat)  $\pm 0.08$  (syst) which is first evidence of CP violation with a 3.1 $\sigma$  significance.



**Figure 1:** Fit results for  $B^0 \to a_1^{\pm} \pi^{\mp}$ . The left plot shows the 3-pion mass highlighting the  $a_1$  ( $a_2$ ) contribution in blue (red). The middle and right plots show the  $\Delta t$  distributions for each flavour tag and its asymmetry, respectively.

## 3. $\phi_3$ with GLW

In the so-called GLW method [7], a theoretically clean measurement of the angle  $\phi_3$  can be obtained from the rate and asymmetry measurements of  $B^- \to D_{CP}^{(*)}K^{(*)-}$  decays, where the  $D^{(*)}$  meson decays to CP-even  $(D_{CP+}^{(*)})$  and CP-odd  $(D_{CP-}^{(*)})$  eigenstates. The method benefits from the interference between the dominant  $b \to c\bar{u}s$  transitions with the corresponding doubly CKM-suppressed  $b \to u\bar{c}s$  transition.

The GLW variables are defined as,

$$R_{CP\pm} = \frac{\mathscr{B}(B^- \to D_{CP\pm}K^-) + \mathscr{B}(B^+ \to D_{CP\pm}K^+)}{\mathscr{B}(B^- \to D^0K^-) + \mathscr{B}(B^+ \to \bar{D}^0K^+)},$$

$$A_{CP\pm} = \frac{\mathscr{B}(B^- \to D_{CP\pm}K^-) - \mathscr{B}(B^+ \to D_{CP\pm}K^+)}{\mathscr{B}(B^- \to D_{CP\pm}K^-) + \mathscr{B}(B^+ \to D_{CP\pm}K^+)},$$
(3.1)

which are related to  $\phi_3$  as

$$CP\text{-even }D_{CP+} \text{ decays}$$

$$CP\text{-odd }D_{CP-} \text{ decays}$$

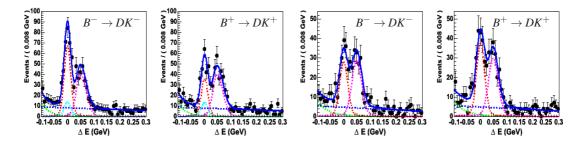
$$R_{CP+} = 1 + r_B^2 + 2r_B \cos \delta_B \cos \phi_3,$$

$$R_{CP-} = 1 + r_B^2 - 2r_B \cos \delta_B \cos \phi_3,$$

$$A_{CP+} = \frac{+2r_B \sin \delta_B \sin \phi_3}{1 + r_B^2 + 2r_B \cos \delta_B \cos \phi_3},$$

$$A_{CP-} = \frac{-2r_B \sin \delta_B \sin \phi_3}{1 + r_B^2 - 2r_B \cos \delta_B \cos \phi_3}.$$

$$(3.2)$$



**Figure 2:** The two left-most plots show the fit results with the CP-even  $D_{CP}$  channels. The red curve shows  $B \to DK$  signal, while the cyan curve shows the charmless  $K^+K^-K^+$  background. The two right-most plots show the fit results with the CP-odd  $D_{CP}$  channels. The pink curve shows the  $B \to D\pi$  background, the green curve shows the  $B\bar{B}$  component and the blue curve shows the continuum contribution.

This analysis has been performed with the final Belle data set containing 772 million  $B\bar{B}$  pairs using the CP-even channels  $D_{CP+} \to \pi^+\pi^-$ ,  $K^+K^-$ , and the CP-odd channels  $D_{CP-} \to K_S^0\pi^0$ ,  $K_S^0\eta$ , shown in Fig. 2. From this measurement, the GLW observables were found to be

$$R_{CP+} = (7.56 \pm 0.51)\%,$$
  $R_{CP-} = (8.29 \pm 0.63)\%,$   $A_{CP+} = (+28.7 \pm 6.0)\%,$   $A_{CP-} = (-12.4 \pm 6.4)\%.$  (3.3)

## 4. $\phi_3$ with ADS

In the so-called ADS method [8],  $B^- \to DK^-$  with  $D \to K^+\pi^-$  and the charge conjugate decays are used. Here, the favoured B decay ( $b \to c$ ) followed by the doubly CKM-suppressed D decay interferes with the suppressed B decay ( $b \to u$ ) followed by the CKM-favoured D decay. The relative similarity of the combined decay amplitudes enhances the possible CP asymmetry.

The ADS variables are defined as,

$$\mathcal{R}_{DK} \equiv \frac{\mathcal{B}([K^{+}\pi^{-}]K^{-}) + \mathcal{B}([K^{-}\pi^{+}]K^{+})}{\mathcal{B}([K^{-}\pi^{+}]K^{-}) + \mathcal{B}([K^{+}\pi^{-}]K^{+})}, 
\mathcal{A}_{DK} \equiv \frac{\mathcal{B}([K^{+}\pi^{-}]K^{-}) - \mathcal{B}([K^{-}\pi^{+}]K^{+})}{\mathcal{B}([K^{+}\pi^{-}]K^{-}) + \mathcal{B}([K^{-}\pi^{+}]K^{+})}, \tag{4.1}$$

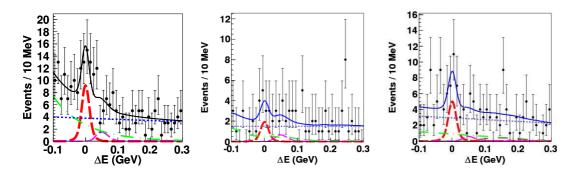
which are related to  $\phi_3$  as

$$\mathcal{R}_{DK} = r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos \phi_3,$$

$$\mathcal{A}_{DK} = \frac{2r_B r_D \sin(\delta_B + \delta_D) \sin \phi_3}{\mathcal{R}_{DK}},$$
(4.2)

where the amplitude ratio  $r_D = A(D^0 \to K^+\pi^-)/A(\bar{D}^0 \to K^+\pi^-)$ , and  $\delta_D$  is the strong phase difference between the two D amplitudes.

This analysis has been performed with the final Belle data set containing 772 million  $B\bar{B}$  pairs using the decays  $D \to K^+\pi^-$  [9],  $D^{*0} \to D[K^+\pi^-]\pi^0$ , and  $D^{*0} \to D[K^+\pi^-]\gamma$ . From this



**Figure 3:** Fit results showing the suppressed  $D_{\text{Sup}} \to K^+\pi^-$  for  $B^- \to DK^-$  (left),  $B^- \to D^{*0}[D\pi^0]K^-$  (middle), and  $B^- \to D^{*0}[D\gamma]K^-$  (right). The red curve shows the signal, the magenta curve the  $D^{(*0)}\pi$  component, the green curve the  $B\bar{B}$  background and the blue curve the continuum.

measurement, the ADS observables were found to be

$$\mathcal{R}_{DK} = (1.63^{+0.44}_{-0.41}^{+0.07}) \times 10^{-2}, \quad \mathcal{R}_{D\pi^0} = (1.0^{+0.8}_{-0.7}^{+0.8}^{+0.1}) \times 10^{-2}, \quad \mathcal{R}_{D\gamma} = (3.6^{+1.4}_{-1.2} \pm 0.2) \times 10^{-2},$$

$$\mathcal{A}_{DK} = -0.39^{+0.26}_{-0.28}^{+0.04}, \quad \mathcal{A}_{D\pi^0} = +0.4^{+1.1}_{-0.7}^{+1.1}^{+0.2}, \quad \mathcal{A}_{D\gamma} = -0.51^{+0.33}_{-0.29} \pm 0.08.$$

$$(4.3)$$

where the first uncertainty is statistical and the second is systematic.

**5.** 
$$B^- \to DK^-, D \to K_S^0 \pi^+ \pi^-$$

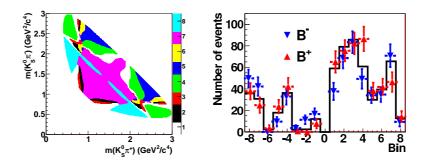
In the so-called GGSZ method [10],  $B^- \to D^{(*)}K^-$  decays where the D decays to the CP eigenstate  $D \to K_S^0 \pi^+ \pi^-$ , are used. One can fit the  $D \to K_S^0 \pi^+ \pi^-$  Dalitz plot with the matrix element  $|\mathscr{M}_{\pm}(m_+^2, m_-^2)|^2 = |f_D(m_+^2, m_-^2) + r_B e^{i(\delta_B \pm \phi_3)} f_D(m_-^2, m_+^2)|^2$ , thereby determining  $\phi_3$  directly in the fit. The amplitude  $f_D$ , which depends on the invariant squared masses  $m_{\pm}(K_S^0 \pi^{\pm})$  is typically parametrised as the coherent sum of 2-body decays via intermediate resonances and also measured in the fit.

This measurement has been performed previously at Belle using 657 million  $B\bar{B}$  pairs [11]. By combining the results of  $B^- \to DK^-$  and  $D^*K^-$ , where  $D^* \to D\pi^0$  and  $D\gamma$ ,  $\phi_3 = (78^{+11}_{-12} \text{ (stat)} \pm 4 \text{ (syst)} \pm 9 \text{ (model)})^\circ$  was obtained. Note that the dominant systematic uncertainty arises from model dependence in the parametrisation of  $f_D$  which would eventually dominate the total uncertainty at LHCb and the next generation B factories.

A new method removing the model uncertainty has recently been developed [12] which involves binning the Dalitz plot and working with the measured number of signal events in each bin instead. This can be compared in a  $\chi^2$  fit with the expected number of events in each bin i,

$$N_i^{\pm} = h_B[K_i + r_B^2 K_{-i} + 2\sqrt{K_i K_{-i}} (x_{\pm} c_i + y_{\pm} s_i)], \tag{5.1}$$

where  $x_{\pm} = r_B \cos(\delta_B \pm \phi_3)$  and  $y_{\pm} = r_B \sin(\delta_B \pm \phi_3)$  are free parameters in the fit, constraining the phase  $\phi_3$ , and  $h_B$  is a normalisation constant. Here,  $K_i$  is the number of events in bin i determined from a flavour-tagged sample  $D^{*\pm} \to D\pi^{\pm}$ , while  $c_i = \langle \cos \Delta \delta_D \rangle_i$  and  $s_i = \langle \sin \Delta \delta_D \rangle_i$  are related to average strong phase difference in bin i and are measured by CLEO [13], but can also be measured at BES-III in the future.



**Figure 4:** The left figure shows the optimised binning where the colours represent different bins. The right figure shows the fitted  $B^- \to DK^-$  yields determined in each Dalitz plot bin as the data points, while the solid curve shows the expected yield in each bin.

Compared to measuring  $|f_D|^2$ , a binned analysis reduces the statistical precision of  $\phi_3$ , but this can be optimised. The advantage of this method is that the optimal binning depends on the model, however  $\phi_3$  does not. Studies show that the precision depends strongly on the amplitude behaviour across the bins. Better precision can be achieved when the phase difference between the  $D^0$  and  $\bar{D}^0$  amplitudes varies as little as possible. The optimised binning was found using the amplitude measured by BaBar [14] and is shown in Fig. 4.

This analysis has been performed with the final Belle data set of 772 million  $B\bar{B}$  pairs which obtained a total signal yield of  $1176\pm43$  events. Following this, the signal yield in the optimised Dalitz plot bins is determined then compared in a  $\chi^2$  fit with the expected signal yield given in Eq. 5.1. A significant CP asymmetry can be seen in Fig. 4 which has a 0.4% probability of being a statistical fluctuation.

The parameters  $x_{\pm}$  and  $y_{\pm}$  are determined in the fit, constraining  $\phi_3$ ,  $r_B$  and  $\delta_B$ ,

$$\phi_3 = (77.3^{+15.1}_{-14.9} \pm 4.2 \pm 4.3)^{\circ},$$

$$r_B = 0.145 \pm 0.030 \pm 0.011 \pm 0.011,$$

$$\delta_B = (129.9 \pm 15.0 \pm 3.9 \pm 4.7)^{\circ},$$
(5.2)

where the first error is statistical, the second systematic and the third is the precision on  $c_i$  and  $s_i$  from CLEO. This is a promising proof of concept as the precision on  $\phi_3$  is comparable to the previous measurement with  $B^- \to DK^-$  only.

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