PROCEEDINGS OF SCIENCE

Universality in soft hadron spectra in pp and e^+e^- collisions

Olga Driga[*][†]
Laboratory SUBATECH, Université de Nantes, France
E-mail: `driga@itep.ru`

The success in understanding dynamics of soft particles inside jets relies on calculations of perturbative QCD, impressive accuracy of which has been widely tested experimentally. However, the theory predicts that inclusive particle spectra inside jets as well as hadron spectra themselves are independent of primary collision energy in the *limiting case of zero particle momentum*, $p \to 0$. In order to test this *universality*, we study world experimental data from pp and $p\bar{p}$ colliders at the collision energies from 200 GeV up to the TeV scale at the LHC. To this end, we elaborate a novel approach by considering extrapolation to zero particle momentum of *ratios* of inclusive charged particle spectra at different collision energies. The ratios exhibit linear behavior with momentum, starting from approximately 0.6 GeV/c, while the extrapolation to zero momentum gives unity. This confirms with accuracy of about 10% that, indeed, the universality for hadron colliders takes place.

At the same time, we demonstrate that for particle momenta below 0.6 GeV/c experimental data break the linear tendency and show up a new behavior.

36th International Conference on High Energy Physics
4-11 July 2012
Melbourne, Australia

---

[*] Speaker.

[†] I am greatly indebted to Valeri Khoze and Wolfgang Ochs for suggesting me the subject of current studies and for their advices and illuminating discussions.

Throughout various stages of completion of this study, I have greatly benefited from discussions with Yuri Kharlov and Maxim Konyushikhin. I am greatly indebted to them for sharing their insights, offering advice, maintaining interest and providing encouragement.

I am also indebted to Yuri Dokshitzer and Andrei Smilga for illuminating discussions.

Finally, let me also thank the organizing committee of 36th International Conference on High Energy Physics for giving me the ability to attend this conference.

PoS(ICHEP2012)271

1. Universality in soft hadron production

Studying multihadron production in relativistic collisions may contribute valuable information about characteristics of partonic branching processes of QCD. These processes originate from the gluon bremsstrahlung and are mathematically described by a sum of infinite number of Feynman graphs. The latter becomes singular (logarithmically large) in the limit of soft emission. It turns out that, driven by the so called *color coherence* phenomenon, large logarithmic terms can be summed up. This result is well understood qualitatively as well as quantitatively [1, 2, 3]. Its experimental confirmation was one of the greatest successes which resulted in dramatic revision of theoretical expectations in soft hadron physics.

A measurement of momentum spectra of particles inside jets is one of basic distributions widely used in experiments to test theoretical predictions for multihadron production in relativistic collisions. Such a measurement, however, is rather difficult to perform. There is yet another prediction that is remarkably simpler. This is inclusive hadron production spectra $E\frac{d^3n}{dp^3}$ in the limit of zero momentum, where E is particle energy, p is particle momentum. In the limit of low momentum, the Born term dominates in the perturbative expansion of a partonic cascade. It is given by

$$E\frac{d^3n}{dp^3} \sim C\,\frac{\alpha_S(p_\perp)}{p_\perp^2}, \qquad p \to 0, \quad p_\perp \to 0, \tag{1.1}$$

where the transverse momentum $p_\perp$ is taken with respect to a collision axis, α_S is running strong coupling constant, while C is a color factor relevant for a minimal partonic process. The latter follows from the fact that a radiated long-wavelength gluon cannot resolve structure of the collision so that its radiation intensity is sensitive only to the total color charge of the underlying process.

The formula (1.1) is remarkable. It shows that at very low momentum inclusive hadron spectra are *independent of the collision energy* $\sqrt{s}$ [4]. This universality was studied in Refs. [4, 5, 6], where the quantity $I_0 = \lim_{p\to 0} E\frac{d^3n}{dp^3}$ for production spectra of different hadrons in e^+e^- and pp experiments was considered. In fact, insufficient number of data points at low momentum combined with steeply falling shapes of the spectra results in ambiguity in choice of a fitting function. Consequently, the extrapolation to the zero momentum point have large uncertainties.

For a verification of the universality of soft hadron spectra, we elaborate a different approach [7]. To this end, instead of fitting the spectra themselves, we calculate their ratios measured by same experiment at different collision energies $\sqrt{s}$. As will become clear soon, such an approach has huge advantage: *whereas the spectra themselves are steeply falling, their ratios change slowly with momentum and exhibit linear behavior in a wide range.* Thus, the ratios of the spectra can be easily and reliably fitted and extrapolated to the zero momentum. The universality takes place if the quantity [1]

$$R(p) = \frac{E\frac{d^3n}{dp^3}(\sqrt{s_1}, p)}{E\frac{d^3n}{dp^3}(\sqrt{s_2}, p)} \tag{1.2}$$

equals to unity at zero momentum:

$$\lim_{p\to 0} R(p) = 1. \tag{1.3}$$

---

[1] By the momentum p here we mean, in fact, the transverse momentum p_T. As long as a corresponding rapidity region is finite, the limit of zero momentum coincides with the limit of zero transverse momentum.

Let us also remark that making ratios of soft hadron spectra of one and the same experiment has two additional advantages:

- the spectra of one experiment are given with same binning parameters over the momentum, so that their ratios can be built as point-by-point ratios;
- several systematic errors and uncertainties of an original measurement cancel (e.g. systematic error of total cross-section measurement).

In practice, the latter, however, means that, for publicly available data, we can exclude only the error of total cross-section measurement, which is usually of about 10%.

2. Ratios of inclusive charged particle spectra and tests of the universality

In what follows, we took the data for *inclusive charged particle spectra* for the following experiments:

- UA1 experiment, $p\bar{p}$ at $\sqrt{s} = 200$, 500 and 900 GeV [8];
- CDF experiment, $p\bar{p}$ at $\sqrt{s} = 630$ and 1800 GeV [9];
- CMS experiment, pp at $\sqrt{s} = 900$ GeV and 7 TeV [10];
- CMS experiment, pp at $\sqrt{s} = 900$ GeV and 7 TeV, older papers [11, 12]; corresponding values will be marked with “prev.” suffix;
- ALICE experiment, $\sqrt{s} = 900$ GeV, 2.76 TeV and 7 TeV [13, 14].

The UA1 and the CDF data points are given in $E\frac{d^3\sigma}{dp^3}$ format. Before making the ratios, we scaled them by a factor $1/\sigma_{\mathrm{inel}}$,

$$E\frac{d^3n}{dp^3} = \frac{1}{\sigma_{\mathrm{inel}}} \times E\frac{d^3\sigma}{dp^3}, \tag{2.1}$$

where total inelastic cross-sections are given in Table 1.

Table 1: Total cross-sections for the UA1 and the CDF experiments.

Exp	$\sqrt{s}$ [GeV]	σ_{tot} [mb]	σ_{inel} [mb]
UA1	200	52	43 ± 4
UA1	500	62	49 ± 2
CDF	630	63	50 ± 2
UA1	900	68	53 ± 4
CDF	1800	74	57 ± 3

The ratios are drawn in Figs. 1–4, with their systematic and statistical errors added in quadrature.

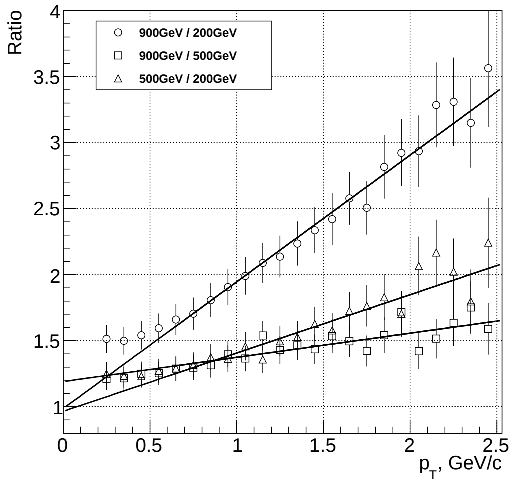

Figure 1: The UA1 ratios.

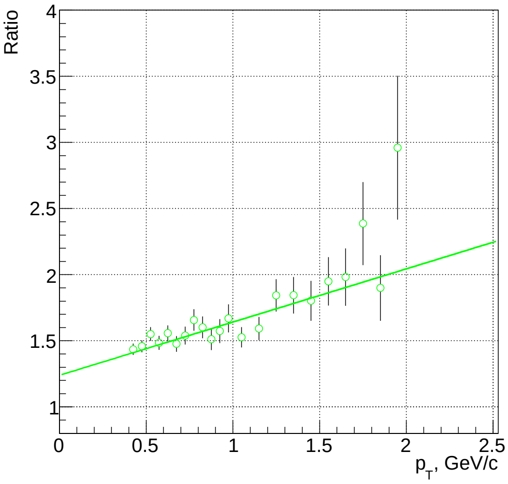

Figure 2: The CDF ratios.

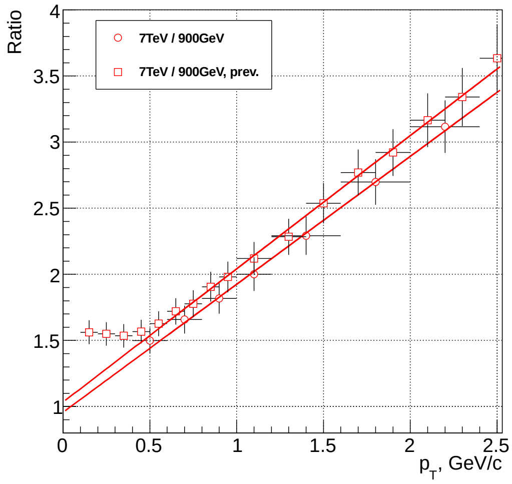

Figure 3: The CMS ratios.

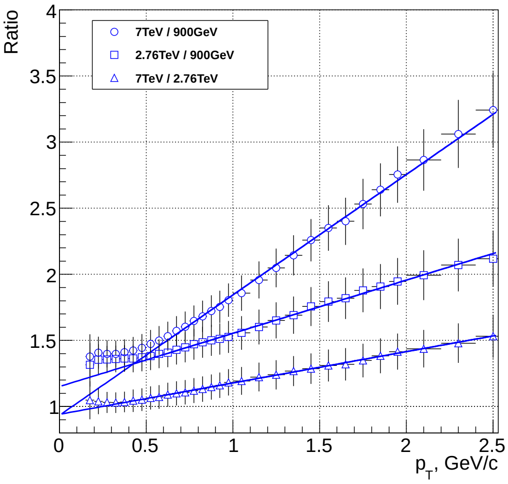

Figure 4: The ALICE ratios.

As one can see, all the ratios, surprisingly, exhibit *linear behavior* with momentum, starting from approximately 0.6 GeV/c. On the figures, we additionally fitted the ratios with linear functions in the region 0.6–2.5 GeV/c. The tendency of the points below 0.6 GeV/c differs from the linear behavior above 0.6 GeV/c and do not go to unity. At the same time, the linear behavior of the points above 0.6 GeV/c does extrapolate well to unity at zero momentum, which is clearly visible from the fits. Table 2 summarizes the fitting parameters of the linear fits

$$R(p_T) = R_0 + Bp_T. \tag{2.2}$$

As one can see from the table, the values of the R_0 parameter, which is nothing else than the extrapolated value at zero momentum, are indeed very close to unity. The average error is about 10%. The biggest deviations from unity present in the ratios with small slopes B which correspond

Table 2: Parameters of linear fits $R(p_T) = R_0 + Bp_T$.

Exp	$\sqrt{s_1}$ [GeV]	$\sqrt{s_2}$ [GeV]	$\sqrt{s_1}/\sqrt{s_2}$	ratio at zero momentum, R_0	slope, B [GeV^{-1}]
UA1	900	500	1.8	1.19 ± 0.08	0.18 ± 0.05
UA1	500	200	2.5	0.96 ± 0.09	0.44 ± 0.07
ALICE	7000	2760	2.54	0.94 ± 0.06	0.24 ± 0.05
CDF	1800	630	2.86	1.24 ± 0.08	0.40 ± 0.08
ALICE	2760	900	3.07	1.15 ± 0.08	0.40 ± 0.06
UA1	900	200	4.5	0.99 ± 0.11	0.96 ± 0.09
CMS	7000	900	7.78	0.96 ± 0.20	0.97 ± 0.17
CMS, prev.	7000	900	7.78	1.03 ± 0.11	1.01 ± 0.09
ALICE	7000	900	7.78	0.93 ± 0.09	0.91 ± 0.07

to small ratios $\sqrt{s_1}/\sqrt{s_2}$. Thus, one can conclude that, indeed, to the extent of experimental uncertainties, the universal behavior of the considered spectra – the independence of the collision energy $\sqrt{s}$ – takes place, provided one considers data points above the 0.6 GeV/c threshold. In particular, one concludes that the universality is valid at the LHC energies. Moreover, it is suggested from the table that the slopes of the linear fits depend on ratios of collision energies $\sqrt{s}_1/\sqrt{s}_2$ and do not depend on the collision energies themselves. The only exception for this rule is seen for the (unofficial) $\sqrt{s} = 2.76$ TeV data of the ALICE experiment.

3. Discussion of the linear behavior

The linear tendency of the spectra ratios in a wide region of momenta is surprising and is not fully understood. Let us show how the linearity in the ratios can be understood "phenomenologically". In the ALICE and the CMS experiments, the spectra are fitted with the Tsallis function, which we, for further convenience, write as

$$f(p_T) = \frac{A}{[1 + p_T/(nT)]^n} \tag{3.1}$$

with three free parameters A, T and n, neglecting any mass parameters. For instance, the values of fitting parameters for the inclusive charged particle spectra of the CMS experiment [11, 12],

$$\begin{array}{lll} \text{900 GeV:} & T = 0.13 \pm 0.01; & n = 7.7 \pm 0.2, \\ \text{7 TeV:} & T = 0.145 \pm 0.005; & n = 6.6 \pm 0.2, \end{array} \tag{3.2}$$

give $\frac{\Delta T}{T} \approx 0.1$, $\frac{\Delta n}{n} \approx 0.2$. Thus, assuming that the parameters T and n are slow functions of collision energy, one expands the ratio $R(p_T)$ into Taylor series with respect to $\frac{\Delta T}{T}$ and $\frac{\Delta n}{n}$:

$$R(p_T) = \frac{f_{7\text{TeV}}(p_T)}{f_{900\text{GeV}}(p_T)} \approx 1 + \frac{\Delta T}{T^2} p_T, \qquad \frac{\Delta T}{T} \ll 1, \quad \frac{\Delta n}{n} \ll 1, \quad p_T \ll \frac{T^2}{\Delta T}. \tag{3.3}$$

Additionally, we assumed that $p_T/(nT) \ll 1$ and omitted the difference between $A_{7\text{TeV}}$ and $A_{900\text{GeV}}$ since their ratio is close to unity.

4. Conclusion

In this study, we elaborated simple and efficient method of particle spectra ratios and showed with accuracy of 10% that particle spectra in pp and $p\bar{p}$ collisions are independent of collision energy in the limiting case of zero momentum. The observed linearity of the ratios above particle transverse momentum 0.6 GeV/c is related to slow evolution of parameters of the spectra with collision energy. In addition, we were able to observe the breaking of the linear tendency at momentum $p_T < 0.6$ GeV/c. The breaking tendency below 0.6 GeV/c is not surprising, since theoretical predictions were made in the approximation of zero masses of all participating partons. If quarks were massless, so as pions, the universality should extend to the zero momentum. Due to nonzero quark masses, this is not the case.

The latter suggests that, in practice, instead of considering the extrapolation of the fits of hadron spectra to zero momentum, it is more appropriate to consider the ratios of those fits taken in a region of momentum starting from about 0.6 GeV/c and linearly extrapolate the ratios to zero momentum point. Moreover, the discovered “mass effects” should be studied with inclusive spectra of identified pions, kaons and protons.

Note also that, as a by-product, a linear fit of a ratio of two spectra allows one to determine the difference in “temperatures” ΔT with a better precision compared to the value which can be extracted as the difference of the temperatures of the Tsallis fits themselves.

References

[1] Y. L. Dokshitzer, V. A. Khoze, A. H. Mueller and S. I. Troian, *“Basics of perturbative QCD”*, Gif-sur-Yvette, France: Ed. Frontieres (1991) 274 p.

[2] V. A. Khoze and W. Ochs, Int. J. Mod. Phys. A **12**, 2949 (1997) [hep-ph/9701421]; V. A. Khoze and W. Ochs, J. Phys. G G **28**, 895 (2002) [hep-ph/0110295]. V. A. Khoze, W. Ochs and J. Wosiek, In *Shifman, M. (ed.): At the frontier of particle physics, vol. 2*, 1101-1194 [hep-ph/0009298].

[3] Y. I. Azimov, Y. L. Dokshitzer, V. A. Khoze and S. I. Troyan, Z. Phys. C **27**, 65 (1985).

[4] W. Ochs, V. A. Khoze and M. G. Ryskin, Eur. Phys. J. C **68**, 141 (2010) [arXiv:1003.2127 [hep-ph]].

[5] W. Ochs, V. A. Khoze and M. G. Ryskin, arXiv:1011.5649 [hep-ph].

[6] W. Ochs, Acta Phys. Polon. B **41**, 2839 (2010) [arXiv:1011.2422 [hep-ph]].

[7] O. Driga, “Analysis of the ALICE calorimeter data and soft hadron production in high energy collisions”, PhD thesis (2012).

[8] C. Albajar *et al.* [UA1 Collaboration], Nucl. Phys. B **335**, 261 (1990).

[9] F. Abe *et al.* [CDF Collaboration], Phys. Rev. Lett. **61**, 1819 (1988).

[10] S. Chatrchyan *et al.* [CMS Collaboration], JHEP **1108**, 086 (2011) [arXiv:1104.3547 [hep-ex]].

[11] V. Khachatryan *et al.* [CMS Collaboration], JHEP **1002**, 041 (2010) [arXiv:1002.0621 [hep-ex]].

[12] V. Khachatryan *et al.* [CMS Collaboration], Phys. Rev. Lett. **105**, 022002 (2010) [arXiv:1005.3299 [hep-ex]].

[13] K. Aamodt *et al.* [ALICE Collaboration], Phys. Lett. B **693**, 53 (2010) [arXiv:1007.0719 [hep-ex]].

[14] K. Aamodt *et al.* [ALICE Collaboration], unpublished.