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Neutrino masses in warped extra-dimensions

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Lepton masses and mixings are re investigated within the framework of warped extra-dimensions. The matter fields and the gauge bosons propagate in the bulk, while the Higgs is assumed to be localized on the IR brane. The following three cases have been considered for neutrino masses: 1) Higher dimensional LLHH operator 2)Dirac neutrino 3) Type 1 see-saw with bulk Majorana mass terms. For the first two cases χ^2 minimization is used to fit the charged lepton and neutrino mixing data to the theory parameters. New contributions to lepton flavour violation as a consequence of having the fermions and gauge bosons in the bulk is studied for all three cases. The LLHH case is characterized by very heavy Kaluza Klein (KK) scales for the charged lepton singlets owing to the extreme localization of their zero modes towards the IR brane. This results in weak flavour constraints at the leading order. However the effective 4-D Yukawa coupling to the KK states are very large in the non-perturbative regime. For the Dirac and the bulk Majorana case good fits can be obtained for $\mathcal{O}(1)$ choices of the bulk mass parameters. The profile equations for the the right handed neutrinos for the bulk Majorana case are solved numerically. Lepton flavour violating rates for both the cases are very large. We invoke the Minimal Flavour Violation ansatz implemented in the RS setup as a means to evade the flavour constraints. Examples of flavour symmetry groups is provided for both cases.

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1. Introduction

The Randall-Sundrum (RS) model is one of the most intriguing solutions to the hierarchy problem [1]. It is characterized by single extra-dimension compactified on an S_1/Z_2 orbifold. Two opposite tension branes, situated at the two orbifold fixed points, are characterized by the UV and IR scales respectively. The metric for the RS background is given as

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \tag{1.1}$$

with $\sigma(y) = k|y|$ where k is the reduced Planck scale. In the original setup, all the SM fields were localized on the IR brane while only gravity propagated in the bulk. The solution to the hierarchy problem is achieved by means of exponential warping of scales.

Fermions in the bulk offers a geometric solution to the flavour hierarchy problem [2]. Localizing the fermion generations at different points in the bulk leads to varying degrees of overlap of the fermion profiles with the Higgs. This can naturally explain the hierarchy in the fermion masses in addition to the mixing parameters by choice of $\mathcal{O}(1)$ parameters in the fundamental theory. In the hadronic sector the implication of bulk fermions on FCNC have been investigated by various authors[3, 4, 5, 6, 7]. The situation with leptons, especially the neutrinos is a bit more involved.

In this work we consider the case where the Higgs is localized on the IR brane while the fermions and the gauge fields are in the bulk. We explore three cases for neutrino mass generation: 1) LLHH operator 2) Dirac neutrinos 3) Type -1 seesaw with bulk Majorana mass terms. The bulk parameters - Dirac masses and the $\mathcal{O}(1)$ Yukawa parameters were varied simultaneously to numerically minimize a χ^2 function, defined as a function of the lepton masses and mixing, to arrive at the best fit regions of the parameter space. The range of bulk parameters which fit the charged lepton data are strongly constrained by flavour considerations.

The theory parameters include the bulk mass parameter and the $\mathcal{O}(1)$ Yukawa couplings while the charged lepton masses, two neutrino mass squared differences and the three mixing angles constitute the observable for the χ^2. The central values of the observables used in our analysis is given below [8, 9]

Table 1: Experimental Data

masses (MeV)	mass-squared (eV^2)	mixing angles
$m_e = 0.51^{+0.0000007}_{-0.0000007}$	$\Delta m^2_{12} = 7.59^{+0.20}_{-0.21} \times 10^{-5}$	$\theta_{12} = 0.59^{+0.02}_{-0.015}$
$m_\mu = 105.6^{+0.000003}_{-0.000003}$	$\Delta m^2_{23} = 2.43^{+0.13}_{-0.13} \times 10^{-3}$	$\theta_{23} = 0.79^{+0.12}_{-0.12}$
$m_\tau = 1776^{+0.00016}_{-0.00016}$		$\theta_{13} = 0.154^{+0.016}_{-0.016}$

The standard definition of χ^2 for N observables is used for the analysis and is given by

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{y_i^{exp} - y_i^{theory}}{\sigma_i} \right)^2 \tag{1.2}$$

where, y_i^{theory} is the value of the i^{th} observable predicted by the model and y_i^{exp} is its corresponding experimental number measured with a uncertainty of σ_i. Since, the values of the charged lepton are measured to a very high accuracy, it is difficult to fit masses to such high accuracy. Thus, we incorporate up to $\sim 1.5\%$ errors in the masses of charged leptons.

The minimization for the χ^2 was performed using MINUIT [10] which looks for a local minimum for a given set of input theory parameters. The scan is repeated by randomly varying the input parameters thus arriving at a global minima. Flavour constraints in both the Dirac and the Majorana cases were evaded by invoking the Minimal Flavour Violation (MFV) ansatz implemented in the RS scenario.[11, 12]. We provide example symmetry groups where the flavor violating constraints can be removed for both the Dirac and the Majorana cases. In section (5) we give examples of flavour groups for the MFV ansatz applied to both the Dirac and the bulk Majorana case. We now discuss the fermion mass fits and the flavour constraints for all the three cases in the following three sections.

2. LLHH operator

The neutrino masses are assumed to be generated by a higher dimensional operator suppressed by the fundamental scale of the theory *i.e* M_{Planck}. In the SM such an operator leads to very small neutrino masses. We study the implications of such an operator in the RS setup. The five dimensional action has the form:

$$\begin{aligned} S &= S_{kin} + S_{Yuk} + S_\nu + S_{higgs} \\ S_{kin} &= \int d^4x \int dy\, \sqrt{-g}\ \left(\bar{L}(iD_M\Gamma^m - m_L)L + \bar{E}(i\Gamma^m D_m - m_E)E + \dots \right) \\ S_{Yuk} &= \int d^4x \int dy\, \sqrt{-g} \left(Y_U \bar{Q} U \tilde{H} + Y_D \bar{Q} D H + Y_E \bar{L} E H \right) \delta(y - \pi R) \\ S_\nu &= \int d^4x \int dy\, \sqrt{-g} \left(\frac{\kappa}{\Lambda^{(5)}} LHLH \right) \delta(y - \pi R) \end{aligned} \tag{2.1}$$

The details of the Kaluza Klein (KK) reduction and the orthonormality relations are presented in the [13]. The effective zero mode mass matrix for the leptons is given as

$$\begin{aligned} (\mathscr{M}_e^{(0,0)})_{ij} &= \frac{v}{\sqrt{2}} (Y_E')_{ij} e^{(1-c_L-c_E)kR\pi} \sqrt{\frac{(0.5-c_{L_i})}{e^{(1-2c_{L_i})\pi kR}-1}} \sqrt{\frac{(0.5-c_{E_j})}{e^{(1-2c_{E_j})\pi kR}-1}}, \\ (\mathscr{M}_\nu^{(0,0)})_{ij} &= \frac{v^2}{2\Lambda^{(5)}} (\kappa')_{ij} e^{(2-c_{L_i}-c_{L_j})kR\pi} \sqrt{\frac{(0.5-c_{L_i})}{e^{(1-2c_{L_i})\pi kR}-1}} \sqrt{\frac{(0.5-c_{L_j})}{e^{(1-2c_{L_j})\pi kR}-1}} \end{aligned} \tag{2.2}$$

where we have defined the $\mathscr{O}(1)$ Yukawa coupling as

$$Y_E' = 2kY_E \ \ ; \ \ \kappa' = 2k\kappa \tag{2.3}$$

The fifteen Yukawa couplings and the six $c_{L,E}$ parameters are varied simultaneously so as to minimize the function in Eq.(1.2). The points which give a χ^2 between 1 and 8 are considered to give a 'good fit' to the data. The range for the scan of the $c_{L,E}$ has been judiciously chosen between 0.82 and 1.0 for bulk doublets and $-5\times 10^7 < c_{E_1} < -0.2$, $-10^8 < c_{E_2} < -8000$ and $-10^9 < c_{E_3} < -9000$ for first, second and third generation charged singlets respectively. A larger democratic range does not change the results significantly. Fig.[1] presents the regions in $c_{L_{1,2,3}}$

and $c_{E_{1,2,3}}$ which have minimum χ^2 assuming normal hierarchy for neutrino masses. The allowed ranges in the $c_{L,E}$ which satisfy the minimum χ^2 requirement are summarized in Table[2]. We find that the lepton doublets are required to be localized very close to the UV brane to fit small neutrino masses. In order to offset this UV localization of the doublets, the zero mode of the charged singlets must be almost localized on the IR brane as is evident from the large negative values of c_{E_i}.

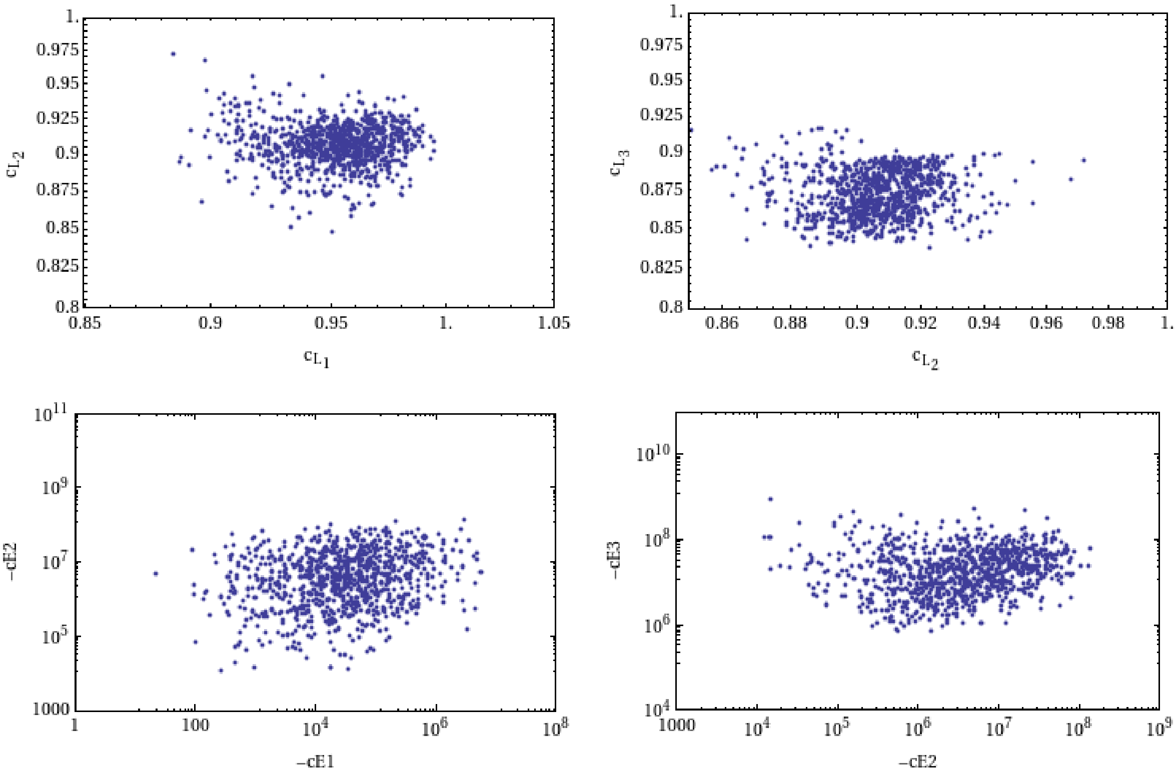

Figure 1: Regions in c_i for the LHLH case which give best fit to lepton masses and mixing. The graphs in the upper row shows the region of parameter space for the bulk masses for doublets which fits small neutrino masses. Neutrino masses are assumed to have normal hierarchy in this analysis. The graphs in the lower row shows the region for the bulk masses for the charged singlets c_{E_i}. We have used log scale for c_{E_i}

The large negative values of the c_{E_i} parameters have some implications in terms of the AdS/CFT correspondence[14, 15]. The CFT interpretation for the bulk scalars has been studied in [16, 15] and for bulk fermions in [17]. The best fit $c_{L,E}$ parameters of LLHH case given in Table[2] leads to an unusual situation where the left handed leptons are almost completely elementary while the right handed singlets are completely composite. This can be understood using the 'holographic basis' of [18]. The composite component for the zero mode of the doublets is proportional to $e^{-(c_L-0.5)kR\pi}$. It goes to 0 when $c_L \to 0.99$ thus making zero modes for the doublets elementary. For the c_E fields however, the elementary component for the zero mode is given as $\sqrt{(c_E-0.5)(c_E+1.5)}e^{-|1.5+c_E|kR\pi}$. Thus the zero mode for the charged singlets are characterized by vanishing elementary component and are completely composite fields. The effective 4-D Yukawa coupling of the zero mode to the KK modes, is given as $Y'_E\sqrt{(0.5-c_E)}$. A problematic feature of these models is that this coupling enters the non-perturbative regime for c_E large and

Table 2: Allowed range for the bulk parameters with minimum χ^2. Neutrino masses have normal hierarchy. Range of first KK scale of the doublet(singlet) $M_L^{(1)}(M_E^{(1)})$ corresponding to the bulk mass parameter $c_L(c_E)$ is also given.

parameter	range	range of $M_L^{(1)}$ (TeV)	parameter	range	range of $M_E^{(1)}$(TeV)
c_{L_1}	0.87-0.995	1.49-1.59	c_{E_1}	-10.0 to -5.0×10^6	7.9-3.9 $\times 10^6$
c_{L_2}	0.86-0.98	1.48-1.58	c_{E_2}	-1.0×10^4 to -1.2×10^8	7.9×10^3-9.5×10^7
c_{L_3}	0.84-0.92	1.47-1.53	c_{E_3}	-7.0×10^5 to -1×10^9	5.5×10^5 7.9×10^8

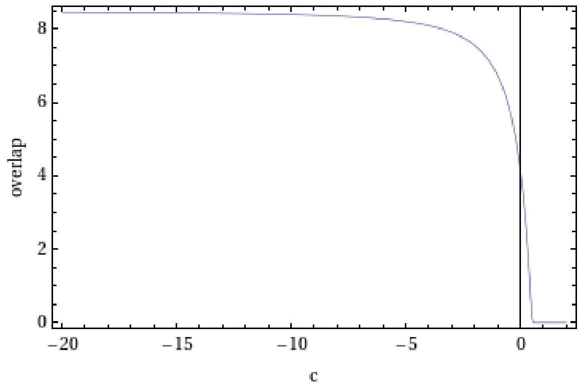

Figure 2: Coupling of two zero mode fermions to Z_1 as a function of bulk mass parameter [19].

negative.

2.1 Flavour Constraints on LLHH case

In the LLHH case where the doublets (singlets) are sufficiently localized towards the UV(IR), the contributions to trilepton decays from graphs like Fig.[7] are highly suppressed. This is due to the fact that for zero mode fermions sufficiently localized towards the IR and UV brane, the coupling with the first KK state of the Z boson is universal as shown in Fig[2]. Other potentially large contributions comes from the large mixing between zero mode charged singlet states and the first KK modes of the lepton doublets; the corresponding Yukawa coupling is very large due to the large negative c_E values. However for a fairly degenerate bulk doublet masses, (c_{L_i}), the contributions are negligible. The contribution to $l_j \to l_i\gamma$ due to loop diagram in Fig.[7] are heavily suppressed owing to the heavy KK mass scales corresponding to the charged singlets. The corresponding masses are in shown in Table[2]. Additionally, the large effective 4-D Yukawa couplings of the charged singlets to the KK modes make it difficult to apply techniques of perturbation theory to calculate one Feynman graph of the form $j \to i\gamma$ due to Higgs in the loop[13].

3. Dirac Neutrinos

Three singlet right handed neutrino are added to the SM spectrum. Global lepton number violating effects are assumed to be highly suppressed.

The bulk and Yukawa actions for this take the form:

$$\begin{aligned} S_{kin} &= \int d^4x \int dy \sqrt{-g} \left(\bar{L}(i\Gamma^M D_M - m_L)L + \bar{E}(i\Gamma^M D_M - m_E)E + \bar{N}(i\Gamma^M D_M - m_N)N \right) \\ S_{yuk} &= \int d^4x \int dy \sqrt{-g} \left(Y_N \bar{L}NH + Y_E \bar{L}EH \right) \delta(y - \pi R), \end{aligned} \tag{3.1}$$

Performing the KK reductions and imposing the orthonormality conditions [13], we arrive at the following expression for the zero mode mass matrix for the charged leptons and the neutrinos

$$\begin{aligned} (\mathscr{M}_e^{(0,0)})_{ij} &= \frac{v}{\sqrt{2}} (Y_E')_{ij} e^{(1-c_{L_i}-c_{E_j})kR\pi} \sqrt{\frac{(0.5-c_{L_i})}{e^{(1-2c_{L_i})\pi kR}-1}} \sqrt{\frac{(0.5-c_{E_j})}{e^{(1-2c_{E_j})\pi kR}-1}} \\ (\mathscr{M}_\nu^{(0,0)})_{ij} &= \frac{v}{\sqrt{2}} (Y_N')_{ij} e^{(1-c_{L_i}-c_{N_j})kR\pi} \sqrt{\frac{(0.5-c_{L_i})}{e^{(1-2c_{L_i})\pi kR}-1}} \sqrt{\frac{(0.5-c_{N_j})}{e^{(1-2c_{N_j})\pi kR}-1}}, \end{aligned} \tag{3.2}$$

where $Y'_{E,N} = 2kY_{E,N}$. A total of 27 parameters, which include 9 c_i and 18 $\mathscr{O}(1)$ Yukawa parameters are simultaneously varied to minimize the χ^2 The doublets (c_{L_i}) and the the charged singlets are varied between 0.02 and 1, while the neutral singlets are varied between between 1 and 1.9. The order one Yukawa couplings, $Y'_{E,N}$, are varied randomly between -4 and 4 with a lower bound $|Y| \gtrsim 0.08$. The points in the bulk mass parameter space which satisfy $0 < \chi^2 < 8$ constitute the best fit regions. Figs.[3,4,5] represents the region of parameter space which satisfies this constraint. Table[3] summarizes the ranges for the fit.

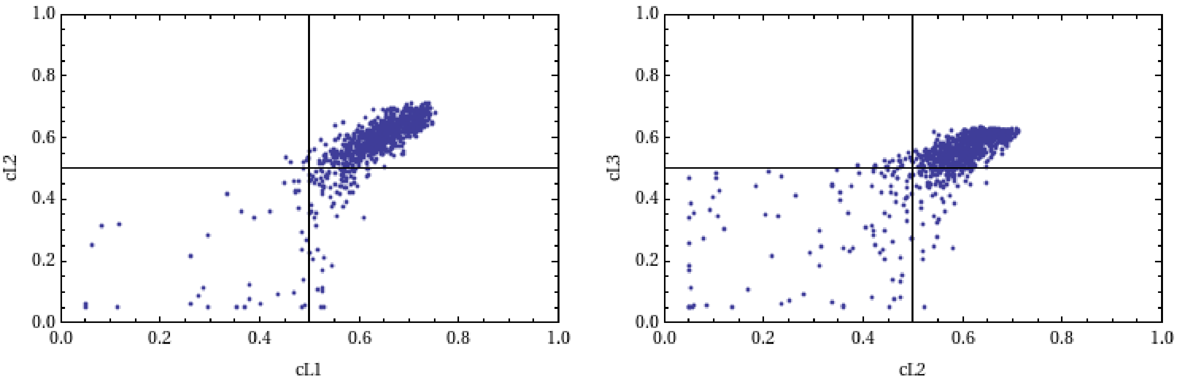

Figure 3: The figures above correspond to the case in which neutrinos are of Dirac type. The points in the above figures correspond to a χ^2 between 1 and 8. The plot represents the parameter space for the bulk masses of the doublets. This case corresponds to the normal hierarchical case.

The Dirac neutrino mass matrix in the RS model seems to fit the data more naturally compared to the *LHLH* discussed in the previous subsection. A large section of the points fall in the regime $c_i > 1/2$ indicating that they are localized closer to the UV brane. This implies that the zero modes of the charged and neutral leptons are elementary components of the CFT owing to their localization closer to the UV brane.

3.1 Flavour constraints on Dirac Neutrinos

In comparison the LLHH case the Dirac case gives a good fit to the leptonic data for a reasonable choice of $\mathscr{O}(1)$ parameters. Flavour considerations however, place very tight constraints

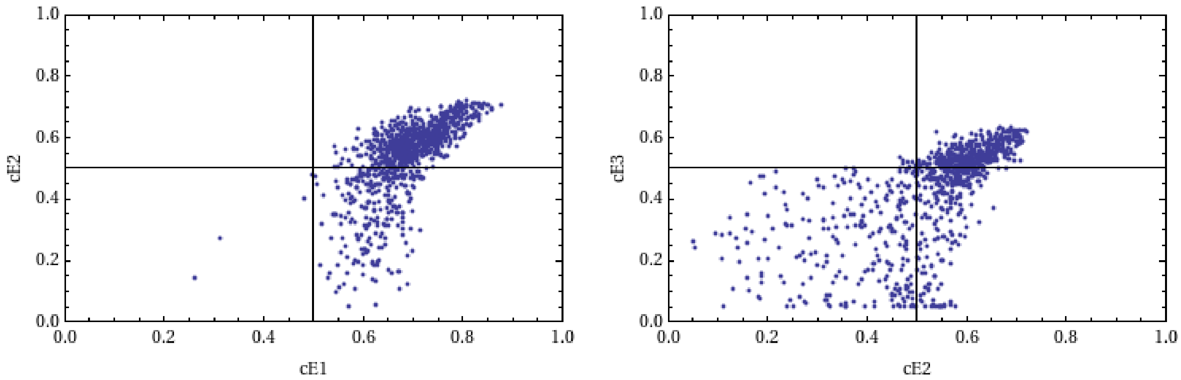

Figure 4: The plot represents the parameter space for the bulk masses of charged singlets.

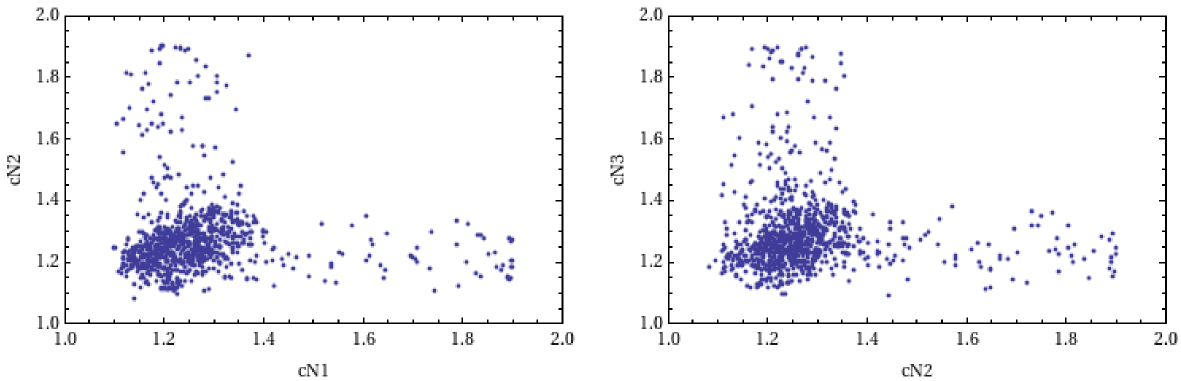

Figure 5: The plot represents the parameter space for the bulk masses of neutrino singlets.

Table 3: Allowed ranges of bulk parameters with normal hierarchy of neutrino masses. The range of first KK scale corresponding to the range of c values is also given.

parameter	range	$M_L^{(1)}$ TeV	parameter	range	$M_E^{(1)}$ TeV	parameter	range	$M_\nu^{(1)}$ TeV
c_{L_1}	0.05-0.76	0.839-1.4	c_{E_1}	0.2-0.88	0.959-1.5	c_{N_1}	1.1-1.9	1.67-2.31
c_{L_2}	0.05-0.72	0.839-1.37	c_{E_2}	0.05-0.73	0.839-1.38	c_{N_2}	1.1-1.9	1.67-2.31
c_{L_3}	0.05-0.64	0.839-1.31	c_{E_3}	0.05-0.64	0.839-1.31	c_{N_3}	1.1-1.9	1.67-2.31

on the parameter space. For the Dirac case the dominant contribution to tree-level decays comes from the tree level diagram in Fig.[7]. Fig.[6] shows the points within the best fit parameter space consistent with all constraints from tree-level processes. The black point is allowed for a KK gauge boson scale of 1.9 TeV, where as the green points are for mass of 3 TeV. The constraints from dipole processes are far more severe. The constraint from $\mu \to e\gamma$ required a KK fermion mass scale $\mathcal{O}(10)$ TeV to suppress it below the current experimental limit of 5.7×10^{-13} [20].

Thus all the points which give a 'small' χ^2 are in conflict with the flavour data for a fermion KK scale $\sim (2-3)$ TeV. In summary, the Dirac case is only viable with very large KK masses or extremely fine tuned fermion mass fits.

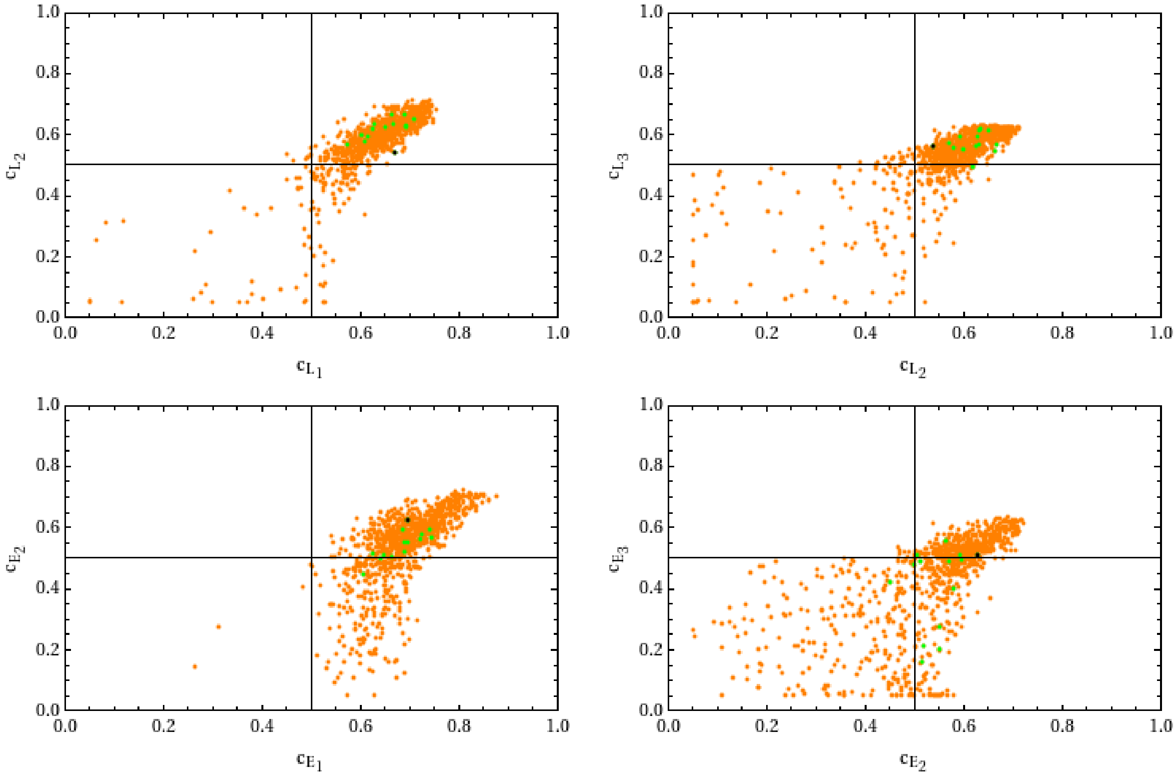

Figure 6: The black dot and the green region represent the parameter space permitted by tree-level constraints for a KK gauge boson scale of 1920 and 3000 GeV respectively

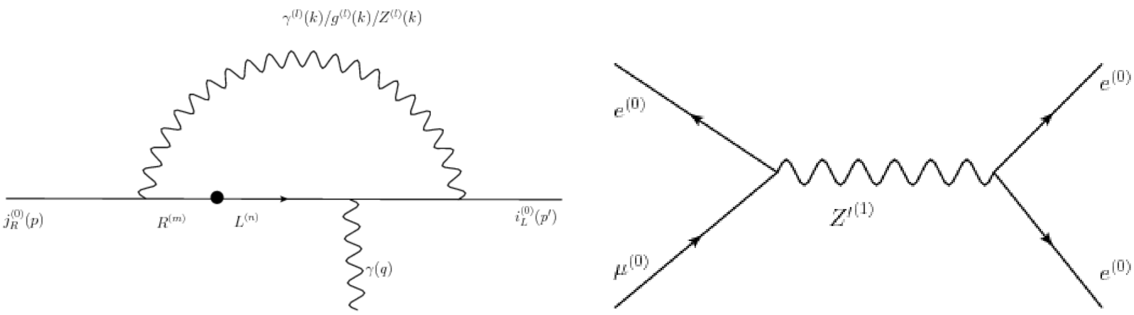

Figure 7: Left panel shows the one loop diagram for $j \to i\gamma$ due to gauge boson in the loop. Right panel shows a tree level graph for flavour violating decays of the form $l_i \to l_j l_k l_k$ due to exchange of first KK state of Z.

4. Bulk Majorana mass term

Lepton number violating bulk Majorana mass terms for the singlet neutrinos are introduced in the bulk in addition to the bulk Dirac mass terms. This leads to an additional parameter in the theory which controls the localization of the right handed neutrinos. At the effective 4-D level, this case is the same as the Type -1 see-saw mechanism for for generating small neutrino masses The case with bulk Majorana mass terms has been first considered in [21]. Scenarios with such mass terms localized on the boundary have been investigated by numerous authors [22, 23, 24, 12]. We extend the analysis of [21] by computing the numerical solutions to the profiles of the right handed neutrinos. The part of the action which contains the singlet right handed neutrinos is given by

$$S_N = \int d^4x \int dy \sqrt{-g} \left(m_M \bar{N} N^c + m_D \bar{N} N + \delta(y - \pi R) Y_N \bar{L} \tilde{H} N \right) \tag{4.1}$$

where $N^c = C_5 \bar{N}^T$ with C_5 being the five-dimensional charge conjugation matrix[1] and $m_M = c_M k$, with k being the reduced Planck scale[2]. The bulk Dirac mass for the right handed neutrino is parametrized as $m_D = c_N k$. As before we consider all the mass parameters to be real. The bulk singlet fields N have the following KK expansions:

$$N_L(x,y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} e^{2\sigma(y)} N_L^{(n)}(x) g_L^{(n)}(y) \; ; \; N_R(x,y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} e^{2\sigma(y)} N_R^{(n)}(x) g_R^{(n)}(y), \tag{4.2}$$

where g_L and g_R are profiles of the singlet neutrinos in the bulk. They follow the following orthonormal conditions

$$\frac{1}{2\pi R} \int_{-\pi R}^{\pi R} dy \, e^{\sigma} \left(g_L^{(n)} g_L^{(m)} + g_R^{(n)} g_R^{(m)} \right) = \delta^{(n,m)} \tag{4.3}$$

Using this, the eigenvalues equations for the $g_{L,R}$ fields become [21]

$$\begin{aligned} (\partial_y + m_D) g_L^{(n)}(y) &= m_n e^{\sigma} g_R^{(n)}(y) - m_M g_R^{(n)}(y) \\ (-\partial_y + m_D) g_R^{(n)}(y) &= m_n e^{\sigma} g_L^{(n)}(y) - m_M g_L^{(n)}(y) \end{aligned} \tag{4.4}$$

where we have assumed the five dimensional wave functions to be real. The system in Eq.(4.4) does not admit a solution consistent with the zero mode mass being 0 [21, 13]. Thus at the effective 4-D level we will have an infinite dimensional Majorana mass matrix. The solutions to Eq.(4.4) are complicated to solve analytically. We have obtained the numerical solutions of $g_{L,R}$ by solving the second order equations derived from Eq.(4.4). We assume g_L to be even under Z_2 and the second order differential equation for g_L takes the form:

$$g_L''(y) - \frac{m_n k R e^{kRy}}{m_n e^{kRy} - c_M k} g_L'(y) - \left(\frac{c_N m_n e^{kRy} k^2}{m_n e^{kRy} - c_M k} + c_N^2 k^2 - \left(m_n e^{kRy} - c_M k \right)^2 \right) R^2 g_L(y) = 0 \tag{4.5}$$

where we have used the notation $m_D = c_N k$ and $m_M = c_M k$ introduced earlier. The primes on $g_L(y)$ derivatives on the profiles. In Fig.[8] we present some sample solutions to Eq.(4.5) for a fixed value of $c_N = 0.58$ while c_M is varied from 0.55 to 1

The charged fermion mass matrix takes the same form as the earlier sections. For the singlet Majorana mass matrix, we assume $c_{N_i} = c_N \forall i$ nd $c_{M_i} = c_M$ for all the three generations. The light neutrino mass matrix takes the form

$$m_\nu^{(0,0)} = Y_N' e^{(1-c_L)kR\pi} g_L(\pi R) (M_R^{-1}) Y_N' e^{(1-c_L)kR\pi} g_L(\pi R) \tag{4.6}$$

where $Y_N' = 2kY_N$. In Table (4), we present two sample points one for inverted hierarchy and another for normal hierarchy, which fit the neutrino masses and mixing angles as well as charged lepton masses. The corresponding Yukawa coupling matrices are presented in Eqs. (4.7,4.8).

[1] C_5 is taken to be C_4.

[2] Majorana mass terms does not have the same interpretation in the bulk as in 4D.

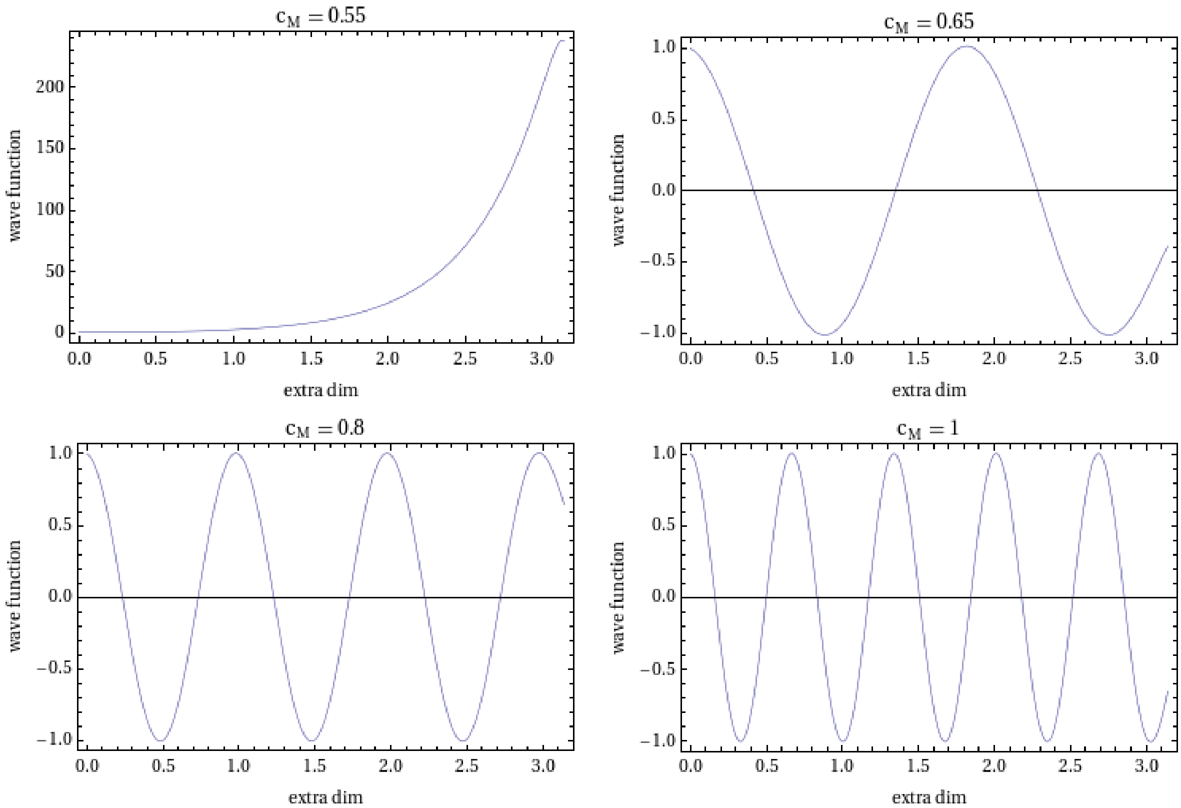

Figure 8: The Figure shows the form of the profile for solution to Eq. for a fixed bulk dirac mass of 0.58 for the right handed neutrinos.

Yukawa parameters for inverted hierarchy

$$Y'_N = \begin{pmatrix} 2.73 & 1.81 & .108 \\ -0.83 & 0.975 & .328 \\ 0.327 & -0.679 & .182 \end{pmatrix} \quad Y'_E = \begin{bmatrix} 3.44 & -0.41 & .87 \\ 0.62 & 1.583 & 0.332 \\ 2.74 & 0.55 & 2.33 \end{bmatrix} \tag{4.7}$$

Yukawa parameters for normal hierarchy

$$Y'_N = \begin{bmatrix} 2.56 & 1.69 & 1.26 \\ -0.795 & 0.927 & 3.89 \\ 0.414 & -0.859 & 2.86 \end{bmatrix} \quad Y'_E = \begin{bmatrix} 2.825 & -0.41 & .87 \\ 0.62 & 1.2008 & 0.332 \\ 2.74 & 0.55 & 2.31 \end{bmatrix} \tag{4.8}$$

4.1 Flavour constraints on scenarios with bulk Majorana mass

The tree-level decays only constrain the parameter space of the bulk doublets and charged singlets as seen in Fig.[6]. Since, the charged lepton mass fitting is independent of any right handed neutrino parameter, the constraints coming from tree-level decays in the Dirac case are applicable in this case as well. The dominant contribution to dipole decays in this case is due to Higgs in the loop. They are calculated for the both the normal and inverted hierarchy cases presented earlier and are given in Table[5]. The branching fractions are evaluated for $M_{KK} \sim 1250$ GeV which is the first KK scale of the doublet.

Table 4: Sample points with corresponding fits of observables for Normal and Inverted Hierarchy schemes in Bulk Majorana case with O(1) Yukawas. The masses are in GeV

Parameter	Normal	Inverted
M_{kk}	161.4	161.4
c_{M_i}	0.55	0.55
$g_L^{(1)}(\pi R)$	3×10^{-13}	1.2×10^{-12}
c_{L_1}	0.58	0.59
c_{L_2}	0.56	0.57
c_{L_3}	0.55	0.55
c_{E_1}	0.735	0.735
c_{E_2}	0.5755	0.575
c_{E_3}	0.501	0.501
c_{N_i}	0.58	0.58
m_e	5.09×10^{-4}	5.08×10^{-4}
m_μ	0.1055	0.1055
m_τ	1.77	1.774
θ_{12}	0.58	0.58
θ_{23}	0.80	0.8
θ_{13}	0.13	0.13
Δm^2_{sol}	7.8×10^{-23}	7.8×10^{-23}
Δm^2_{atm}	2.4×10^{-21}	2.4×10^{-21}

Table 5: BR for dipole decays for the case with bulk Majorana mass for the points in Table 4

Hierarchy	BR($\mu\to e\gamma$)	BR($\tau\to\mu\gamma$)	BR($\tau\to e\gamma$)
Inverted	2.4×10^{-5}	1.9×10^{-5}	7.6×10^{-6}
Normal	1.4×10^{-5}	3.4×10^{-5}	1.3×10^{-5}

5. Flavour symmetries

The strong misalignment between the Yukawa coupling matrix and bulk mass terms which determine the profile is the cause of the large flavor violating transitions leading to strong restrictions on these models. This necessitates the need for imposing flavour symmetries to ease the constraints on the model. We adapt the Minimal Flavour violation ansatz which reduces the misalignment by demanding an alignment between the Yukawa matrices and the bulk parameters. The basic premise of the MFV ansatz is that the Yukawa couplings are the only sources of flavor violation. In the RS setting this would require that the bulk mass terms should now be expressed in terms of the Yukawa couplings [11]. The exact expression would depend on the particle content and the flavor symmetry assumed.

5.1 Dirac Neutrino Case

In the presence of right handed neutrinos the flavour group is $SU(3)_L \times SU(3)_E \times SU(3)_N$; the lepton number is conserved. The Y_E transforms as $Y_E \to (3,\bar{3},1)$ and Y_N transforms as $Y_N \to (3,1,\bar{3})$. The Yukawa couplings are aligned with the five dimensional bulk mass matrices. The bulk masses can be expressed in terms of the Yukawas as

$$c_L = a_1 I + a_2 Y'_E Y'^{\dagger}_E + a_3 Y'_N Y'^{\dagger}_N \quad c_E = b Y'^{\dagger}_E Y'_E \quad c_N = c Y'^{\dagger}_N Y'_N \tag{5.1}$$

where a,b,c $\in \Re$ and $Y'_{E,N}$ are as defined earlier as $Y'_{E,N} = 2kY_{E,N}$. Owing to the flavor symmetry we work in a basis in which Y'_E is diagonal. We then rotate Y'_N by the PMNS matrix *i.e,* writing $Y'_N \to V_{PMNS}\text{Diag}(Y'_N)$

The simplest Yukawa combination transforming as (8,1,1) under the flavour group is given as

$$\Delta = Y'_N Y'^{\dagger}_N \tag{5.2}$$

Thus the BR for $\mu \to e\gamma$, which is the most constrained is given as [12]

$$BR(\mu \to e\gamma) = 4 \times 10^{-8}\, (Y'_N Y'^{\dagger}_N)^2_{12} \left(\frac{3\text{TeV}}{M_{KK}}\right)^4 \tag{5.3}$$

For a appropriate choice of $\mathscr{O}(1)$ parameter Y'_N, consistent with the neutrino mixing data, the (1,2) element of Δ which is responsible for $\mu \to e\gamma$ can be made small enough to be consistent with the experimental bound for a fermion KK mass of around 3 TeV.

5.2 Bulk Majorana mass term

Owing to the presence of a bulk Majorana mass term, we choose the flavour group for the lagrangian in Eq.(4.1) to be $SU(3)_L \times SU(3)_E \times O(3)_N$. Y_E transforms as $Y_E \to (3,\bar{3},1)$ and Y_N transforms as $Y_N \to (3,1,3)$. The bulk Majorana term $\bar{N}^c N$ transforms as $(1,1,6)$ under this flavour group. In terms of the dimensionless Yukawa couplings, $Y'_{E,N}$ the bulk mass parameters can be expressed as

$$c_L = a_1 I + a_2 Y'_E Y'^{\dagger}_E + a_3 Y'_N Y'^{T}_N \quad c_E = 1 + b Y'^{\dagger}_E Y'_E \quad c_N = 1 + c Y'^{T}_N Y'_N \quad c_M = d I_{3\times 3} \tag{5.4}$$

where a,b,c,d $\in \Re$. Working in a basis in which Y'_E is diagonal and $Y'_N = V_{PMNS}\text{Diag}(Y'_N)$, with suitable choices of $\mathscr{O}(1)$ parameter Y'_N the MFV ansatz can ease the stringent constraints from flavour changing processes.

6. Summary and Outlook

The set up of Randall-Sundrum provides a natural framework to understand the fermion mass hierarchy and mixing angles. The quark sector has been explored in considerable detail. The neutrino sector offers various possibilities as far as Dirac or Majorana nature of the neutrino are concerned. In this work we quantify the parameter space of both the $\mathscr{O}(1)$ (dimensionless) Yukawa couplings as well as the bulk mass parameters which determine the fits to the charged lepton and neutrino mixing data

We considered the RS setup with the Higgs localized on the IR boundary while the fermions and the gauge bosons are in the bulk. Three models of neutrino mass generation were considered (a) The LH LH higher dimensional operator (b) The Dirac case and the (c) Majorana case.

While the LHLH case did not admit a fit to the lepton and neutrino data with $\mathcal{O}(1)$ choice of bulk mass parameters the Dirac and the Majorana cases were favourable as far as choice of bulk mass parameters is concerned. The LHLH case is characterized by weak flavour violating constraints at the leading order. However the large Yukawa coupling between the zero mode fermions and the KK modes make it unattractive from the perturbation theory point of view. For the Dirac case the most stringent constraint comes from the loop decays like like $\mu \to e + \gamma$ which requires the KK states of fermions to be considerably heavy. For the bulk Majorana case too, the points we have considered display strong constraints from leptonic flavor violation and are ruled out. They can be alleviated by invoking the MFV ansatz and is found to considerably lower the KK scales. A more detailed analysis of the Majorana case will be presented in an upcoming publication[25].

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