Methods for evaluating physical processes in strong external fields at e^+e^- colliders: Furry picture and quasi-classical approach

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Future linear colliders designs, ILC and CLIC, are expected to be powerful machines for the discovery of Physics Beyond the Standard Model and subsequent precision studies. However, due to the intense beams (high luminosity, high energy), strong electromagnetic fields occur in the beam-beam interaction region. In the context of precision high energy physics, the presence of such strong fields may yield sensitive corrections to the observed electron-positron processes. The Furry picture of quantum states gives a conceptually simple tool to treat physics processes in an external field. A generalization of the quasi-classical operator method (QOM) as an approximation is considered too.

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1. Introduction

After the first successful data runs of the LHC a completely new energy region at the Terascale energy frontier has been touched and already one new boson, a Higgs boson, has been discovered. Thanks to the excellent luminosity and detector performance at the LHC, already a rather precise and consistent mass measurement of the new particle has been achieved,

$$m_{\text{Higgs}} \sim 126 \, \text{GeV} \, [1, 2]. \tag{1.1}$$

Such a mass for a Higgs boson is perfectly in agreement with mass predictions from high precision analyses in the electroweak sector. However, in order to clearly manifest whether the discovered particle is the Standard Model (SM) Higgs boson or whether the candidate points to an extended model as, for instance, Supersymmetry, a precise measurement of all couplings, the total width and branching ratios is necessary. Therefore corresponding measurements at an e^+e^- collider are highly desirable.

Designs for a planned future e^+e^- linear collider (LC) are set up to reach high $\sqrt{s}$ up to 1-1.5 TeV for the ILC and about 3 TeV for CLIC, with a very high luminosity in the range of 10^{-34}-10^{-35} cm^{-2}s^{-1} [3, 4].

The key arguments for a linear lepton collider are, for instance, the clean environment and the availability of polarized beams in the initial state, as well as rather low background rates compared with corresponding rates at a hadron collider. These facts permit in principle a full reconstruction of the observed processes and enable unprecedented high precision measurements. Therefore the LC is expected to perform high sensitivity for precision physics in the Higgs boson, top quark and electroweak gauge bosons sectors and in both direct, as well as indirect searches for new physics Beyond the Standard Model (BSM). Thus, the planned linear collider physics potential can strikingly contribute and even extend the high energy physics range of the LHC.

In order to reach the high luminosity required by the comprehensive physics program, the LC applies strongly squeezed and intense bunches of electrons and positrons at the Interaction Point (IP). At the ILC, for instance, about a factor 1000 more intense beams, i.e. more e^-/e^+ per pulse, will be achieved than at the SLC. In order to optimize the outcome of such high precision physics at a LC, one needs to know, however, in detail all processes occurring at the IP.

Each charged bunch generates an intense collective electromagnetic field, that is felt by the interacting particles from the oncoming beam. Due to its intensity, this bunch field can be considered as one external electromagnetic field as a whole rather than to be composed by single photons interacting with the colliding leptons.

We study the impact of such external fields on the actual physics processes in electroweak and BSM physics. To achieve this goal, a full comprehension of QED effects in an external field (non linear QED) is required.

Processes in external electromagnetic fields have caused interest since the beginning of quantum electrodynamics, with the paradox of electron quantum tunneling in an arbitrary high potential barrier, observed by Klein in 1929 [5]. Sauter (1931) [6] showed that this effect depends exponentially on the intensity of the field in the barrier. Schwinger (1951) [7] interpreted this paradox via the concept of a critical field: the Schwinger critical field ($E_{cr} = m_e^2/e \simeq 1.3 \cdot 10^{18}$ V/m, $B_{cr} = m_e^2/e \simeq 4.41 \cdot 10^9$ T, using natural units) corresponds to the intensity of an electromagnetic

field at which the vacuum spontaneously creates electron-positron pairs. The vacuum that is due to the external field full of *virtual* particles becomes unstable and transforms into a more stable charged vacuum via producing *real* particles [8].

The critical field can be reached in nature on the surface of pulsars and magnetars [9] and close to superheavy nuclei [10]. Such conditions are very difficult to reproduce in a laboratory. In recent years, new experiments have been developed and designed where the Schwinger critical field condition could be achieved in the boosted rest frame of a test electron. In the context of low energy physics, this is for example the case for newly designed intense lasers like XFEL at DESY [11]. Regarding high energy physics (HEP), the first experiment to study the strong field regime of nonlinear QED was the E-144 experiment at SLAC [12], in which 46.6 GeV electrons were shot through an intense laser. A second example of sources for non linear QED effects in HEP experiments are, as explained above, the interaction points (IPs) at future linear collider designs, ILC [3] and CLIC [4]. The Schwinger condition may be fulfilled, indeed, in the rest frame of the ultrarelativistic particles that scatter at the IP.

Quantum processes in intense electromagnetic fields have been studied in different research areas like intense plasma and laser physics [13] (for a detailed review see also [14]), using the so called Furry picture (FP) [15]. According to the FP, the external field is treated as a classical object that modifies the equations of motions. However, an analogous appropriate study is still lacking for the case of linear colliders, and this is the aim of our studies.

In these proceedings, we will address the question whether electroweak physics processes may be affected by the presence of the very intense external electromagnetic fields at the IPs at colliders and whether these effects can be easily incorporated in the calculations.

In section 2 we will introduce the parameters that need to be considered in general in the presence of an external electromagnetic field, describing also the generated fields at the interaction points of a future linear collider; in section 3 we introduce the Furry picture formalism and explain how to apply it to the discussed processes; in section 4 we sketch the Baĭer-Katkov quasi-classical operator method as a possible alternative in the case of ultrarelativistic particles; we conclude in section 5.

2. Intense fields at linear colliders

In order to perform the high precision physics program planned for future linear colliders, very high luminosity L is needed. Therefore extremely squeezed e^+ and e^- bunches are required,

$$L \propto \frac{N_{e^+}N_{e^-}}{\sigma_x \sigma_y},$$

where $N_{e^\pm}$ is the number of $e^\pm$ per bunch and σ_x, σ_y are the transversal dimensions of the bunch propagating along z.

Each of the dense bunches ($N_{e^\pm} \sim 10^{10}$ and σ_x, $\sigma_y \sim$ 1-10^3 nm) can be regarded as an electromagnetic current generating a collective strong electromagnetic field at the IP.

Correspondingly, the colliding particles at the IP will see a superposition of the collective fields originating from the two beams. Due to the boosted particle frame, these fields can be approximated by two almost anticollinear constant crossed fields; each particle will mainly see

the constant crossed field from the oncoming bunch [16]. A constant crossed field is the limit for infinite period of a plane wave field with momentum $k = \frac{c}{\omega}$, it has a trivial spatial dependence $A^\mu(k \cdot x) = a^\mu k \cdot x$ and its electric (**E**) and magnetic (**B**) components are orthogonal and equal in magnitude:

$$\mathbf{E} \perp \mathbf{B}, \quad |\mathbf{E}| = |\mathbf{B}|. \tag{2.1}$$

Due to momentum transfer, the presence of external electromagnetic field permits processes that are kinematically not allowed, as, for instance, beamstrahlung (i.e. bremsstrahlung in the electromagnetic field of a relativistic particle bunch) and coherent pair production. These external fields can also affect the rate of the allowed processes like incoherent pair production.

At previous accelerators LEP and SLC these effects have been considered and estimated with some approximations. Beamstrahlung and coherent pair production have been treated via the Baĭer-Katkov quasi-classical operator method (QOM) [17] while incoherent pair production was described via the equivalent photon approximation (EPA) [18].

However, effects from strong external fields do not only affect the previously mentioned "background" processes at the IP, but they may also have direct consequences on the electroweak SM or BSM processes that are in the focus of future colliders. To our knowledge, this problem has never been addressed at colliders. Only some topics have been discussed in the context of laser physics ([19, 14] and references therein), astroparticle physics [9] and decays in extremely intense fields [20, 21].

Therefore a correct knowledge of the QED effects within a strong external field environment is required in order to optimize the physics program with the expected precision.

For the description of the external electromagnetic field at the IP, it is useful to introduce four Lorentz and gauge invariants that one can compose with the external field strength tensor $F_{\mu\nu}$ and the momenta of the propagating particle p_μ. The considered propagating particle can also be a photon. The probabilities of the processes with a *single* initial particle in a general external field (ex. beamstrahlung, coherent pair production) depend on the following quantities [22]:

$$x = \frac{e\sqrt{(A_\mu)^2}}{m} = \frac{eE}{m\omega} \tag{2.2}$$

$$\chi = \frac{e}{m^3}\sqrt{(F_{\mu\nu}p^\mu)^2} \tag{2.3}$$

$$F = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \mathbf{B}^2 - \mathbf{E}^2 \tag{2.4}$$

$$|G| = |\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}| = |-\mathbf{E} \cdot \mathbf{B}| \tag{2.5}$$

where m and e denote the mass and the charge of the propagating particle (for an initial photon one uses the mass and charge of the electron) and $\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$.

The parameter x represents the work done by the external field in a Compton length $\lambda_C = eF\hbar/mc$ in units of the energy $\hbar\omega$ of the quanta of the external field. Therefore, for $x \ll 1$, a low number of photons from the external field are expected to interfere with the interacting particles, while for $x \gtrsim 1$, multiphoton processes are favoured with a nonlinear dependence on the external field. Therefore x is called the classical (since it does not depend on $\hbar$) nonlinearity parameter.

The χ parameter, often also called Υ in the literature, represents in units of mc^2 the work performed by the field over the Compton length in the particle rest system. It parametrizes the

Machine	LEP II	SLC	ILC-1TeV	CLIC-3TeV
Energy (GeV)	94.5	46.6	500	1500
N (10^{10})	334	4	2	0.37
σ_x, σ_y (μm)	190, 3	2.1, 0.9	0.49, 0.002	0.045, 0.001
σ_z (mm)	20	1.1	0.15	0.044
$\chi_{average}$	0.00015	0.001	0.27	3.34

Table 1: Lepton colliders parameters. N is the number of leptons per bunch, σ_x, σ_y are the transversal dimensions of the bunches, σ_z presents the longitudinal dimension. E is the energy of the particles in the bunches. The parameters for ILC-1TeV are taken from a 2011 dataset.

magnitude of the quantum nonlinear effects and is called the quantum nonlinearity parameter [14]. In particular, in the case of ultrarelativistic particles, it describes the intensity of the external field in the particle frame in units of the Schwinger critical field:

$$\chi = \frac{\gamma_L E}{E_{cr}} = \frac{\gamma_L B}{B_{cr}}$$

where γ_L denotes the Lorentz factor and E, B the electric and magnetic components in the laboratory frame. Highly energetic initial particles can see a critical regime also in less intense fields in the laboratory frame, as long as $\chi \sim O(1)$. This value would correspond to an external field of the order of the Schwinger critical field, at which the vacuum is polarized. The parameters F and $|G|$ instead describe respectively the relative magnitude and orientation between **E** and **B**.

The probability W of a process with one initial particle in a constant background field depends in general only on $\chi, F, |G|$ since correspondingly $x \gg 1$ [22]. According to (2.1), (2.4) and (2.5), one has $F, |G| = 0$ for a constant crossed field. In the case of an ultrarelativistic particle ($p^0 \gg m_e$) in a relatively weak field compared to E_{cr}, one has $|F|, |G| \ll \min(1, \chi^2)$ [22], confirming that a crossed field is a good approximation for the field seen by the colliding particles at the IP of LCs. One can generalize the above picture also to processes with two initial particles in an external field. As a consequence, the probabilities of processes at the IPs simplify $W(\chi, F, |G|) \simeq W(\chi, 0, 0)$, depending effectively only on the intensity of the external field. Moreover, constant crossed fields allow simpler analytical calculations and integrations and have been object of recent studies [23].

The χ parameter varies during the collision since the bunches distort under the pinch effect and the disruption effects. According to [16], the average value for χ in a gaussian bunch is given by:

$$\chi_{average} \approx \frac{5}{6} \frac{N r_e^2 \gamma_L}{\alpha_{em} \sigma_z (\sigma_x + \sigma_y)} \tag{2.6}$$

where N is the number of leptons per oncoming bunch, α_{em} the fine structure constant, r_e the Compton radius, σ_x, σ_y the transversal dimensions of the bunches, σ_z the longitudinal dimension. Using the IP beam-beam simulation program CAIN [24] one obtains the values presented in Tab. 1 for ILC and CLIC, while for LEP II and SLC we used the approximations (2.6).

It is clear that the electromagnetic fields at the IPs of future linear colliders would be much

	ILC-1TeV	CLIC-3TeV
# of coherent pairs	~ 0	$6.8 \cdot 10^8$
# of incoherent pairs	$3.9 \cdot 10^5$	$3.8 \cdot 10^5$

Table 2: Pair production processes at the IP regions at ILC and CLIC.

more intense than in the previous lepton accelerators[1], with values of χ up to order $O(1)$. Such a value for χ corresponds to the polarization of the vacuum. This unstable vacuum requires incorporation within the calculations in a non-trivial way. A study of the effectiveness of the previously mentioned quasi-classical and EPA approximations for processes in this physical case should be undertaken.

The expected e^+e^- pair productions at linear colliders are listed in Tab. 2.

3. The Furry picture and its application

The intense external field experienced by each particle at the IP is characterized by a high photon density and a corresponding wave function overlap, so that it can be seen as a *classical* external background field. For physics in intense fields, the effects of such a classical external field are taken into account *exactly* through the so called *Furry picture* or representation (FP) of quantum states [15]. The main idea of FP is to find the exact solutions of the equation of motion in the external field, taking into account the latter non perturbatively. Then, one applies the solutions in the Feynman-Schwinger-Tomonaga S-matrix theory [25] to calculate the probabilities of the physical processes. We propose to use FP also to calculate the probabilities of processes at the IP of linear colliders. In the following, we briefly review the FP technique [26].

In the usual Interaction (or Dirac) picture the time dependence is shared between the state vectors and the observables. In particular, the Hamiltonian is given by

$$\mathscr{H} = \mathscr{H}_0 + \mathscr{V} \tag{3.1}$$

where $\mathscr{H}_0$ is the time-independent unperturbed Hamiltonian, describing the time evolution of observables, and $\mathscr{V}$ is the interaction Hamiltonian, that regulates the time dependence of the states. In QED, $\mathscr{V}$ represents the gauge interactions between fermions and photons. The eigenstates of $\mathscr{H}_0$ are assumed to be the free states of the particle in the vacuum.

In the FP the Hamiltonian is given by

$$\mathscr{H} = \mathscr{H}_0 + \mathscr{H}_{ext} + \mathscr{V} = \mathscr{H}_B + \mathscr{V} \tag{3.2}$$

where $\mathscr{H}_{ext}$ represents the interaction of the fermions with the external *classical* field. In the FP the considered state vectors are the bound states of the fermions in the external field, eigenstates of $\mathscr{H}_B$. The FP bound eigenstates are related by a canonical transformation to the free particle states of the Interaction picture [15], they obey to different commutation relations but, in the limit of null

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[1]At the LHC, the less dense bunches do not allow to reach the really high critical field corresponding to proton pair production (10^{24} V/m).

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external field, the usual commutation relations are recovered. The corresponding QED Lagrangian can be written as:

$$\mathscr{L} = \bar{\psi}(i\partial\!\!\!/ - e A\!\!\!/_{ext} - m)\psi - \frac{1}{4}FF - e\bar{\psi}A\!\!\!/\psi \tag{3.3}$$

where A^{μ}_{ext} is the classical external field and its interaction term is distinguished from the usual gauge interaction term. Note that there is no kinetic therm $\frac{1}{4}F_{ext}F_{ext}$ since A^{μ}_{ext} is a classical external background field, not a dynamical field.

From the Lagrangian (3.3) one derives the modified Dirac equation:

$$(i\partial\!\!\!/ - e A\!\!\!/_{ext} - m)\psi = 0 \tag{3.4}$$

the solution of which was found by Volkov in the '30s [27]:

$$\Psi^V_p(k \cdot x) = \frac{1}{\sqrt{(2\pi)^3 2\varepsilon_p}} E_p(k \cdot x)\, u(p) \tag{3.5}$$

with

$$E_p(k \cdot x) \equiv \left(1 - \frac{e A\!\!\!/_{ext} k\!\!\!/}{2(k \cdot p)}\right) \exp\left[-ip \cdot x - i \int_0^{(k \cdot x)} \left[\frac{e(A_{ext}(\phi) \cdot p)}{(k \cdot p)} - \frac{e^2 A_{ext}(\phi)^2}{2(k \cdot p)}\right] d\phi\right], \tag{3.6}$$

where k is the momentum of the external field, p and ε_p the canonical momentum and energy of the fermion while $u(p)$ is the usual Dirac spinor solution.

The solution (3.5) is valid for a vast class of classical external fields. However, the precise expression is known only for few configurations like the plane wave electromagnetic field, the crossed electromagnetic field and the Coulomb field. This solution takes entirely into account the effects of the external electromagnetic field on the fermion. The Ψ_V solutions constitute an orthogonal and complete system [28], for a review [29]. Solutions of analogue equations of motion for charged scalars and vector bosons in an external field have been found, see [20] for a review.

Ψ_V and the analogue scalar and vector solutions can be used in perturbation theory to build new Feynman rules and diagrams in order to describe processes in an external field.

The described solutions of the modified Dirac equation (3.4), of the analogue modified Klein-Gordon equation etc. can be used in perturbation theory to build new Feynman rules and diagrams in order to describe processes in an external field.

In particular, here we give the QED FP-Feynman rules at the tree-level, built out of solution (3.5). The fermion two-point function in coordinate space is given by, see Fig. (1):

$$G(x,x') = \frac{1}{(2\pi)^4} \int_{-\infty}^{+\infty} d^4p\, E_p(k \cdot x) \frac{p\!\!\!/ + m}{p^2 - m^2} \overline{E}_p(k \cdot x') e^{ip \cdot (x' - x)}, \tag{3.7}$$

where the usual fermion propagator is sandwiched between E_p factors ($\overline{E}_p = E_p^\dagger \gamma^0$) coming from the Volkov solutions. It is interesting to note that there is a non trivial dependence on the coordinates x, x' within which the fermion propagates, instead of their difference $x' - x$ [25].

Often in laser physics [13], the FP fermion line is interpreted considering a bare fermion "dressed" by an arbitrary number of photons emitted or absorbed from the laser, Fig. 2.

Figure 1: The FP fermion propagator. The double line represents the Volkov solution.
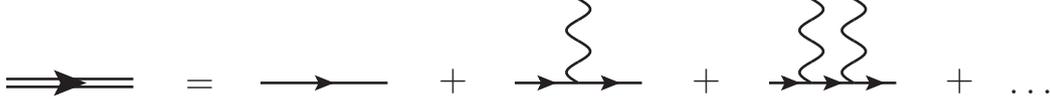
**Figure 2:** Interpretation of the electron propagator derived from the Volkov solution.

The QED vertex in Fig. (3), instead, is given by:

$$-ie\,\gamma_\mu^e = -ie\,(2\pi)^4 \sum_{r=-\infty}^{+\infty} \overline{E}_{p_f}(r)\gamma_\mu E_{p_i}(r)\ \delta^4(p_f + k_f - p_i - rk). \qquad (3.8)$$

Each term of the sum is given by the usual Dirac matrix γ^μ, sandwiched between the factors E_p, $\overline{E}_p$ and multiplied by a δ-function with a momentum conservation law as argument. The latter contains a term $-rk$ that represents the momentum exchanged with the external field.

In the case of a constant crossed field the sum in the vertex becomes an integral:

$$-ie\,\gamma_\mu^e = -ie\,(2\pi)^4 \int_{-\infty}^{+\infty} dr\ \overline{E}_{p_f}(r)\gamma_\mu E_{p_i}(r)\ \delta^4(p_f + k_f - p_i - rk). \qquad (3.9)$$
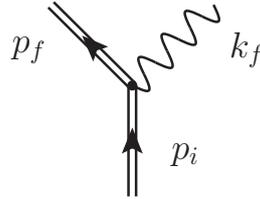

Figure 3: The FP QED vertex.

Initial and final (anti)fermions are described by the usual Dirac spinors $u_p, \bar{u}_p, v_p, \bar{v}_p$ since the E_p factors have been grouped from the Volkov spinors in Feynman rules (3.8) and (3.9). The photon propagator at tree level is unchanged, since the photon has null charge and it does not interact directly to the external field.

The above described Feynman rules are the tool to build every Feynman diagram that is needed, at each order in perturbation expansion. For example, beamstrahlung and coherent pair production can be drawn as FP processes at the 1st-order in perturbation theory, see Fig. (4).

(a) Beamstrahlung

(b) Coherent pair production
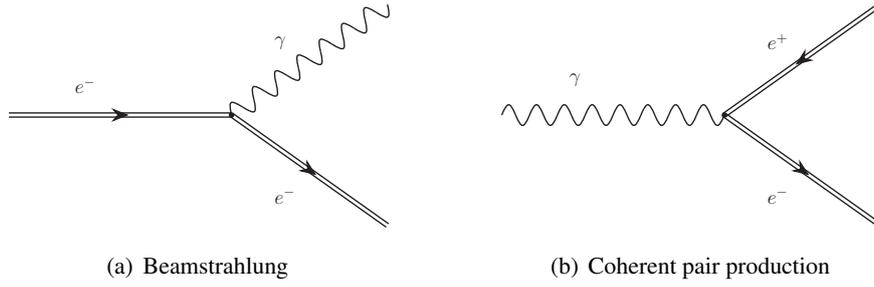
Figure 4: 1st-order FP processes.

Writing down the amplitudes of these two processes, one obtains an expression with a sum coming from the modified QED vertex (3.8). This leads to a naïve interpretation [30] for the new Feynman diagram: it can be seen as sum over all the Feynman graphs characterized with the emission or absorption of r photons from the external[2] see Fig. (5).
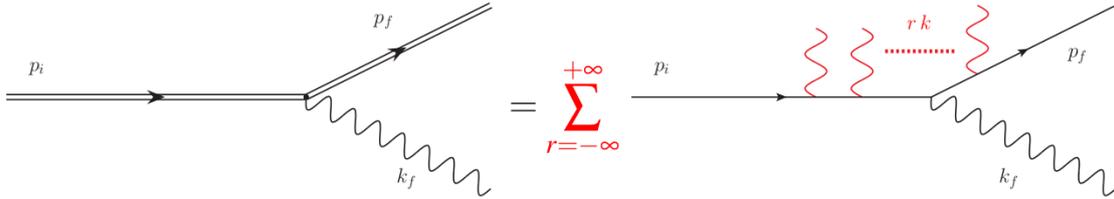
Figure 5: Naïve interpretation of FP beamstrahlung diagram.

This interpretation shows that even if the photons of the external field were not considered at the beginning, they "appear" through the quantum interaction of the fermion with the external field encoded in the Volkov solution.

Typically, performing probability calculations within the FP, one has to to handle integrals over Airy or Bessel functions coming from the E_p factors, that can be simplified using the integral-representation properties of these special functions.

As shortly described in Sec. 2, each colliding particle sees at the IP the superposition of two external fields, but usually only the field from the oncoming bunch is considered. In order to take fully into account the effect of the intense fields of *both* colliding charge bunches at the IP of a collider, new particle wavefunctions are required. These are obtained by solving the Dirac equation minimally coupled to the fields of two, non-collinear, constant crossed fields. Such new solutions will lead to different particle process transition probabilities. Technically, calculations using the new solutions may prove simpler than those in one constant crossed field [31].

For an expression of beamstrahlung process through the Furry picture in *N collinear* constant crossed fields, see [32]. A new EM solver/generic event generator software, `IPStrong`, is being

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[2]In the case of a constant crossed field, and hence of a QED vertex with an integral, one can interpret saying that there has been the absorption/emission either of one photon with momentum rk or of r photons with momentum k.

developed to model the strong field processes at the IP at linear colliders with χ and the FP-cross sections as inputs [31].

4. Alternative: generalized quasi-classical operator method (QOM)

The conceptually simple FP recipe can lead to relatively complex analytical calculations already at the first order, due to the –already mentioned– multidimensional integrations over special functions and polynomials. Only a few tree-level two-vertices processes [33, 19] and one-loop 2-points amplitudes [28] are known, some of them are computed exploiting the optical theorem, see also, [9] and [20].

Asymptotic approximations of results in the FP involving highly energetic particles ('ultra-relativistic', i.e. Lorentz factor $\gamma_L \gg 1$) that are present, for instance, in the IP region of a linear collider, are equivalent to calculations in a fully quasi-classical approximation [22].

Therefore, Baǐer and Katkov invented an effective method for the calculation of such processes in an external field using the a quasi-classical approach from the beginning. Their alternative procedure is well-known as quasi-classical operator method (QOM) [17], for a review [25] and [34]. The QOM is particularly powerful when considering ultrarelativistic initial state particles and its results are implemented in `CAIN` and `GuineaPig` [35] to estimate beamstrahlung and coherent pair production at linear colliders.

The QOM has been originally applied to the case of radiation from a charged particle in an external magnetic field **B** (synchrotron radiation) [17]. The field is considered stationary and $B = |\mathbf{B}| \ll B_{cr}$. From the classical theory the Larmor frequency ω_0 and the peak frequency of the quasi-continuous spectrum ω are such that:

$$\omega_0 \approx \frac{|e|B}{\varepsilon}, \qquad \omega \sim \omega_0 \left(\frac{\varepsilon}{m}\right)^3 \tag{4.1}$$

where ε is the energy of the electron.

In this process one can identify two quantum effects: the quantum propagation of the electron and the quantum recoil on the electron due to the photon emission. The relevance of a quantum effect is encoded by the commutation relations between the operators and by the corresponding dynamical variables in the uncertainty relations.

Exploiting the non-commutativity between the velocity components of the electron in **B**, one obtains the corresponding uncertainty relations

$$\Delta v_i \Delta v_k \sim \frac{e\hbar B}{\varepsilon^2} = \frac{B}{B_{cr}\gamma^2} = \frac{\hbar\omega_0}{\varepsilon}, \tag{4.2}$$

where $\hbar\omega_0$ is the unit energy interval between the possible electron levels in motion in the field **B**. Relations (4.1) and (4.2) show that the motion is increasingly classical for increasing energy ε.

The non-commutativity between the electron and the emitted photon dynamical variables is of order

$$\frac{\hbar\omega}{\varepsilon}. \tag{4.3}$$

Considering however, $\chi \sim \frac{\hbar\omega}{\varepsilon} \gtrsim 1$, it is obvious that the classical theory cannot be applied for the quantum recoil of the emitted photon energy $\hbar\omega$ [17].

Due to (4.2) and (4.3), the key idea of QOM is to consider the motion of the electron as being classical right from the beginning, whereas the quantum recoil from photon emission is not neglected in the calculation of the amplitude. Therefore this method is called *quasi-classical*.

In order to implement the quasi-classical approach, the amplitudes are written in terms of *operators* instead of the corresponding dynamical variables. In particular, the electron solutions of equation of motion in an external field are not written in terms of Volkov solutions, but in the symbolic operator form [25] (operators are denoted by ˆ):

$$\psi(x) = \frac{1}{\sqrt{2\hat{H}}} \, u(\hat{P}) \, e^{-\frac{i}{\hbar}\hat{H}t} \phi(\mathbf{x}) \tag{4.4}$$

where $\phi(x)$ is the classical wavefunction of a spinless particle and

$$u(\hat{P}) = \frac{1}{\sqrt{\hat{H}+m}} \begin{pmatrix} \hat{H}+m \\ \sigma \cdot \hat{\mathbf{P}} \end{pmatrix} \tag{4.5}$$

is the usual Dirac spinor $u(p)$ incorporating the kinematic momentum operator $\hat{\mathbf{P}} = \hat{\mathbf{p}} - e\mathbf{A}$ of the electron within the vector potential $\mathbf{A}$ as well as the energy operator operator $\hat{H} = \sqrt{\hat{\mathbf{P}}^2 + m^2}$. Note, $\hat{\mathbf{P}}$ has not to be misinterpreted as the canonical (or generalized) momentum $\hat{\mathbf{p}}$ that appears also in the Volkov solutions. In (4.4) the information about the external field is implicitly encoded using $\hat{\mathbf{P}}$. The arbitrary two-component spinor w satisfies $w^*w = 1$.

The transition matrix element has the form:

$$U_{fi} = \frac{e\sqrt{2\pi}}{\sqrt{\hbar\omega}} \int dt \, \langle f| \frac{u_f^\dagger(\hat{P})}{\sqrt{2\hat{H}}} (\alpha_i \cdot \varepsilon_\mathbf{i}^*) \frac{u_i(\hat{P})}{\sqrt{2\hat{H}}} \, |i\rangle \, e^{i\omega t} \tag{4.6}$$

where the bra $\langle f|$ and the ket $|i\rangle$ are solutions of the Klein-Gordon equation in the given field, $\alpha_i = \gamma^0\gamma_i$ ($i = 1,2,3$), γ_μ are the Dirac matrices and ε_μ the photon polarization vector.

In the probability calculations the operators are kept up to a later stage where further commutation relations may appear. According to the previous prescriptions, the commutation relations of velocity components of the electron moving in an external field are neglected, while the commutation relations between electron and photon operators are left untouched.

Eventually, one obtains an expression composed by a product of commuting operators, so that that they can be substituted by the corresponding classical values (c-numbers). The simplification comes in that the relatively complicated expressions derived from the exact solutions in an external wave do not appear explicitly while the simple expression for the classical trajectory of the electron in the field is inserted.

The results by Baĭer and Katkov on synchrotron radiation, coherent pair production and annihilation of a pair into a photon [17] are consistent with the ones obtained by Nikishov and Ritus [30] and previously by Klepikov [36]. In the case of the beamstrahlung it has been shown that the transition probabilities obtained using the FP and the QOM are asymptotically identical in the ultra-relativistic limit [37]. The QOM has been successfully applied to processes happening in media, in crystals with strong inter-lattice fields and also in super strong fields (common in astrophysics) [34], [38].

The QOM method could in principle be generalized to processes other than only the photon radiation and cross-symmetric ones. Our present idea is to understand whether it is possible to

apply this kind of approach also to higher order processes, the main object of studies of future linear colliders, as it is possible with the Furry picture, and understand whether it is effective and practical. The process $e^-e^+ \rightarrow \mu^-\mu^+$ is currently under study in the quasi-classical approach. This process has already been studied in Born approximation [39] and in the context of laser-driven reaction from positronium using Volkov solutions [19].

Another promising operator approach, based on the quasi-classical Green's function of the Dirac equation in an arbitrary is being developed lately in the context of laser and atomic physics [40, 41].

5. Conclusions and outlook

The planned linear collider will produce physics processes in the environment of very intense electromagnetic fields possibly exceeding the critical field introduced by Schwinger in the rest frame of the colliding electrons and positrons.

The unstable vacuum present at the interaction points might lead to a regime of nonlinear Quantum Electrodynamics, affecting the processes in the IP area. Such conditions therefore motivate to calculate all probabilities of the physics processes under fully consideration of the external electromagnetic fields affecting the vacuum.

At previous lepton colliders, the much weaker external electromagnetic fields at the IPs did not needed to be considered apart for background processes: the first order background processes as beamstrahlung and coherent pair production, the second order incoherent pair production as well. At future linear colliders the external fields would be orders of magnitude higher so an estimate of the effects on all the processes is requested. As we have shown, indeed, the χ parameter, that encodes the dependence of the probabilities on the intensity of the external field at the IP, is up to 3 orders of magnitude higher at ILC and CLIC than at LEP. In particular at CLIC-3TeV, we would have $\chi_{av} \sim 3.34$, describing a critical regime.

A formally exact method to consider processes in a *classical* electromagnetic environment is given by the Furry picture of quantum states. The interaction with the external field is taken into account not perturbatively and separately from the usual gauge interactions since fully incorporated in modified equations of motion, the solutions of which are utilized in the usual S-matrix formalism.

The quasi-classical operator method (QOM) offers an alternative to FP in the case of ultrarelativistic initial states; a generalization of this method to two vertex processes is now under study.

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