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Interplay of IR-Improved DGLAP-CS Theory and NLO Parton Shower MC Precision

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We present the interplay between the new IR-improved DGLAP-CS theory and the precision of NLO parton shower/ME matched MC's as it is realized by the new MC Herwiri1.031 in interface to MC@NLO. We discuss phenomenological implications using comparisons with recent LHC data on single heavy gauge boson production.

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1. Introduction

With the recent announcement [1] of an Englert-Brout-Higgs (EBH) [2] candidate boson after the start-up and successful running of the LHC, the era of precision QCD, wherein the total precision tag is 1% or better, is upon us. The attendant need for exact, amplitude-based resummation of large higher order effects is now more paramount, in view of the expected role of precision comparison between theory and experiment in determining the detailed properties of the newly discovered EBH boson candidate. It has been argued elsewhere [3, 4] that such resummation allows one to have better than 1% theoretical precision as a realistic goal in such comparisons. Here, we present the status of this approach to precision QCD for the LHC with the attendant IR-improved DGLAP-CS [5, 6] theory [7, 8] realization via HERWIRI1.031 [9] in the HERWIG6.5 [10] environment in interplay with NLO exact, matrix element matched parton shower MC precision issues. We employ the MC@NLO [11] methodology to realize the attendant exact, NLO matrix element matched parton shower MC realizations in comparisons with recent LHC data.

In the discussion we continue the strategy of building on existing platforms to develop and realize a path toward precision QCD for the physics of the LHC. We exhibit a union of the new IR-improved DGLAP-CS theory and MC@NLO. We are also pursuing the implementation [12] of the new IR-improved DGLAP-CS theory for HERWIG++ [13], HERWIRI++, for PYTHIA8 [14] and for SHERPA [15], as well as the corresponding NLO ME/parton shower matching realizations in the POWHEG [16] framework – one of the strongest cross checks on theoretical precision is the difference between two independent realizations of the attendant theoretical calculation.

We set the stage for the proper exposition of the interplay between the NLO ME matched parton shower MC precision and the new IR-improved DGLAP-CS theory in the next section by showing how the latter theory follows naturally in the effort to obtain a provable precision from our approach [4] to precision LHC physics. We review this latter approach in the next section as well. We then turn in Section 3 to the applications to the recent data on single heavy gauge boson production at the LHC with an eye on the analyses in Refs. [9] of the analogous processes at the Tevatron. We will focus in this discussion on the single $Z/\gamma*$ production and decay to lepton pairs for definiteness. The other heavy gauge boson processes will be taken up elsewhere [12].

2. Brief Recapitulation

The starting point for our discussion may be taken as the fully differential representation

$$d\sigma = \sum_{i,j} \int dx_1 dx_2 F_i(x_1) F_j(x_2) d\hat{\sigma}_{\rm res}(x_1 x_2 s) \tag{2.1}$$

of a hard LHC scattering process using a standard notation so that the $\{F_j\}$ and $d\hat{\sigma}_{\rm res}$ are the respective parton densities(PDFs) and reduced resummed hard differential cross section, where the resummation is for all large EW and QCD higher order corrections in order to achieve a total precision tag of 1% or better for the total theoretical precision of (2.1). The proof of the correctness of the value of the total theoretical precision $\Delta\sigma_{\rm th}$ of (2.1) is the basic issue for precision QCD for the LHC . This precision can be represented as follows:$\Delta\sigma_{\rm th} = \Delta F \oplus \Delta\hat{\sigma}_{\rm res}$ where ΔA is the contribution of the uncertainty on the quantity A to $\Delta\sigma_{\rm th}$[1]. The proof of the correctness of the value of the total theoretical precision $\Delta\sigma_{\rm th}$ is essential for validation of the application of a given theoretical prediction to precision experimental observations for the signals and the backgrounds for both Standard Model(SM) and new physics (NP) studies, and more specifically for the overall normalization of the cross sections in such studies. We cannot emphasize too much that NP can be missed if a calculation with an unknown value of $\Delta\sigma_{\rm th}$ is used for the attendant studies. We note that here $\Delta\sigma_{\rm th}$ is the total theoretical uncertainty that comes from the physical precision contribution and the technical precision contribution [17]: the physical precision contribution, $\Delta\sigma_{\rm th}^{\rm phys}$, arises from such sources as missing graphs, approximations to graphs, truncations,....; the technical precision contribution, $\Delta\sigma_{\rm th}^{\rm tech}$, arises from such sources as bugs in codes, numerical rounding errors, convergence issues, etc. The total theoretical error follows from

$$\Delta\sigma_{\rm th} = \Delta\sigma_{\rm th}^{\rm phys} \oplus \Delta\sigma_{\rm th}^{\rm tech}. \tag{2.2}$$

[1]Here, we discuss the situation in which the two errors in the equation for $\Delta\sigma_{\rm th}$ are independent for definiteness; the equation for it has to be modified accordingly when they are not.

As a general rule, the desired value for $\Delta\sigma_{\rm th}$, which depends on the specific requirements of the observations, should fulfill $\Delta\sigma_{\rm th} \leq f\Delta\sigma_{\rm expt}$. Here $\Delta\sigma_{\rm expt}$ is the respective experimental error and $f \lesssim \frac{1}{2}$ so that the theoretical uncertainty does not significantly adversely affect the analysis of the data for physics studies.

With the goal of realizing such precision in a provable way, we have developed the QCD $\otimes$ QED resummation theory in Refs. [4] for the reduced cross section in (2.1) and for the resummation of the evolution of the parton densities therein as well. Mainly because the theory in Refs. [4] is not widely known, we recapitulate it here briefly. The master formula for our resummation theory may be identified as

$$d\bar{\sigma}_{\rm res} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \prod_{j_1=1}^{n} \frac{d^3k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{m} \frac{d^3k'_{j_2}}{k'_{j_2}} \int \frac{d^4y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum k'_{j_2})+D_{\rm QCED}} \tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3p_2}{p_2^0}\frac{d^3q_2}{q_2^0}, \tag{2.3}$$

where $d\bar{\sigma}_{\rm res}$ is either the reduced cross section $d\hat{\sigma}_{\rm res}$ or the differential rate associated to a DGLAP-CS [5, 6] kernel involved in the evolution of the $\{F_j\}$ and where the *new* (YFS-style [18]) *non-Abelian* residuals $\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$ have n hard gluons and m hard photons and we show the final state with two hard final partons with momenta p_2, q_2 specified for a generic $2f$ final state for definiteness. The infrared functions $\rm SUM_{IR}(QCED)$, $D_{\rm QCED}$ are defined in Refs. [4, 7, 8]. This simultaneous resummation of QED and QCD large IR effects is exact.

We have shown in Refs. [7–9] that the methods in Refs. [19, 20] give approximations to our hard gluon residuals $\hat{\bar{\beta}}_n$; for, the methods in Refs. [19, 20], unlike the master formula in (2.3), are not exact results. The threshold-resummation methods in Refs. [19], using the result that, for any function $f(z)$,

$$\left|\int_0^1 dz z^{n-1} f(z)\right| \leq (\frac{1}{n}) \max_{z\in[0,1]} |f(z)|,$$

drop non-singular contributions to the cross section at $z \to 1$ in resumming the logs in n-Mellin space. The SCET theory in Refs. [20] drops terms of $\mathcal{O}(\lambda)$ at the level of the amplitude, where $\lambda = \sqrt{\Lambda/Q}$ for a process with the hard scale Q with $\Lambda \sim .3$GeV so that, for $Q \sim 100$GeV, $\lambda \cong 5.5\%$. The known equivalence of the two approaches implies that the errors in the threshold resummation must be similar. We can only use these approaches as a guide to our new non-Abelian residuals as we develop results for the sub-1% precision regime.

As it is explained in Refs. [4], the new non-Abelian residuals $\tilde{\bar{\beta}}_{m,n}$ allow rigorous shower/ME matching via their shower subtracted analogs: $\tilde{\bar{\beta}}_{m,n} \to \hat{\bar{\beta}}_{m,n}$ where the $\hat{\bar{\beta}}_{m,n}$ have had all effects in the showers associated to the $\{F_j\}$ removed from them and this naturally brings us to the attendant evolution of the $\{F_j\}$. For a strict control on the theoretical precision in (2.1), we need both the resummation of the reduced cross section and that of the latter evolution.

When the QCD restriction of the formula in (2.3) is applied to the calculation of the kernels, P_{AB}, in the DGLAP-CS theory itself, we get an improvement of the IR limit of these kernels, an IR-improved DGLAP-CS theory [7–9] with new resummed kernels $P^{\rm exp}_{AB}$, which are reproduced here for completeness:

$$
\begin{aligned}
P^{\exp}_{qq}(z) &= C_F F_{\rm YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z) \right],\\
P^{\exp}_{Gq}(z) &= C_F F_{\rm YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q},\\
P^{\exp}_{GG}(z) &= 2C_G F_{\rm YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \{ \frac{1-z}{z} z^{\gamma_G} + \frac{z}{1-z}(1-z)^{\gamma_G}\\
&\qquad + \frac{1}{2}(z^{1+\gamma_G}(1-z) + z(1-z)^{1+\gamma_G}) - f_G(\gamma_G)\delta(1-z)\},\\
P^{\exp}_{qG}(z) &= F_{\rm YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \frac{1}{2}\{ z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G}\},
\end{aligned}
\tag{2.4}
$$

where the superscript "exp" indicates that the kernel has been resummed as predicted by Eq. (2.3) when it is restricted to QCD alone, where the YFS [18] infrared factor is given by $F_{\rm YFS}(a) = e^{-C_E a}/\Gamma(1+a)$ where C_E is Euler's constant and where the respective resummation functions $\gamma_A,\ \delta_A,\ f_A,\ A = q,\ G$ are given in Refs. [7,8] [2]. $C_F(C_G)$ is the quadratic Casimir invariant for the quark(gluon) color representation respectively. From these new kernels we get a new resummed scheme for the PDFs and the reduced cross section:

$$
\begin{aligned}
F_j,\ \hat\sigma &\rightarrow F'_j,\ \hat\sigma' \text{ for}\\
P_{Gq}(z) &\rightarrow P^{\exp}_{Gq}(z), \text{etc.},
\end{aligned}
\tag{2.5}
$$

with the same value for σ in (2.1) with improved MC stability [9] – we do not need an IR cut-off 'k_0' parameter in the attendant parton shower MC based on the new kernels. Note that, while the degrees of freedom below the IR cut-offs in the usual showers are dropped in those showers, in the showers in HERWIRI1.031, as one can see from (2.3), these degrees of freedom are integrated over and included in the calculation in the process of generating the Gribov-Lipatov exponents γ_A in (2.4). The new kernels agree with the usual kernels at $\mathcal{O}(\alpha_s)$ as the differences between them start in $\mathcal{O}(\alpha_s^2)$, so that the NLO matching formulas in the MC@NLO and POWHEG frameworks apply directly to the new kernels for exact NLO ME/shower matching.

In Fig. 1 we show the basic physical idea of Bloch and Nordsieck [23] underlying the new kernels: an accelerated charge generates a coherent state of very soft massless quanta of the re-

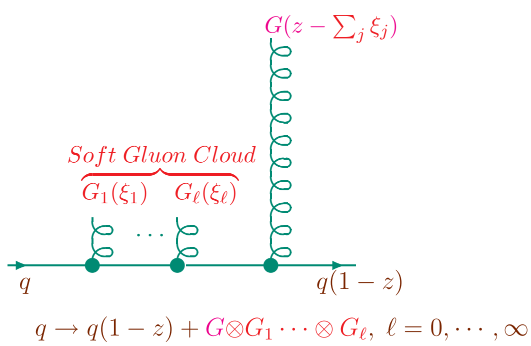

Figure 1: Bloch-Nordsieck soft quanta for an accelerated charge.

spective gauge field so that one cannot know which of the infinity of possible states one has made in the splitting process $q(1) \rightarrow q(1-z) + G \otimes G_1 \cdots \otimes G_\ell,\ \ell = 0, \cdots, \infty$ illustrated in Fig. 1. The

[2] The improvement in Eq. (2.4) should be distinguished from the resummation in parton density evolution for the "$z \rightarrow 0$" Regge regime – see for example Ref. [21, 22]. This latter improvement must also be taken into account for precision LHC predictions.

new kernels take this effect into account by resumming the terms $\mathscr{O}\left((\alpha_s \ln(\frac{q^2}{\Lambda^2})\ln(1-z))^n\right)$ when $z \to 1$ is the IR limit. From (2.5) and (2.1), one can see that when the usual kernels are used these terms are generated order-by-order in the solution for the cross section σ in (2.1). Our resumming of these terms enhances the convergence of the representation in (2.1) for a given order of exactness in the in input perturbative components therein. In the next section we illustrate this last remark in the context of the comparison of NLO parton shower/matrix element matched predictions to recent LHC data.

3. Interplay of NLO Shower/ME Precision and IR-Improved DGLAP-CS Theory

Here, we compare new MC HERWIRI1.031 [9] with HERWIG6.510, both with and without the MC@NLO [11] exact $\mathscr{O}(\alpha_s)$ correction to illustrate the interplay between the attendant precision in NLO ME matched parton shower MC's and the new IR-improvement for the kernels realized in Herwiri1.031, where we use the new LHC data for our baseline for the comparison.

For the single $Z/\gamma*$ production at the LHC, we show in Fig. 2 in panel (a) the comparison between the MC predictions and the CMS rapidity data [25] and in panel (b) the analogous comparison with the ATLAS P_T data, where the rapidity data are the combined $e^+e^- - \mu^-\mu^+$ results and the p_T data are those for the bare e^+e^- case, as these are the data that correspond to the theoretical framework of our simulations – we do not as yet have complete realization of all the corrections involved in the other ATLAS data in Ref. [26]. These results should be viewed with an eye on our

(a) **(b)**

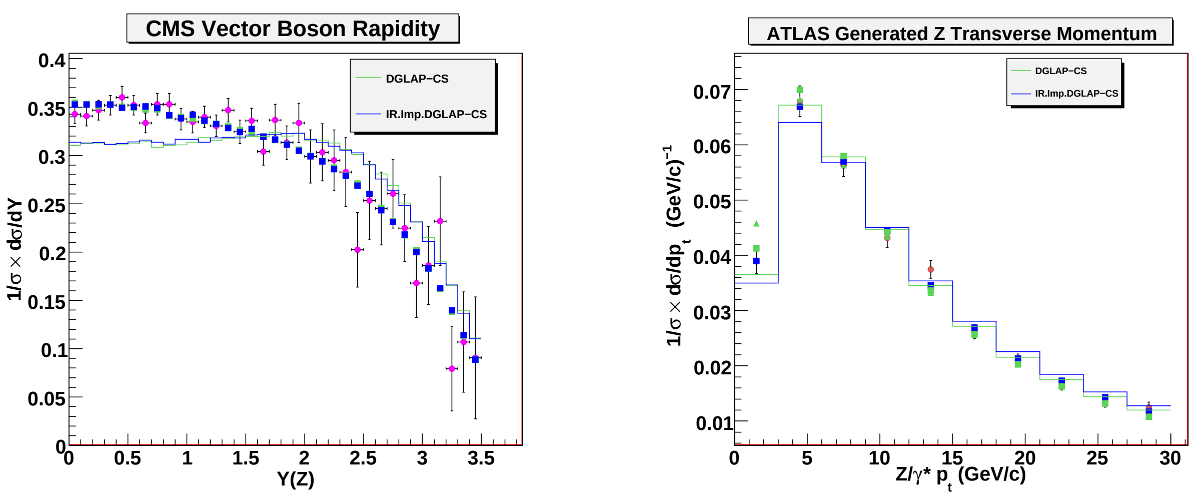

Figure 2: Comparison with LHC data: (a), CMS rapidity data on (Z/γ^*) production to e^+e^-, $\mu^+\mu^-$ pairs, the circular dots are the data, the green(blue) lines are HERWIG6.510(HERWIRI1.031); (b), ATLAS p_T spectrum data on (Z/γ^*) production to (bare) e^+e^- pairs, the circular dots are the data, the blue(green) lines are HERWIRI1.031(HERWIG6.510). In both (a) and (b) the blue(green) squares are MC@NLO/HERWIRI1.031(HERWIG6.510(PTRMS = 2.2GeV)). In (b), the green triangles are MC@NLO/HERWIG6.510(PTRMS =0). These are otherwise untuned theoretical results.

analysis in Ref. [9] of the FNAL data on the single Z/γ^* production in $p\bar{p}$ collisions at 1.96 TeV.

In Fig. 11 of the second paper in Ref. [9], we showed that, when the intrinsic rms p_T parameter PTRMS is set to 0 in HERWIG6.5, the simulations for MC@NLO/HERWIG6.510 give a good fit to the CDF rapidity distribution data [28] therein but they do not give a satisfactory fit to the D0 p_T distribution data [29] therein whereas the results for MC@NLO/HERWIRI1.031 give good fits to both sets of data with the PTRMS $= 0$. Here PTRMS corresponds to an intrinsic Gaussian distribution in p_T. The authors of HERWIG [27] have emphasized that to get good fits to both sets of data, one may set PTRMS $\cong 2$ GeV. Thus, in analyzing the new LHC data, we have set PTRMS $= 2.2$GeV in our HERWIG6.510 simulations while we continue to set $PTRMS = 0$ in our HERWIRI simulations.

Turning now with this perspective to the results in Fig. 2, we see confirmation of the finding of the HERWIG authors. To get a good fit to both the CMS rapidity data and the ATLAS p_T data, one needs to set PTRMS $\cong$ 2GeV [30] in the MC@NLO/HERWIG6510 simulations. We again see that at LHC one gets a good fit to the data for both the rapidity and the p_T spectra in the MC@NLO/HERWIRI1.031 simulations with PTRMS $= 0$. In quantitative terms, the χ^2/d.o.f. for the rapidity data and p_T data are (.72,.72)((.70,1.37)) for the MC@NLO/HERWIRI1.031(MC@NLO/HERWIG 6510(PTRMS=2.2GeV)) simulations. For the MC@NLO/HERWIG6510(PTRMS=0) simulations the corresponding results are (.70,2.23).

The usual DGLAP-CS kernels require the introduction of a hard intrinsic Gaussian spread in p_T inside the proton to reproduce the LHC data on the p_T distribution of the Z/γ^* in the pp collisions whereas the IR-improved kernels give a better fit to the data without the introduction of such. This hard PTRMS is entirely ad hoc; it is in contradiction with the results of all successful models of the proton wave-function [31], wherein the scale of it is $\lesssim$ .4GeV. More importantly, it contradicts the known experimental observation of precocious Bjorken scaling [32, 33], where the SLAC-MIT experiments show that Bjorken scaling occurs already at $Q^2 = 1_+$ GeV^2 for $Q^2 = -q^2$ with q the 4-momentum transfer from the electron to the proton in the famous deep inelastic electron-proton scattering process whereas, if the proton constituents really had a Gaussian intrinsic p_T distribution with PTRMS $\cong$ 2GeV, these observations would not be possible. What can now argue is that the ad hoc PTRMS $\cong$ 2.2GeV value is a phenomenological representation of the more fundamental dynamics realized by the IR-improved DGLAP-CS theory. Is it possible to tell the difference between the two representations of the data in Fig. 2?

Physically, one expects that more detailed observations should be able to distinguish the two. Indeed, in Fig. 3 we show the MC@NLO/HERWIRI 1.031(blue squares) and MC@NLO/HERWIG6510(PTRMS=2.2GeV) (green squares) predictions for the Z/γ^* mass spectrum when the decay lepton pairs are required to satisfy the LHC type requirement that their transverse momenta $\{p_T^\ell, p_T^{\bar{\ell}}\}$ exceed 20 GeV. As the peaks differ by 2.2%, the high precision data such as the LHC

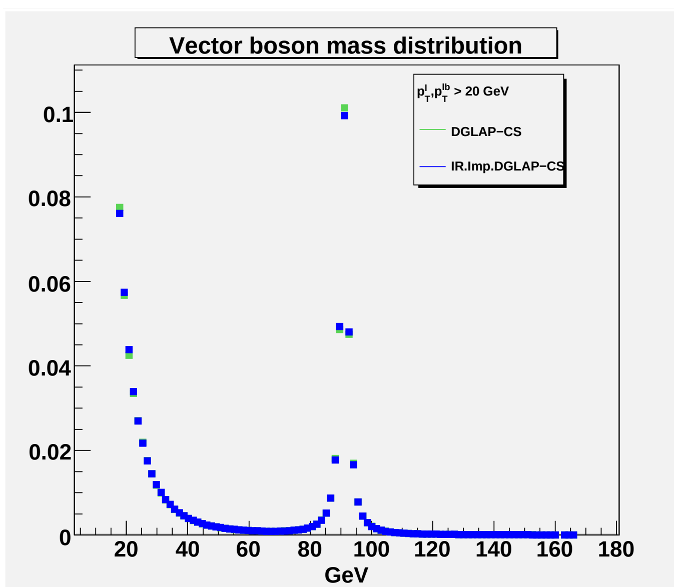

Figure 3: Normalized vector boson mass spectrum at the LHC for $p_T(\text{lepton}) > 20$ GeV.

ATLAS and CMS experiments will have (each already has over 5×10^6 lepton pairs) will allow

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one to distinguish between the two sets of theoretical predictions. Other such detailed observations may also reveal the differences between the two representations of parton shower physics and we will pursue these elsewhere [12]. In closing, two of us (A.M. and B.F.L.W.) thank Prof. Ignatios Antoniadis for the support and kind hospitality of the CERN TH Unit while part of this work was completed.

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