Charm mixing at Belle

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We report recent charm mixing results using $D^0 \rightarrow K^0_S \pi^+ \pi^-$ and $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ based on the data produced by the KEKB collider and collected with the Belle detector.

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†on behalf of the Belle collaboration.
1. Introduction

In $D^0$-$\bar{D}^0$ mixing phenomenon, the neutral charm mesons oscillate with each other by a mixture of two mass eigenstates $|D_{1,2}\rangle = p|D^0\rangle + \pm|\bar{D}^0\rangle$, where $|p|^2 + |q|^2 = 1$. The physical eigenstates must have different masses $m_1$, $m_2$ and widths $\Gamma_1$, $\Gamma_2$. The mixing parameters $x, y$ define the masses and widths differences of the two mass eigenstates $x = (m_1 - m_2)/\Gamma$ and $y = (\Gamma_1 - \Gamma_2)/\Gamma$, where $\Gamma = (\Gamma_1 + \Gamma_2)/2$. The charm mixing rate is predicted to be small in standard model (SM). So measuring of mixing parameters and searching for CP violation is important for determining the physics beyond SM. Different $D^0$ decay modes show sensitivity to different combination of $x$ and $y$. We report here two of them.

2. $D^0 \rightarrow K^0_S\pi^+\pi^-$

In self conjugated decay $D^0 \rightarrow K^0_S\pi^+\pi^-$, the initially produced $D^0$ or $\bar{D}^0$’s decay amplitudes can be expressed as a function of time

$$
\mathcal{M}(m^2_+, m^2_-, t) = \frac{1}{2} (e_1(t) + e_2(t)) \mathcal{A}(m^2_+, m^2_-) + \frac{q}{2p} (e_1(t) - e_2(t)) \overline{\mathcal{A}}(m^2_+, m^2_-), \tag{2.1}
$$

$$
\overline{\mathcal{M}}(m^2_+, m^2_-, t) = \frac{1}{2} (e_1(t) + e_2(t)) \overline{\mathcal{A}}(m^2_+, m^2_-) + \frac{p}{2q} (e_1(t) - e_2(t)) \mathcal{A}(m^2_+, m^2_-), \tag{2.2}
$$

where $\mathcal{A}$ and $\overline{\mathcal{A}}$ are the decay amplitudes for $D^0$ and $\bar{D}^0$ as a function of Dalitz-plot variables $(m^2_+, m^2_-) = (m^2_{K^0\pi^+}, m^2_{K^0\pi^-})$, and $e_{1,2}(t) = e^{-i(m^2_{1,2} - \Gamma_{1,2}t)/2}$. Assuming CP is conserved, $p/q = 1$ and $\overline{\mathcal{A}}(m^2_+, m^2_-) = \mathcal{A}(m^2_+, m^2_-)$. Upon squaring $\mathcal{M}$ and $\overline{\mathcal{M}}$, one can obtain the decay rates for $D^0$ and $\bar{D}^0$, where neutral mesons oscillate with each other through the terms $\cosh(y\Gamma t)$, $\sinh(y\Gamma t)$, $\cos(x\Gamma t)$, and $\sin(x\Gamma t)$. So a time-dependent Dalitz-plot analysis of $D^0 \rightarrow K^0_S\pi^+\pi^-$ allows one to measure $x$ and $y$ directly, which is developed by CLEO [1] and extended by Belle [2] and BaBar [3].

We select the $D^0$ and $\bar{D}^0$ events by decay chain $D^{*+} \rightarrow D^0\pi^+ + D^0 \rightarrow K^0_S\pi^+\pi^-$. The flavor of the neutral D meson is tagged by the charge of slow pion $\pi^+_s$. The final states are fully reconstructed via two kinematic variables: the invariant mass of $K^0_S\pi^+$ and $K^0_S\pi^-$. We project the flight length to momentum vector and transform it to $D^0$ center-of-mass system to obtain the $D^0$ decay time $t$ and its uncertainty $\sigma_t$. To suppress the combinatorial background and events from B decays, we required $D^{*+}$ momentum in the center-of-mass (CM) frame to be greater than 2.5 GeV/c and 3.1 GeV/c for $\Upsilon(4S)$ and $\Upsilon(5S)$ data respectively. Two observables are used to determine the yield of signal and backgrounds: the invariant mass of $D^0$ daughters $M = m_{K^0_S\pi^+\pi^-}$ and the energy released from $D^{*+}$ decay $Q = m_{K^0_S\pi^+\pi^-} - m_{K^0_S\pi^-\pi^+} - m_{\pi^+}$. The Dalitz distribution of $D^0 \rightarrow K^0_S\pi^+\pi^-$ are expressed as a sum of quasi-two-body amplitudes. We adopt 12 intermediate resonances for the P and D wave, K-matrix parameterization for the $\pi\pi$ S-wave [4] and the same description as [5] for the $K^0_S\pi$ S wave. We try optional models by using different combinations of alternative formalism and by adding or removing resonances with small fractions. The final DP parameterization is optimized according likelihood and $\chi^2$ test.

To extract the mixing parameters $x$ and $y$, an unbinned maximum likelihood fit is performed. We parameterize the signal’s PDF in a normalized form.
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$$p_{\text{sig}}(m_{-}^{2},m_{+}^{2},t_{i}) = \frac{\int_{0}^{+\infty} dt' R(t_{i} - t') |\mathcal{M}(m_{-}^{2},m_{+}^{2},t',t')|^{2} \varepsilon(m_{-}^{2},m_{+}^{2})}{\int_{0}^{+\infty} dt \int_{D} dm_{-}^{2} dm_{+}^{2} |\mathcal{M}(m_{-}^{2},m_{+}^{2},t)|^{2} \varepsilon(m_{-}^{2},m_{+}^{2})},$$

where $i$ runs over all selected event candidates, $R(t_{i} - t')$ is the resolution function for $D^{0}$ decay time, the efficiency $\varepsilon(m_{-}^{2},m_{+}^{2})$ is described by a cubic polynomial with symmetry for $m_{-}^{2}$ and $m_{+}^{2}$ obtained from Monte Carlo fit.

The distribution for combinatorial background is determined in $M$ sideband. The random $\pi^{+}$ background contains a mixture of $D^{0}$ and $\bar{D}^{0}$ decays. The Dalitz amplitude can be written as $p_{\text{rnd}}(m_{-}^{2},m_{+}^{2}) = (1 - f_{w})|\mathcal{A}(m_{-}^{2},m_{+}^{2})|^{2} + f_{w}|\mathcal{A}(m_{-}^{2},m_{+}^{2})|^{2}$, where $f_{w}$ is a fraction of wrong charge tagged events in this background. We extracted $f_{w} = 0.511 \pm 0.003$ from $Q$ sideband. The fitting procedure is validated with fully simulated Monte Carlo experiments with background included.

![Figure 1: Dalitz-plot distribution, decay time, and $m_{-}^{2},m_{+}^{2}$ projections for Data fit.](image)

The fit results of data are shown in Fig. 1. The fit gives $x = (0.56 \pm 0.19)\%$, $y = (0.30 \pm 0.15)\%$, $\tau_{D^{0}} = (410.3 \pm 0.45)\,\text{fs}$, the latter one consistent with world average [5]. There are two categories
of systematic uncertainties arising from experiment and $D^0$ decay model (summarized in Table 1). The dominant contributions for experimental and Dalitz model sources are from the fitting performances and the parameterizations of angular dependence respectively.

**Table 1:** The sources of experimental systematic uncertainty. The positive and negative errors are added in quadrature separately.

<table>
<thead>
<tr>
<th>Source</th>
<th>$(\Delta x) \times 10^{-4}$</th>
<th>$(\Delta y) \times 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time resolution of signal</td>
<td>-1.39</td>
<td>-0.92</td>
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<tr>
<td>Combinatorial background time’s PDF</td>
<td>+1.74</td>
<td>+1.65</td>
</tr>
<tr>
<td>Errors on combinatorial time parameters</td>
<td>±0.77</td>
<td>±1.57</td>
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<tr>
<td>Backgrounds’ Dalitz dependence on time</td>
<td>-4.76</td>
<td>-3.55</td>
</tr>
<tr>
<td>Fraction of wrong tagged events $f_w$</td>
<td>-0.67</td>
<td>-0.45</td>
</tr>
<tr>
<td>Efficiency</td>
<td>-1.13</td>
<td>-2.09</td>
</tr>
<tr>
<td>Best Candidate selection</td>
<td>+1.05</td>
<td>+1.87</td>
</tr>
<tr>
<td>$K^*(892)$ DCS/CF reduced by 5%</td>
<td>-7.28</td>
<td>+2.29</td>
</tr>
<tr>
<td>$K_2^*(1430)$ DCS/CF reduced by 5%</td>
<td>+1.71</td>
<td>-0.67</td>
</tr>
<tr>
<td>Normalization of random background N</td>
<td>±0.27</td>
<td>±0.13</td>
</tr>
<tr>
<td>Normalization of combinatorial background</td>
<td>±0.13</td>
<td>±0.24</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>+2.78</td>
<td>+3.74</td>
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</table>

**Table 2:** The sources of modeling systematic uncertainty. The positive and negative errors are added in quadrature separately.

<table>
<thead>
<tr>
<th>Fit model</th>
<th>$(\Delta x) \times 10^{-4}$</th>
<th>$(\Delta y) \times 10^{-4}$</th>
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<tbody>
<tr>
<td>Form factors $F_r, F_D$</td>
<td>+4.05</td>
<td>+2.35</td>
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<td>Resonances’ widths dependence $\Gamma(q^2)$</td>
<td>+3.33</td>
<td>-1.61</td>
</tr>
<tr>
<td>Remove $K^*(1680)^+$</td>
<td>-1.78</td>
<td>-3.02</td>
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<tr>
<td>Remove $K^*(1410)^\pm$</td>
<td>-1.16</td>
<td>-3.62</td>
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<tr>
<td>Remove $\rho(1450)$</td>
<td>+2.13</td>
<td>+0.30</td>
</tr>
<tr>
<td>K-matrix formalism</td>
<td>-2.16</td>
<td>+1.79</td>
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<tr>
<td>Angular dependence</td>
<td>-8.46</td>
<td>-3.86</td>
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<td>Resonances’ $M$ &amp; $\Gamma$ errors</td>
<td>±1.40</td>
<td>±1.21</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>-9.09</td>
<td>+5.21</td>
</tr>
</tbody>
</table>

3. $D^0 \rightarrow K^+K^-; \pi^+\pi^-$

Another evaluation method of $D^0 - \bar{D}^0$ mixing is measuring the lifetime difference between CP even decays and its CP odd decays

$$y_{CP} = \frac{\tau(D^0 \rightarrow K^-\pi^+)}{\tau(D^0 \rightarrow h^+h^-)} - 1,$$

(3.1)

where $h$ denotes $K$ or $\pi$. The $y_{CP}$ is equal to $y$ in the absence of CP violation.
If CP is violated in charm decays, the CP violating parameter can be defined as the lifetime asymmetry between $D^0$ and $\bar{D}^0$ to the same CP eigenstate.

$$A_\Gamma = \frac{\tau(D^0 \rightarrow K^- K^+) - \tau(D^0 \rightarrow K^+ K^-)}{\tau(D^0 \rightarrow K^- K^+) + \tau(D^0 \rightarrow K^+ K^-)}$$  \hspace{1cm} \text{(3.2)}$$

To extract the proper time in different decays, we parameterized the proper decay time distribution by

$$f(t) = \frac{N}{\tau} \int e^{-t'/\tau} R(t-t') \, dt' + B(t).$$  \hspace{1cm} \text{(3.3)}$$

The background distribution $B(t)$ is estimated by a fit to the sideband events. To account for the observed dependence between $D^0$ proper time mean value and $D^0$ polar angle $\theta^*$, we performed the fit in each bins of $\cos \theta^*$, where $\theta^*$ is the $D^0$ polar angle in CMS.

Fig. 2 shows the results of the fits in bins of $\cos \theta^*$ for $y_{CP}$, $A_\Gamma$ and $D^0$ lifetime $\tau$. We obtained the average $y_{CP} = (1.11 \pm 0.22 \pm 0.11)\%$, $A_\Gamma = (-0.03 \pm 0.20 \pm 0.08)\%$, and $\tau_{po} = (408.46 \pm 0.54) \text{fs}$ by a least square fit to a constant. The measured $D^0$ lifetime $\tau$ is consistent with the world average [5].

![Figure 2](image_url)\hspace{1cm} Figure 2: Results of $y_{CP}$, $A_\Gamma$ and $\tau_{po}$ in simultaneous fits in bins of $\cos \theta^*$ (points with error bars. Top/bottom plots are for data collected by 3-layer/4-layer silicon detector respectively.)

### 4. Summary

In summary, using time-dependent Dalitz plot analysis of $D^0 \rightarrow K_s^0 \pi^+ \pi^-$, we measure

$$x = (0.56 \pm 0.19^{+0.03+0.06}_{-0.09-0.09})\%,$$

$$y = (0.30 \pm 0.15^{+0.04+0.03}_{-0.05-0.06})\%. \hspace{1cm} \text{(4.2)}$$
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which is most precise to date and consistent with previous measurements \cite{2} \cite{3}.

In $D^0 \rightarrow K^+ K^-, \pi^+ \pi^-$, we obtain

$$\gamma_{CP} = (1.11 \pm 0.22 \pm 0.11)\%,$$

$$A_{\Gamma} = (-0.03 \pm 0.20 \pm 0.08)\%.$$ (4.3) (4.4)

The significance of $\gamma_{CP} \neq 0$ confirmed evidence for $D^0 \rightarrow \bar{D}^0$ mixing, and the result for $A_{\Gamma}$ is consistent with no CP violation.

References