## Study of QCD in gamma gamma to pseudoscalar meson pair processes

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We have measured a series of exclusive meson-pair productions in two-photon collision at the Belle experiment. Above around 3 GeV of two-photon invariant mass, the measured cross sections and angular distributions are compared with perturbative and non-perturbative QCD calculations.

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Figure 1: Factorization of the process $\gamma \gamma \rightarrow M M^{\prime}$ by perturbative QCD (left) and Handbag picture (right).

Exclusive meson-pair production in two-photon collision, $\gamma \gamma \rightarrow M M^{\prime}$ provides useful information for study of perturbative and non-perturbative QCD. From theoretical viewpoint, two-photon process is attractive because of the absence of strong interactions in the initial state and the possibility of calculating $\gamma \gamma \rightarrow q \bar{q}$ amplitudes.

Brodsky and Lepage (BL) [山] have computed the amplitude for the $\gamma \gamma \rightarrow M M^{\prime}$ process for the first time (Fig.[ll left). Their perturbative QCD calculation is obtained by factorizing the amplitude into two components,

$$
\begin{equation*}
\mathscr{M}_{\lambda_{1} \lambda_{2}}\left(s, \theta^{*}\right)=\int_{0}^{1} \int_{0}^{1} d x d y \phi_{M}\left(x, Q_{x}\right) \phi_{M^{\prime}}\left(y, Q_{y}\right) T_{\lambda_{1} \lambda_{2}}\left(x, y, \theta^{*}\right) \tag{1}
\end{equation*}
$$

where $\phi_{M}\left(x, Q_{x}\right)$ is a single-meson distribution amplitude for a meson $M$, the probability amplitude for finding valence partons in the meson, each carrying some fraction $x$ of the meson's momentum. $Q_{x}$ is the typical momentum transfer in the process, $\sim \min (x, 1-x) \sqrt{s}\left|\sin \theta^{*}\right|$ with meson scattering angle $\theta^{*}$ in the two-photon c.m.s. By the sum rule the overall normalization is fixed as $\int_{0}^{1} d x \phi_{M}\left(x, Q_{x}\right)=f_{M} / 2 \sqrt{3}$ where $f_{M}$ is the decay constant for a meson $M . T_{\lambda_{1} \lambda_{2}}$ is a hard scattering amplitude for $\gamma_{\lambda_{1}} \gamma_{\lambda_{2}} \rightarrow q \bar{q} q \bar{q}$ with photon helicities $\lambda_{1}$ and $\lambda_{2}$.

For mesons with zero helicity leading term calculation gives the following dependence on $s$ and scattering angle $\theta^{*}$ :

$$
\begin{equation*}
\frac{d \sigma}{d\left|\cos \theta^{*}\right|}=16 \pi \alpha^{2} \frac{\left|F_{M}(s)\right|^{2}}{s}\left\{\frac{\left[\left(e_{1}-e_{2}\right)^{2}\right]^{2}}{\left(1-\cos ^{2} \theta^{*}\right)^{2}}+\frac{2\left(e_{1} e_{2}\right)\left[\left(e_{1}-e_{2}\right)^{2}\right]}{1-\cos ^{2} \theta^{*}} g\left(\theta^{*}\right)+2\left(e_{1} e_{2}\right)^{2} g^{2}\left(\theta^{*}\right)\right\}, \tag{2}
\end{equation*}
$$

where $e_{1}$ and $e_{2}$ are the quark charges. Under the assumption that $\phi_{K}$ and $\phi_{\pi}$ are similar in shape, the differential cross section ratio depends only on the meson decay constants $f_{K}^{4} / f_{\pi}^{4}$ for the charged mode. Benayoun and Chernyak (BC) [】] employ different wave functions for $\phi_{\pi}(x)$ and $\phi_{K}(x)$ taking into account $\mathrm{SU}(3)$ symmetry breaking effects. Next-to-leading order calculation is done by Duplančić et al. [B]].

The Handbag model by Diehl, Kroll and Vogt (DKV) [4] predicted the differential cross section for the $\gamma \gamma \rightarrow M M^{\prime}$ process as

$$
\begin{equation*}
\frac{d \sigma}{d\left|\cos \theta^{*}\right|}\left(\gamma \gamma \rightarrow M M^{\prime}\right)=\frac{8 \pi \alpha^{2}}{s} \frac{1}{\sin ^{4} \theta^{*}}\left|R_{M M^{\prime}}(s)\right|^{2}, \tag{3}
\end{equation*}
$$

where the transition amplitude is expressed as a hard scattering $\gamma \gamma \rightarrow q \bar{q}$ times a form factor
 nitude of the cross sections between various modes, while it does not give absolute magnitude of the cross section.


Figure 2: Angular dependence of the normalized differential cross section for $\gamma \gamma \rightarrow K^{+} K^{-}$and $\pi^{+} \pi^{-}$(left) and $K_{S} K_{S}$ (right). Solid curves are $\sin ^{-4} \theta^{*}$ dependence. Blue curves show prediction by BC.


Figure 3: Angular dependence of the normalized differential cross section: (a) $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$, (c) $\eta \pi^{0}$, and (d) $\eta \eta$. (a): Dotted (solid) curves show $\sin ^{-4} \theta^{*}$ (fit to $\sin ^{-4} \theta^{*}+b \cos \theta^{*}$ ) dependence. $W$ dependence of $b$ is shown in (b). (c): Curves show $\sin ^{-4} \theta^{*}$. (d): Dotted (solid) curves show $\sin ^{-6} \theta^{*}\left(\sin ^{-4} \theta^{*}\right)$ dependence.
 and $\eta \eta$ [ 9$]$ processes. The results are compared with perturbative and non perturbative QCD predictions.

## 1. Angular Dependence of Differential Cross Section

In Equation ( $\mathbb{\square}$ ) the first term is dominant for charged pair mode, and the angular distribution is thus expected to have $\sim \sin ^{-4} \theta^{*}$ dependence. But for neutral pair mode for which the first term vanishes the angular dependence is directly determined by the shape of $g\left(\theta^{*}\right)$ and the value of

Table 1：Angular dependence of differential cross sections in comparison with $\sin ^{-4} \theta^{*}$ dependence．

| mode | $\sin ^{-4} \theta^{*}$ | energy range | $\left\|\cos \theta^{*}\right\|$ range | reference |
| :---: | :---: | :---: | :---: | :---: |
| $\pi^{+} \pi^{-}$ | Match well． | 3．0－4．1 | $<0.6$ | ［ 5 ］ |
| $K^{+} K^{-}$ | Match well． | 3．0－4．1 | $<0.6$ | ［ $\sqrt{\text { ］}}$ |
| $K_{S} K_{S}$ | Consistent． | 2．4－3．3 | $<0.6$ | ［目］ |
| $\pi^{0} \pi^{0}$ | $\sin ^{-4} \theta^{*}+b \cos \theta^{*} \text { better. }$ <br> Approaches $\sin ^{-4} \theta^{*}$ above 3.1 GeV ． | 2．4－4．1 ${ }^{\dagger}$ | $<0.8$ | ［7］ |
| $\eta \pi^{0}$ | Good agreement above 2.7 GeV ． | 3．1－4．1 | $<0.8$ | ［8］ |
| $\eta \eta$ | Poor agreement． <br> $\sin ^{-6} \boldsymbol{\theta}^{*}$ better above 3.0 GeV ． | 2．4－3．3 | $<0.9$ | ［9］ |

$\dagger \chi_{c J}$ region， $3.3-3.6 \mathrm{GeV}$ is excluded．
$F_{M}(s)$ ，which depend on incalculable factor $\phi_{M}$ ．On the other hand，the handbag model predicts $\sin ^{-4} \theta^{*}$ dependence for large $t$ both for charged and neutral meson pairs．

The measured angular dependence are consistent with $\sin ^{-4} \theta^{*}$ around 3 GeV or higher energy region except $\eta \eta$ mode．Figure $\square$ shows the measured angular dependence for $\pi^{+} \pi^{-}, K^{+} K^{-}$，and $K_{S} K_{S}$ ．For $\gamma \gamma \rightarrow \pi^{0} \pi^{0}$ and $\eta \eta, \sin ^{-4} \theta^{*}+b \cos \theta^{*}$ and $\sin ^{-6} \theta^{*}$ dependence，respectively，show better agreement than $\sin ^{-4}$ while $\eta \pi^{0}$ is in agreement with $\sin ^{-4} \theta^{*}$ above 2.7 GeV （Fig．［3）． Comparison with $\sin ^{-4} \theta^{*}$ dependence is summarized in Table $\boldsymbol{m}$ ．

## 2．Energy Dependence of Cross Section and ratio of Cross Sections

It is found that existing calculations do not agree with absolute normalization of the cross sections even with next－to－leading－order term［B］．However，power－low dependence of cross sec－ tion $\sigma_{0} \sim W^{-n}$ and their ratio，summarized in Table $\square$ ，provide useful information to test QCD predictions．

Figure $\mathbb{T}^{(F i g u r e} \sqrt{5}$ ）shows cross sections integrated over sensitive angular region for $\gamma \gamma \rightarrow$ $K_{S} K_{S}$ and $\pi^{0} \pi^{0}\left(\eta \pi^{0}\right.$ and $\eta \eta$ ）and their ratios to charged（ $\pi^{0} \pi^{0}$ ）mode．The range of all measured $n$ value，from 7 to 10 ，is not far above the asymptotic pQCD prediction of 6 ［［⿴囗 $]$ ．At present energies，the leading term may be small and dominated by the first power correction，therefore energy dependence can be much steeper，$n \sim 10$［［］］．Cross section ratio，$\sigma_{0}\left(K^{+} K^{-}\right) / \sigma_{0}\left(\pi^{+} \pi^{-}\right)$ is constant in present energy region，while neutral－to－charged ratios，$\sigma_{0}\left(K_{S} K_{S}\right) / \sigma_{0}\left(K^{+} K^{-}\right)$and $\sigma_{0}\left(\pi^{0} \pi^{0}\right) / \sigma_{0}\left(\pi^{+} \pi^{-}\right)$seem to approach constant．Cross sections for $\eta \pi^{0}, \eta \eta, K^{+} K^{-}$，and $K_{S} K_{S}$ satisfy well $S U(3)$ relation in Handbag approach［Ш］］．Further discussion can be found in［Ш2］］．

## References

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Figure 4: Left: (a) Cross section for $\gamma \gamma \rightarrow K_{S} K_{S}$, (b) its ratio to $K^{+} K^{-}, \sigma_{0}\left(K_{S} K_{S}\right) / \sigma_{0}\left(K^{+} K^{-}\right)$ and (c) $\sigma_{0}\left(K^{+} K^{-}\right) / \sigma_{0}\left(\pi^{+} \pi^{-}\right)$. Right: (a) Cross section for $\pi^{0} \pi^{0}$ and $\pi^{+} \pi^{-}$and (b) their ratio $\sigma_{0}\left(\pi^{0} \pi^{0}\right) / \sigma_{0}\left(\pi^{+} \pi^{-}\right)$. Curves in (a) show fits to $W^{-n}$. Fit result to a constant value is shown for $\sigma_{0}\left(K^{+} K^{-}\right) / \sigma_{0}\left(\pi^{+} \pi^{-}\right)$and $\sigma_{0}\left(\pi^{0} \pi^{0}\right) / \sigma_{0}\left(\pi^{+} \pi^{-}\right)$.


Figure 5: Cross section and its ratio to $\pi^{0} \pi^{0}$ for $\eta \pi^{0}$ (left) and $\eta \eta$ (right). In (a), $\sigma_{0}\left(\pi^{0} \pi^{0}\right)$ is shown in open squares. Curves are fit result to $W^{-n}$ dependence.

Table 2：The value of $n$ of $\sigma_{0} \propto W^{-n}$ in various reactions fitted in the $W$ and $\left|\cos \theta^{*}\right|$ ranges indicated and the ratio of the cross sections in comparison with QCD predictions．The first and second errors are statistical and systematic，respectively．

| Process | $n$ or $\sigma_{0}$ ratio | $W(\mathrm{GeV})$ | $\left\|\cos \theta^{*}\right\|$ | BL［四］ | BC［目］ | DKV［田］ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\pi^{+} \pi^{-}$ | $7.9 \pm 0.4 \pm 1.5$ | $3.0-4.1$ | $<0.6$ | 6 | 6 |  |
| $K^{+} K^{-}$ | $7.3 \pm 0.3 \pm 1.5$ | $3.0-4.1$ | $<0.6$ | 6 | 6 |  |
| $K_{S}^{0} K_{S}^{0}$ | $10.5 \pm 0.6 \pm 0.5$ | $2.4-4.0^{1}$ | $<0.6$ | 6 | 10 |  |
| $\pi^{0} \pi^{0}$ | $8.0 \pm 0.5 \pm 0.4$ | $3.1-4.1^{\dagger}$ | $<0.8$ | 6 | 10 |  |
| $\eta \pi^{0}$ | $10.5 \pm 1.2 \pm 0.5$ | $3.1-4.1$ | $<0.8$ | 6 | 10 |  |
| $\eta \eta$ | $7.8 \pm 0.6 \pm 0.4$ | $2.4-3.3$ | $<0.8$ | 6 | 10 |  |
| $K^{+} K^{-} / \pi^{+} \pi^{-}$ | $0.89 \pm 0.04 \pm 0.15$ | $3.0-4.1$ | $<0.6$ | 2.3 | 1.06 |  |
| $K_{S} K_{S} / K^{+} K^{-}$ | $\sim 0.13$ to $\sim 0.01$ | $2.4-4.0$ | $<0.6$ |  | 0.005 | $2 / 25$ |
| $\pi^{0} \pi^{0} / \pi^{+} \pi^{-}$ | $0.32 \pm 0.03 \pm 0.06$ | $3.1-4.1$ | $<0.6$ |  | $0.04-0.07$ | 0.5 |
| $\eta \pi^{0} / \pi^{0} \pi^{0}$ | $0.48 \pm 0.05 \pm 0.04$ | $3.1-4.0$ | $<0.8$ | $0.24 R_{f}\left(0.46 R_{f}\right)^{\ddagger}$ |  |  |
| $\eta \eta / \pi^{0} \pi^{0}$ | $0.37 \pm 0.02 \pm 0.03$ | $2.4-3.3$ | $<0.8$ | $0.36 R_{f}^{2}\left(0.62 R_{f}^{2}\right)^{\ddagger}$ |  |  |
| $\dagger \chi_{c J}$ region，3．3－3．6 GeV is excluded． |  |  |  |  |  |  |
| $\eta$ |  |  |  |  |  |  |

$\ddagger \eta$ meson as a pure $\mathrm{SU}(3)$ octet（mixture of octet and singlet with $\left.\theta_{p}=-18^{\circ}\right), R_{f}=f_{\eta}^{2} / f_{\pi^{0}}^{2}$ ．
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[^0]:    36th International Conference on High Energy Physics,
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