We briefly review the existing $\psi(2S)$ data taken at RHIC, the Tevatron and the LHC. We systematically compare them with colour-singlet-model predictions as a function of the center-of-mass energy, of the quarkonium rapidity and of the quarkonium transverse momentum. The overall agreement is good except for large transverse momenta. This points at the existence of large NNLO corrections or at colour-octet dominance.
1. Introduction

The physics of quarkonium production in high-energy hadron collisions has triggered much investigation and debate since the preliminary data of prompt $\psi(2S)$ production at the Tevatron were released by the CDF collaboration in 1994 [1]. It uncovered an obvious discrepancy between the measured $P_T$ spectrum and the one predicted by the simple application of pQCD, dubbed the colour-singlet model (CSM): the spectrum of the data was harder with a clear excess of the $\psi(2S)$ at large $P_T$. This observation was confirmed in the final publication in 1997 [2] and similar observations were also made for the $J/\psi$ — once the feed-down from the $\chi_c$ could be subtracted [3] — and for the $\Upsilon$ family [4] — albeit to a less extent.

Until not too long ago, the most popular explanation of these discrepancies was provided by an enhancement of the yield due to non-perturbative transitions between the colour-octet state produced at short distances and the colour-singlet mesonic states (for reviews see [5]). However, it is clear nowadays that one cannot neglect the $\alpha_5^S$ and $\alpha_5^S$ corrections to the CSM [6] if one wants to confront the prediction of the CSM with the data for the $P_T$ spectrum of $J/\psi$ and $\Upsilon$ produced in high-energy hadron collisions [7, 8, 9, 10, 11]. In fact, this indicates that a factorised description of high-$P_T$ quarkonia beyond leading power in $m_Q^2/P_T^2$ [12] may be more suitable than NRQCD [13].

QCD corrections also have a dramatic impact on the predictions of polarisation observables. At LO, the $J/\psi$ and $\Upsilon$ inclusively produced are predicted to be predominantly transversely polarised — in the helicity frame —, whereas they are predicted to be longitudinally polarised at NLO at large $P_T$ [9, 10]. This is also true for instance when they are produced in association with a photon [14, 15]. The sole known case where the quarkonium polarisation is not altered by NLO corrections is when they are produced with a $Z$-boson [16].

On the other hand, in recent works [17, 18], we have shown that the CSM alone is sufficient to account for the magnitude of the $P_T$-integrated cross section and its dependence in rapidity, $d\sigma/dy$, at RHIC, Tevatron and LHC energies. In other words, there is no need for additional contributions at low $P_T$ and the energy dependence is well reproduced.

We consider here the $\sqrt{s}$, $P_T$ and $y$ dependences of the $\psi(2S)$ yield at various colliders. We present various comparisons between the existing data and the yield at LO, NLO and sometimes including some dominant contributions at $\alpha_5^S$ (NNLO$^*$) when possible. It should be stressed that the NLO code for the CSM at high energies and low $P_T$ is not stable likely owing to large contributions of the loop corrections which can become negative at low $P_T$. NLO and NNLO$^*$ are thus only computed for sufficiently large $P_T$.

After having briefly described how the $\psi(2S)$ cross sections are evaluated in the CSM, we compare them with the existing data. As we shall see, the overall agreement is good except for large transverse momenta.

2. Cross-section evaluation in the CSM

Details about the evaluation of quarkonium production cross section at LO in the CSM can be found in [6]. As regards the evaluation of the yield at NLO accuracy, we have used the code of Campbell, Maltoni and Tramontano [7]. In the CSM, the cross section to produce a $\psi(2S)$ is obtained along the same lines as for the $J/\psi$ ground state with the mere change of the value of
the wave function at the origin $R(0)$, which is meant to account for all the non-perturbative and relativistic effects. For the $J/\psi$, $|R(0)|^2$ can be taken as 1.01 GeV$^3$; here, we took 0.67 GeV$^3$ for the $\psi(2S)$ [17].

The expected impact of NNLO QCD corrections for increasing $P_T$ is investigated by evaluating what we call the NNLO* yield. A complete discussion can be found in [10, 11]. In a few words, we anticipate that the NNLO* yield encompasses the kinematically-enhanced topologies which open up at $\alpha_3^5$ with a $P_T^{-4}$ fall off of $d\sigma/dP_T^2$. Let us emphasise that we do not foresee significant modifications of the $P_T$ dependence at N$^3$LO and further. At NNLO, the $P_T^{-4}$ fall-off of the new NNLO topologies is the slowest possible. Above $\alpha_3^5$, the common wisdom about the decreasing impact of further QCD corrections should then hold. One expects a $K$ factor multiplying the NNLO yield to be independent of $P_T$ and to be of the order of unity. A further enhancement between the NNLO and N$^3$LO results by an order of magnitude would be the sign that the expansion in $\alpha_s$ does not converge and would force us to reconsider our understanding of quarkonium production.

As in [10, 11], the NNLO* yield is evaluated thanks to a slightly tuned version of the automated code MADONIA [19]. The uncertainty bands at LO and NLO are obtained from the combined variations of the charm-quark mass ($m_c = 1.5 \pm 0.1$ GeV), the factorisation $\mu_F$ and the renormalisation $\mu_R$ scales chosen in the couples $((0.75,0.75); (1,1); (1,2); (2,1); (2,2)) \times m_T$ with $m_T^2 = 4m_D^2 + P_T^2$. The band for the NNLO* is obtained using a combined variation of $m_c$ ($m_c = 1.5 \pm 0.1$ GeV), of the scales ($0.5m_T < \mu_R = \mu_F < 2m_T$) and of $s_{ij}^{\text{min}}$, the infrared cut-off on the invariant mass of any pair of any light partons in the reaction $(2.25 < s_{ij}^{\text{min}} < 9.00 \text{GeV}^2)$. For the parton densities, we have used the LO set CTEQ6.L and the NLO set CTEQ6.M [20] and have taken $|R_{\psi(2S)}(0)|^2 = 0.67 \text{ GeV}^3$ and $\text{Br}(\psi(2S) \rightarrow \ell^+ \ell^-) = 0.0075$.

3. Result and comparison with existing data

3.1 $P_T$-integrated yields

Now, we compare the $\sqrt{s}$ dependence of $d\sigma/dy|_{y=0}$ obtained at LO in the CSM and that obtained from the PHENIX data ($\sqrt{s} = 200$ GeV) [21], the CDF data ($\sqrt{s} = 1960$ GeV) [22] and rescaled LHCb data ($\sqrt{s} = 7000$ GeV) [23]. To be precise, the CDF data only$^1$ covers the region $P_T > 2$ GeV, for which $\sigma(|y| < 0.6, P_T > 2 \text{ GeV}) \times \text{Br} = 2.6 \pm 0.1$ nb. We have assumed the same $P_T$ dependence as the inclusive $J/\psi$ for $P_T < 2$ GeV [25] and we have obtained $\sigma(|y| < 0.6, P_T < 2 \text{ GeV}) \times \text{Br} = 2.1 \pm 0.1$ nb. In addition, we have rescaled the forward LHCb data assuming the same $y$ dependence as the LO CSM, i.e. using a factor 1.5 for $d\sigma/dy|_{y=0}/(d\sigma/dy)|_{2.0<y<4.5}$.

As one has obtained for the $J/\psi$ and the $Y(nS)$ [18], one does not observe in Fig. 1 any surplus with respect to the CSM predictions. In fact, at high energy, the LO yield tends to be above the experimental data.

As mentioned above, we are unfortunately not able to give, for the time being, corresponding predictions at NLO since these are not well behaved at high energies. This is maybe due to the contributions of the loop corrections which change sign for $P_T$ of the order of the quark masses. This calls for the resummation of initial state radiations (ISR). Yet, at RHIC energies, we have observed in [17] that the NLO yield lies in the lower range of the LO uncertainties (see Fig. 2a).

$^1$On the way, it is instructive to keep in mind that 45% of the $J/\psi$ yield lies below $P_T = 2$ GeV.
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Figure 1: Data vs LO CSM for the $P_T$-integrated $\psi(2S)$ production cross section at $y = 0$ as a function of the cms energy. Data are from [21, 22, 23].

Figure 2: Data vs (N)LO CSM for the $P_T$-integrated direct-quarkonium production cross section as a function of the $\psi(2S)$ rapidity. Data are from [21, 23].

It is therefore sound to expect that the slight overestimation of the LO with respect to the present data would be reduced at NLO once ISR can be resummed and stable results can be presented. Fig. 2a and Fig. 2b shows the rapidity dependence of the CSM predictions at Tevatron and LHC energies. Note that, for now, there is no measurement of the $\psi(2S)$ yield for low enough $P_T$ to derive a $P_T$-integrated cross section for $y \simeq 0$ at the LHC.

3.2 $P_T$-differential yields

Now, we move onto the discussion of the $P_T$ dependence of the cross section. As we discussed in the introduction, it is clear that the LO CSM cannot be sufficient since the leading (and even sub-leading) $P_T$ topologies are missing. Except for low $P_T$, where the results are not stable, the NLO yield can nowadays be computed easily and compared to the data. It is shown at RHIC and Tevatron energies in Fig. 3a and Fig. 3b; one clearly sees a different $P_T$ dependence with respect to the LO yield. It is clearly harder and the discrepancy with the experimental data is reduced, although it is still significant when $P_T$ gets large.

Indeed, at large $P_T$, one expects the $P_T^2$ kinematic enhancement of some NNLO topologies to take over the extra $\alpha_s$ suppression compared to the NLO. This is why the NNLO$^*$ bands in Fig. 3a-3d show a $P_T$ spectrum which is even harder and much closer to the data. Yet, we note a small gap
between the upper limit of the NNLO* band and the data at the largest available $P_T$ in Fig. 3b-3d.

If the CSM at NNLO is indeed the physically relevant production mechanism at large $P_T$, namely that the charmonia are produced at large $P_T$ along with two hard partons (in a sense, two jets), the corresponding theoretical predictions would be involved. Even with a full NNLO, one would have to face large theoretical uncertainties due to the factorisation scale through five powers of $\alpha_s$. In the coming years, it is not clear that one could make precise and definitive comparisons between data and theory as far as the $P_T$ dependence of the yield is concerned. It may thus be more fruitful to also analyse new observables such as associated production.

Figure 3: $\psi(2S)$ data vs CSM predictions at various orders for the $P_T$ dependence of the differential cross section at RHIC, Tevatron and LHC energies. Data are from [21, 22, 23, 24].

For the time being, the most one can tell from the data-theory comparison in the framework of the CSM and the approximate NNLO* is that below, say $P_T = 15$ GeV, the upper uncertainty band of the NNLO* agrees with the data – the same observation also holds for the $J/\psi$ [11, 26, 27] while, for the $\Upsilon(nS)$, the agreement is clearly much better [10, 28]. Whether this has some physical meaning is a question that would only be answered once one has a full NNLO computation and/or once the LHC experimental collaborations release polarisation measurement for prompt $\psi(2S)$ which would be precise enough to rule out the predictions from either the CSM or from the CO dominance.

4. Conclusion

We have compared the existing data of $\psi(2S)$ production at RHIC, the Tevatron and the LHC
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with predictions from the CSM for the $P_T$, $\sqrt{s}$, $y$ dependences of the yield. The $\sqrt{s}$ and $y$ dependences are well reproduced at LO and NLO, where it is available. It is therefore worth re-investigating [29, 30, 31] the possibilities to constrain gluon PDFs with low $P_T$ quarkonium data.

As regards the $P_T$ differential cross section, the upper bound of the NNLO $^\star$ CSM predictions of the $P_T$ differential cross section is getting closer to the experimental data, especially at mid $P_T$, as previously found for $\Upsilon$ [10, 28] and for $J/\psi$ [11, 26, 27]. However, the NNLO $^\star$ evaluation is not a complete NNLO calculation. It is affected by logarithms of an infrared cut-off and its effect might not vanish with increasing $P_T$ as quickly as one has anticipated. A full NNLO evaluation of the cross section in the CSM is therefore eagerly awaited.

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