

Axiology

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Axions might play a crucial role for the solution of the strong CP problem and explanation of cold dark matter in the universe. In addition they may find applications in the formulation of inflationary models for the early universe and can serve as candidates for quintessence. We show that all these phenomena can be described within a single framework exhibiting a specific pattern of mass scales: the axionic see-saw. We also discuss the role of supersymmetry (susy) in this axionic system in two specific examples: weak scale susy in the (multi) TeV range and tele-susy with a breakdown scale coinciding with the decay constant of the QCD axion: $f_a \sim 10^{11} - 10^{12}$ GeV.

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1. Introduction

Axions are well motivated candidates for physics Beyond the Standard Model (BSM) of particle physics [1]. Most notably they have played an important role in the discussion of three specific topics (the three “useful” axions):

- the solution of the strong CP problem [2, 3, 4, 5, 6],
- the formulation of natural inflation [7, 8],
- candidates for quintessence [9, 10, 11, 12].

Moreover, the (so called invisible) QCD axion that has been postulated in the context of the strong CP problem provides a perfect candidate for Cold Dark Matter (CDM) [1, 14, 13, 15]. This constitutes an alternative to the WIMP paradigm for CDM in other approaches to physics BSM, such as supersymmetry (susy), which also address the hierarchy problem of the weak scale compared to the Planck scale. Susy solves the hierarchy problem by postulating new supersymmetric partners at a scale not too far above the weak scale.

Axion scenarios are such that do not add anything new to the solution of the hierarchy problem. Here we consider axions and supersymmetry as two independent approaches to physics BSM. It is known that they can coexist in meaningful ways [16], but they do not necessarily need each other. In the present paper we shall analyze this coexistence and discuss possible relations between the mass scales of these theories, as *e.g.* the scale of supersymmetry breakdown and the axion decay constant.

All attempts of physics BSM would ultimately need an ultraviolet completion which we assume to be string theory. Axionic particles are abundant in string theory [17, 18] and the axions mentioned above can be integrated in the scheme. Generically the axion decay constant f_a is expected to be of order of the string scale M_{string} . Axions are perturbatively massless and receive masses via nonperturbative effects. Depending on the size of these effects we can have a variety of mass scales, as *e.g.* from 10^{13} GeV for the inflationary axion down to 10^{-32} eV for the quintessential axion. As we said earlier, generically one would expect the axion decay constants to be of order of the string scale; however, the scale of a successful QCD axion is found to be hierarchically smaller: $f_a \sim 10^9 - 10^{12}$ GeV [19]. Thus string theory needs a mechanism to lower the scale of the QCD axion.

This brings us back to supersymmetry. String theories seem to need some amount of supersymmetry for internal consistency and the absence of tachyons. We do not know the scale of supersymmetry breakdown, but we expect it to be sufficiently small compared to the string scale, supported by explicit string theory constructions towards the Minimal Supersymmetric Standard Model (MSSM) [20]. Of course, this is all a question of experimental verification and we hope that experiments at the LHC will provide useful hints for the nature of physics BSM. Unfortunately, no sign of new physics has been seen there, except for the recently announced evidence for the existence of the Standard Model (SM) Higgs boson at approximately 125 GeV. Let us suppose that the Higgs mass is in this range. For the MSSM this mass scale is rather high, indicating a high scale of susy breakdown at the multi-TeV range [21]. On the contrary, for the SM without supersymmetry

this Higgs mass is rather low. Evolution of the Higgs self coupling λ would lead to negative values at scales small compared to the Planck scale [22, 23]. This would indicate a vacuum instability that would require new physics BSM (*e.g.* supersymmetry) at this scale. For a Higgs mass of 125 GeV the mass range of instability is $10^8 - 10^{15}$ GeV [23] and coincides with the scale of the QCD axion. Is this an accident? Supersymmetry at this scale could be a reason for vanishing λ via a shift symmetry as noted in [24], that is a property of many successful model constructions in (heterotic) string theory [20].

In the paper [25] we speculate that this is an indication for the presence of a remote supersymmetry (tele-susy) somewhere in the range between multi-TeV and the axion scale. Many axion properties are independent of the scale of supersymmetry breakdown and meaningful axion models can be constructed within this range.

The outline of the paper is as follows. In section 2 we summarize the properties of the three well motivated axions in detail. We then present a discussion of axion potentials in the multi-axion case to obtain a unified scheme in section 3. Such a scheme requires the consideration of at least four axions, as will be discussed in section 4. Section 5 is devoted to the interpretation of the unified scheme and its possible connection to the scale of supersymmetry breakdown. Conclusions and outlook will follow in section 6.

2. Axion properties

Our present understanding of the cosmological evolution of the universe is best described by the Λ CDM model, complemented with the inflationary paradigm. The Λ CDM model accounts for the observed present day accelerated expansion in terms of a tiny cosmological constant Λ , as well as the observed existence of CDM in the universe. Additionally, inflation provides the seeds for the formation of the large scale structures that we observe today. However, CDM, inflation and present day acceleration, still demand a compelling explanation. Is it possible to find a single origin for these three crucial ingredients of the most successful cosmological model known to date? In this note we will argue in favor of a common origin for these ingredients in terms of axion-like particles.

The existence of axions was first postulated to solve the strong CP problem of QCD [2, 3, 4]. They are Pseudo Nambu-Goldstone Bosons (PNGBs), associated with the spontaneous breaking of a global ‘‘Peccei-Quinn’’ $U(1)_{\text{PQ}}$ symmetry at scale f_a , which acquire a mass via QCD anomaly (instanton) effects. The decay constant f_a and mass of the QCD axion are constrained by observations, leaving only a parameter window for the QCD invisible axion: $10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}$ and $10^{-6} \text{ eV} \lesssim m_a \lesssim 10^{-3} \text{ eV}$ [19]. Although very light, axions are legitimate candidates to constitute CDM, since they could have been produced non-thermally in the early universe. Furthermore, they satisfy the two criteria necessary for CDM, *i.e.* (a) they are effectively collisionless (their long range interaction is gravitational) and (b) if their mass is low, a very cold Bose-Einstein condensate of primordial axions could populate the universe today to provide the required dark matter energy density.

There is compelling evidence that the early universe underwent a period of accelerated expansion, known as ‘‘inflation’’ [26]. Although several different models for inflation exist, they all share the nontrivial requirement of flatness of the inflaton potential. This is mandatory (a) for inflation

to take place and (b) to match the primordial perturbations indicated by the Cosmic Microwave Background (CMB) anisotropies [28].

One way to obtain a very flat potential is precisely to use a PNCB as the inflaton, as was proposed in natural inflation [7, 8]. In this model, the potential of the inflaton field has a particular form, resulting from explicit breaking of a shift symmetry. This symmetry guarantees the flatness of the potential and protects it from too large radiative corrections. Depending on the value of f_{infl} , the model falls into the large field ($f_{\text{infl}} > M_{\text{P}}$) (where $M_{\text{P}} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass) or small field ($f_{\text{infl}} < M_{\text{P}}$) classification scheme that has been applied to inflationary models. However, the model agrees with the recent observations of the CMB anisotropy by WMAP7+BAO+ H_0 (+SPT) [29] for $f_{\text{infl}} \gtrsim 4M_{\text{P}}$.

While the presently observed acceleration of the universe can be accounted for by a tiny cosmological constant corresponding to an equation of state¹ of $w = -1$, other equations of state with $-1 < w < -1/3$ are not yet excluded by observations. Indeed, another possibility for dark energy is quintessence [30], where the vacuum energy is not constant but instead relaxes slowly due to the evolution of a scalar field (*e.g.* quintessence). Moreover, the current vacuum energy density of the classical quintessential field must be of order $E_{\text{vac}} \sim 0.003$ eV. This requires the mass of the quintessence to be extremely small, $m_{\text{Q}} \sim 10^{-32}$ eV. Such a light scalar field has the generic problem of driving long-range fifth forces, which are at odds with observations. The standard solution to this problem involves PNCBs with shift symmetries, which forbid higher dimensional operators, rendering the quintessence field stable against radiative corrections. Thus axion fields turn out to be natural candidates to explain the present day acceleration of the universe too.

According to the above discussion, a natural explanation for the origin of all three basic ingredients of the Λ CDM model can be found in terms of axion particles, with the additional bonus of providing a solution to the strong CP problem. In the letter [25] we consider this possibility and show that a minimal number of four axions is enough to fulfill all requirements. We motivate our multi-axion system in the context of string theory, where several axions are present in generic compactifications of the theory. It is known that in string theory the common values for the (individual) axions' decay constants turn out to be $f_a \gtrsim 10^{16}$ GeV [17]. However, there exist mechanisms within heterotic and type IIB string theories, which allow for much lower values for the decay constants [31, 32].

Let us note that our approach [25] is different to that of the axiverse [18] and N-flation [33] since we do not expect a plenitude of axions. Therefore our approach is rather minimal in the sense that we require a limited number of axions, four, to have the required properties to provide a natural origin for the early and present day acceleration as well as a CDM candidate (which includes a solution to the strong CP problem).

3. Potentials for axion cosmology

Let us begin by briefly reviewing some applications of axion physics in cosmology, including the possibility to describe inflation via axions (natural inflation) or to obtain candidates for dark energy within the quintessence scenario. This will set up the stage in order to investigate richer

¹The equation of state for the cosmological matter is $p = w\rho$ where ρ is the energy density and p is its pressure.

possibilities, in an attempt to account for all the above cosmological phenomena within a single framework.

3.1 Two-axion models

In cosmology nearly flat potentials are of crucial importance. A very attractive way to protect the flatness of a potential is by invoking a shift symmetry. Such symmetries naturally arise in the presence of pseudoscalar fields, such as axions. Then, shallow potentials may be generated by the breaking of such global symmetries. In single field models, the explicit breaking of a shift symmetry gives rise to a potential of the form

$$V(\theta) = \Lambda^4 \left[1 - \cos\left(\frac{\theta}{f}\right) \right], \quad (3.1)$$

where θ is an axion with the shift symmetry $\theta \rightarrow \theta + \text{constant}$ and f its scale. As it was advocated in [34, 11], it is often plausible to consider more than one axion in order to meet certain observational constraints. Let us consider the simplest possibility of two axions related to two shift symmetries. The corresponding potential resulting from their breaking is a direct generalization of Eq. (3.1) and it is given by

$$V(\theta, \rho) = \Lambda_1^4 \left[1 - \cos\left(\frac{\theta}{f_1} + \frac{\rho}{g_1}\right) \right] + \Lambda_2^4 \left[1 - \cos\left(\frac{\theta}{f_2} + \frac{\rho}{g_2}\right) \right], \quad (3.2)$$

where the two axions are denoted as θ and ρ . Clearly, the mass matrix in the (θ, ρ) basis is not diagonal and it is easy to see, *e.g.* by calculating its determinant, that a flat direction exists when the condition

$$\frac{f_1}{g_1} = \frac{f_2}{g_2}, \quad (3.3)$$

is satisfied. It is then straightforward to determine the physical fields which correspond to linear combinations of θ and ρ . Since in the following we shall perform such an analysis for cases with more axions, here we just state some qualitative features of this procedure. Let us stress that the scales Λ_i should exhibit large hierarchies in order to account for the relevant phenomena. This fact obviates the need for exact eigenvectors of the mass matrix (which in some cases are very complicated); indeed, approximate eigenvectors turn out to be enough in our framework. According to the above, assuming without loss of generality that $\Lambda_1 \gg \Lambda_2$, the potential may be written in terms of the physical fields, say $\tilde{\theta}$ and $\tilde{\rho}$, as

$$V(\tilde{\theta}, \tilde{\rho}) = \Lambda_1^4 \left[1 - \cos\left(\frac{\tilde{\theta}}{f_{\tilde{\theta}}}\right) \right] + \Lambda_2^4 \left[1 - \cos\left(c(f_i, g_i) \tilde{\theta} + \frac{\tilde{\rho}}{f_{\tilde{\rho}}}\right) \right]. \quad (3.4)$$

This form of the potential decouples the physical fields, in the sense that $\tilde{\theta}$ is a heavy field, while $\tilde{\rho}$ is light when the symmetry condition (3.3) is nearly satisfied and therefore it has a high effective scale $f_{\tilde{\rho}}$. This feature may be utilized in cosmological applications. Indeed, in [34] the above mechanism was invoked to account for inflation, identifying the light axion with the inflaton. Furthermore, in [11, 12] a different application of the above procedure led to the identification of the two axions as a candidate for quintessence (termed quintaxion) and a QCD axion dark matter candidate.

According to the above it is reasonable to pose the question whether it is possible to use such mechanisms to account for inflation, dark matter and quintessence at the same time. Evidently, an affirmative answer to this question would involve at least three axions. Therefore we begin our pursuit for an answer with the minimal set up of exactly three axions. As we shall see, this set up will prove inadequate to accommodate the QCD axion, however it will pave the road to overcome the difficulties by considering a fourth axion. The latter possibility is investigated in the next section.

3.2 Three-axion model

Let us consider three axion fields denoted as θ , ρ and a . The first two are considered to be hidden sector axions, while the latter one is the model-independent axion present in all string compactifications. According to the previous discussion a potential for the above fields, generated by the breaking of the associated shift symmetries, has the following form

$$V(\theta, \rho, a) = \sum_{i=1}^3 V_i = \sum_{i=1}^3 \Lambda_i^4 \left[1 - \cos\left(\frac{\theta}{f_i} + \frac{\rho}{g_i} + \frac{a}{h_i}\right) \right]. \quad (3.5)$$

Let us first determine some general properties of this potential. First of all, since the axion a is the model-independent one, it couples universally and therefore it is natural to set $h_1 = h_2 = h_3 \equiv h$. The determinant of the mass matrix which has been obtained by expanding the potential around the minimum $\theta = \rho = a = 0$ is given by

$$\det M^2 = \frac{\Lambda_1^4 \Lambda_2^4 \Lambda_3^4}{(f_1 f_2 f_3 g_1 g_2 g_3 h)^2} \left[f_1 f_2 g_3 (g_1 - g_2) + f_2 f_3 g_1 (g_2 - g_3) + f_1 f_3 g_2 (g_3 - g_1) \right]^2. \quad (3.6)$$

It may be easily verified that for $f_1 = f_2 = f_3$ or $g_1 = g_2 = g_3$, flat directions are obtained.

The procedure one may follow in order to determine the physical fields is to find the eigenvalues of the mass matrix, whose eigenvectors will be the physical fields and rewrite the potential in terms of these eigenvectors but this procedure is complicated when three or four axions are present. However, the hierarchy among the scales Λ_i (*i.e.* $\Lambda_1 \gg \Lambda_2 \gg \Lambda_3$) facilitates our analysis, since one may instead determine approximate eigenvectors of the potential by applying certain manipulations to the potential itself. In order to make the procedure more illuminating we assume at this stage that $f_1 = f_3$ and $g_2 = g_3$.

Let us now define the new fields φ , χ and ψ such that the field φ is the normalized linear combination of θ , ρ and a fields and is orthogonal to χ and ψ , while the latter are not orthogonal to each other. However, we can orthogonalize them by defining two new fields, χ_1 and ψ_1 .

Thus at the end of the day, collecting the various terms, the potentials take the form:

$$\begin{aligned} V_1 &= \Lambda_1^4 \left[1 - \cos\left(P_1 \varphi\right) \right], \\ V_2 &= \Lambda_2^4 \left[1 - \cos\left(P_2 \varphi + Q_2 \chi_1\right) \right], \\ V_3 &= \Lambda_3^4 \left[1 - \cos\left(P_3 \varphi + Q_3 \chi_1 + R_3 \psi_1\right) \right], \end{aligned} \quad (3.7)$$

where P_i , Q_i and R_i are explicit complicated functions of f_i , g_i and h . According to the above, we have a clear separation among the three fields, in a heavy, a semi-heavy (or semi-light) and a light field. We can confirm this by looking at the masses and decay constants of the physical fields. Indeed, the masses turn out to be

$$\begin{aligned} m_\phi^2 &\simeq \left(\frac{1}{f_1^2} + \frac{1}{g_1^2} + \frac{1}{h^2} \right) \Lambda_1^4, \\ m_{\chi_1}^2 &\simeq \frac{A}{f_2^2 g_2^2 (g_1^2 h^2 + f_1^2 (g_1^2 + h^2))^2} \Lambda_2^4, \\ m_{\psi_1}^2 &\simeq \frac{(f_1 - f_2)^2 (g_2 - g_1)^2 (f_1^2 + g_1^2) (f_1^2 + h^2)}{A} \Lambda_3^4, \end{aligned}$$

where $A = (g_1^2 g_2 (f_2 - f_1) + f_1^2 f_2 (g_2 - g_1))^2 (f_1^2 + h^2) + (f_1^2 + g_1^2) (f_1^2 f_2 (g_1 - g_2) + (f_2 g_1 - f_1 g_2) h^2)^2$. The decay constants for the axions that appear above are given by

$$\begin{aligned} f_\phi &= \frac{f_1 g_1 h}{\sqrt{g_1^2 h^2 + f_1^2 (g_1^2 + h^2)}}, \\ f_{\chi_1} &= \frac{f_2 g_2 (g_1^2 h^2 + f_1^2 (g_1^2 + h^2))}{\sqrt{A}}, \\ f_{\psi_1} &= \frac{\sqrt{A}}{|f_1 - f_2| |g_2 - g_1| \sqrt{f_1^2 + g_1^2} \sqrt{f_1^2 + h^2}}. \end{aligned} \quad (3.8)$$

Having at hand the masses and decay constants of the physical fields in terms of the parameters appearing in the scalar potential, let us now discuss their possible interpretation. First of all, it is clear that nearly flat potentials may be assigned to the fields χ_1 and ψ_1 . On the other hand, there is one more field left, the ϕ , which definitely cannot be related to the QCD axion, since the scale Λ_1 is the largest one in the model. According to the above, and keeping in mind that $\Lambda_1 \gg \Lambda_2 \gg \Lambda_3$, the only reasonable possibility is to consider ϕ to be a heavy auxiliary axion, while attempting to identify χ_1 with the inflaton and ψ_1 with the quintaxion. Whether such an interpretation is plausible depends on the possibility to meet the bound

$$f_{\text{infl}} \gtrsim 4 M_{\text{P}}, \quad (3.9)$$

for the inflaton as well as the condition on the quintaxion

$$f_{\text{QA}} \simeq M_{\text{P}}, \quad (3.10)$$

for appropriate scales Λ_i . There are indeed sets of values for the parameters f_i and g_i which satisfy the above requirements. Indicatively, we suggest the following possible set of values:

Parameter	Value
h	M_{P}
$f_1 = f_3$	$0.15 M_{\text{P}}$
$g_2 = g_3$	$0.125 M_{\text{P}}$
f_2	$0.125 M_{\text{P}}$
g_1	$0.15 M_{\text{P}}$

Here we assigned to h the value M_{P} , which is expected for the model-independent axion in string compactifications. Moreover, all the scales f_i and g_i are subplanckian as they should. Substituting these values into Eq. (3.8) we obtain the following results:

$$f_{\varphi} \simeq 0.1 M_{\text{P}}, \quad f_{\chi_1} \simeq 5.0 M_{\text{P}}, \quad f_{\psi_1} \simeq 1.1 M_{\text{P}}, \quad (3.11)$$

which is indeed in the desired range. The masses of the physical fields in units of M_{P}^{-2} , turn out to be

$$m_{\varphi}^2 = 10^2 \Lambda_1^4, \quad m_{\chi_1}^2 = 0.04 \Lambda_2^4, \quad m_{\psi_1}^2 = 0.83 \Lambda_3^4. \quad (3.12)$$

The above masses depend on the scales Λ_i . It is now time to specify these scales. In particular, the smallest scale Λ_3 , related to quintessence, should be of the order of 0.003 eV, corresponding to the vacuum energy of the universe today. Then, according to Eq. (3.12), the mass of the quintaxion is of the order 10^{-32} eV, which is an acceptable small value. Furthermore, if we assume that Λ_2 , corresponding to the inflation scale, is at the GUT scale of 10^{15-16} GeV, we obtain an inflaton mass of the order 10^{11-13} GeV.

Thus we conclude that a potential of the type (3.5) involving three axions offers the possibility of accounting for inflation and quintessence in the presence of a heavy spectator field. Although this is already an interesting result we shall not delve into further details. Instead we shall proceed to the treatment of a four-axion case which will prove to be richer and to allow for better interpretations.

4. Four-axion case

In order to go one step further in our interpretation and try to account also for the QCD axion, let us follow the most straightforward way, which amounts to considering a fourth axion. Denoting the four axions as θ , ρ , ϕ and a , the corresponding potential has the form

$$V = \sum_{i=1}^4 V_i = \sum_{i=1}^4 \Lambda_i^4 \left[1 - \cos \left(\frac{\theta}{f_i} + \frac{\rho}{g_i} + \frac{\phi}{h_i} + \frac{a}{h} \right) \right], \quad (4.1)$$

where, as before, we consider a to be the model independent axion with universal scale h . The necessary procedure in order to determine the physical fields, their decay constants and their masses closely follows the three-axion case. Here we explain in a qualitative way the procedure and we do not present all the intermediate steps, since they do not add anything substantial to the physics of the problem.

The mass matrix in the present case is a 4×4 one and it is of course non-diagonal. Its determinant may be easily computed but it does not directly acquire an illuminating form. Thus, without loss of generality, we assume that $f_1 = f_2 = f_4$, $g_2 = g_3 = g_4$ and $h_1 = h_2 = h_3$. Then the determinant of the mass matrix is given by

$$\det M^2 = \frac{\Lambda_1^4 \Lambda_2^4 \Lambda_3^4 \Lambda_4^4}{(f_1 f_3 g_1 g_2 h_1 h_4 h)^2} (f_1 - f_3)^2 (g_1 - g_2)^2 (h_1 - h_4)^2, \quad (4.2)$$

which directly shows that the conditions for the three expected flat directions simplify to $f_1 = f_3$, $g_1 = g_2$ and $h_1 = h_4$.

A simple way to unveil the nearly flat directions of the potential in the present case follows a three step orthogonalization. Having started with the fields (θ, ρ, ϕ, a) , in a first step we define a set of linear combinations of them, say $(\theta^{(1)}, \rho^{(1)}, \phi^{(1)}, a^{(1)})$, such that $\theta^{(1)}$, the argument in the cosine of the first term in the potential (4.1), is orthogonal to all the rest of the redefined fields. In the second step, we leave the field $\theta^{(1)}$ (and therefore V_1) unchanged and we construct the fields $\rho^{(2)}, \phi^{(2)}$ and $a^{(2)}$, which are linear combinations of $\rho^{(1)}, \phi^{(1)}$ and $a^{(1)}$ such that $\rho^{(2)}$ is orthogonal to $\phi^{(2)}, a^{(2)}$ and now the second term in the potential V_2 , depends only on $\theta^{(1)}$ and $\rho^{(2)}$, while the others depend on all fields. In the third and final step the fields $\theta^{(1)}$ and $\rho^{(2)}$ (and therefore the terms V_1 and V_2) are left untouched, while out of $\phi^{(2)}$ and $a^{(2)}$ we construct linear combination of $\phi^{(3)}$ and $a^{(3)}$ such that $\phi^{(3)}$ is orthogonal to $a^{(3)}$ and the potential terms are such that V_3 depends on the three fields $\theta^{(1)}, \rho^{(2)}, \phi^{(3)}$, while the last term V_4 depends on all the fields. Thus at the end of the day, collecting the various terms and adopting the lighter notation $(\theta^{(1)}, \rho^{(2)}, \phi^{(3)}, a^{(3)}) \rightarrow (\varphi, \chi, \psi, \omega)$ for the physical fields, the potentials take the form:

$$\begin{aligned} V_1 &= \Lambda_1^4 \left[1 - \cos(A_1 \varphi) \right], \\ V_2 &= \Lambda_2^4 \left[1 - \cos(A_2 \varphi + B_2 \chi) \right], \\ V_3 &= \Lambda_3^4 \left[1 - \cos(A_3 \varphi + B_3 \chi + C_3 \psi) \right], \\ V_4 &= \Lambda_4^4 \left[1 - \cos(A_4 \varphi + B_4 \chi + C_4 \psi + D_4 \omega) \right], \end{aligned} \quad (4.3)$$

for some specific constants A_i, B_i, C_i and D_i , being functions of f_i, g_i, h_i and h . We also assume the hierarchy $\Lambda_1 \gg \Lambda_2 \gg \Lambda_3 \gg \Lambda_4$.

The decay constants and the masses of the physical fields are functions of the parameters f_i, g_i, h_i and h and we shall calculate them below for some specific values of the parameters.

Let us choose the following value set of subplanckian scales

Parameter	Value
h	M_{P}
$f_1 = f_2 = f_4$	$0.75 M_{\text{P}}$
$g_2 = g_3 = g_4$	$0.66 M_{\text{P}}$
$h_1 = h_2 = h_3$	$0.75 M_{\text{P}}$
f_3	$0.0000003 M_{\text{P}}$
g_1	$0.75 M_{\text{P}}$
h_4	$0.40 M_{\text{P}}$

Substituting these values in the expressions for the decay constants we obtain

$$f_\varphi \simeq 0.4 M_{\text{P}}, \quad f_\chi \simeq 4.9 M_{\text{P}}, \quad f_\psi \simeq 0.5 \times 10^{-6} M_{\text{P}}, \quad f_\omega \simeq 1.0 M_{\text{P}}. \quad (4.4)$$

It is directly observed that the resulting values are in principle appropriate to describe an auxiliary axion φ , an inflaton χ , a QCD axion ψ and a quintaxion ω . Indeed the bounds (3.9) and (3.10) are met, as well as the QCD axion bound

$$10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV} \quad \Leftrightarrow \quad 10^{-9} M_{\text{P}} \lesssim f_a \lesssim 10^{-6} M_{\text{P}}. \quad (4.5)$$

Whether the above interpretation is really true depends also on the corresponding energy scales. We have already assumed the hierarchy $\Lambda_1 \gg \Lambda_2 \gg \Lambda_3 \gg \Lambda_4$. The smallest scale Λ_4 should be of the order of 0.003 eV, since it is related to the vacuum energy. Moreover, the next to smallest scale Λ_3 should be the QCD scale (~ 200 MeV). Finally, Λ_2 is identified with the inflation scale, which we assume that it corresponds to the GUT scale of 10^{15-16} GeV, while the largest one, Λ_1 , is taken to be the Planck scale. These identifications are in accord with the interpretation of the axions which we suggested above. Before discussing further their cosmological evolution, let us also compute their masses. These turn out to be of the following orders

$$m_\phi \simeq 10^{18} \text{ GeV}, \quad m_\chi \simeq 10^{12} \text{ GeV}, \quad m_\psi \simeq 10^{-4} \text{ eV}, \quad m_\omega \simeq 10^{-32} \text{ eV}. \quad (4.6)$$

We observe the desired hierarchy between the masses. The quintaxion has an extremely small mass of the order of 10^{-32} eV, while the QCD axion mass of order 10^{-4} eV lies within its acceptable window. Finally, the inflaton mass is of the order of 10^{12} GeV.

5. Interpretation

5.1 Axionic see-saw

Let us discuss some interesting relations among the scales which are relevant in the present framework. A diagrammatic companion illustrating the ensuing discussion appears in Fig. 1. We start with the following benchmark relation between the Planck scale M_P , the weak scale M_{weak} , the vacuum energy E_{vac} and the mass of the quintaxion m_{QA} :

$$E_{\text{vac}} \sim \frac{M_{\text{weak}}^2}{M_P}, \quad m_{\text{QA}} \sim \frac{E_{\text{vac}}}{M_P}. \quad (5.1)$$

The weak scale is $\mathcal{O}(\text{TeV})$, while as we have already mentioned, $E_{\text{vac}} \sim 0.003$ eV and $m_{\text{QA}} \sim 10^{-32}$ eV. Moreover, we invoke the hidden sector of heterotic string compactifications in order to relate the weak scale to higher energy scales. Indeed, a dynamical mechanism such as hidden sector gaugino condensation may account for supersymmetry breaking with a gravitino mass at the (multi) TeV scale. In that case the corresponding relation between the scales is

$$M_{\text{weak}} \sim \frac{\Lambda_h^3}{M_P^2}, \quad (5.2)$$

where $\Lambda_h \sim 10^{13}$ GeV is the hidden sector scale, obtained as $\Lambda_h \sim M_{\text{GUT}}^2/M_P$. In the framework we present here, there are four scales Λ_i , $i = 1, \dots, 4$ which are hierarchical as $\Lambda_1 \gg \Lambda_2 \gg \Lambda_3 \gg \Lambda_4$. The highest scale is naturally associated with the Planck scale, *i.e.* $\Lambda_1 \sim M_P$. Moreover, the second highest scale, the scale of inflation, is identified with the GUT scale, *i.e.* $\Lambda_2 \sim M_{\text{GUT}} \sim 10^{15-16}$ GeV. We can directly state that the inflaton mass is $m_{\text{infl}} \sim \Lambda_2^2/M_P \sim 10^{12-14}$ GeV. Note that this mass is at the intermediate energy scale of the hidden sector, *i.e.* $m_{\text{infl}} \sim \Lambda_h$. Moreover, the smallest scale is identified with the vacuum energy, *i.e.* $\Lambda_4 \sim E_{\text{vac}}$, which is related to the other scales via the relations (5.1). Finally, the remaining scale is identified with the QCD scale, $\Lambda_3 \sim \Lambda_{\text{QCD}}$, while the corresponding axion has a mass of the order $m_a \sim \Lambda_{\text{QCD}}^2/f_a \sim E_{\text{vac}}$, with an invisible axion scale f_a of order 10^{10-11} GeV. The latter scale, say Λ_{int} , can be also related to the rest as follows. Having

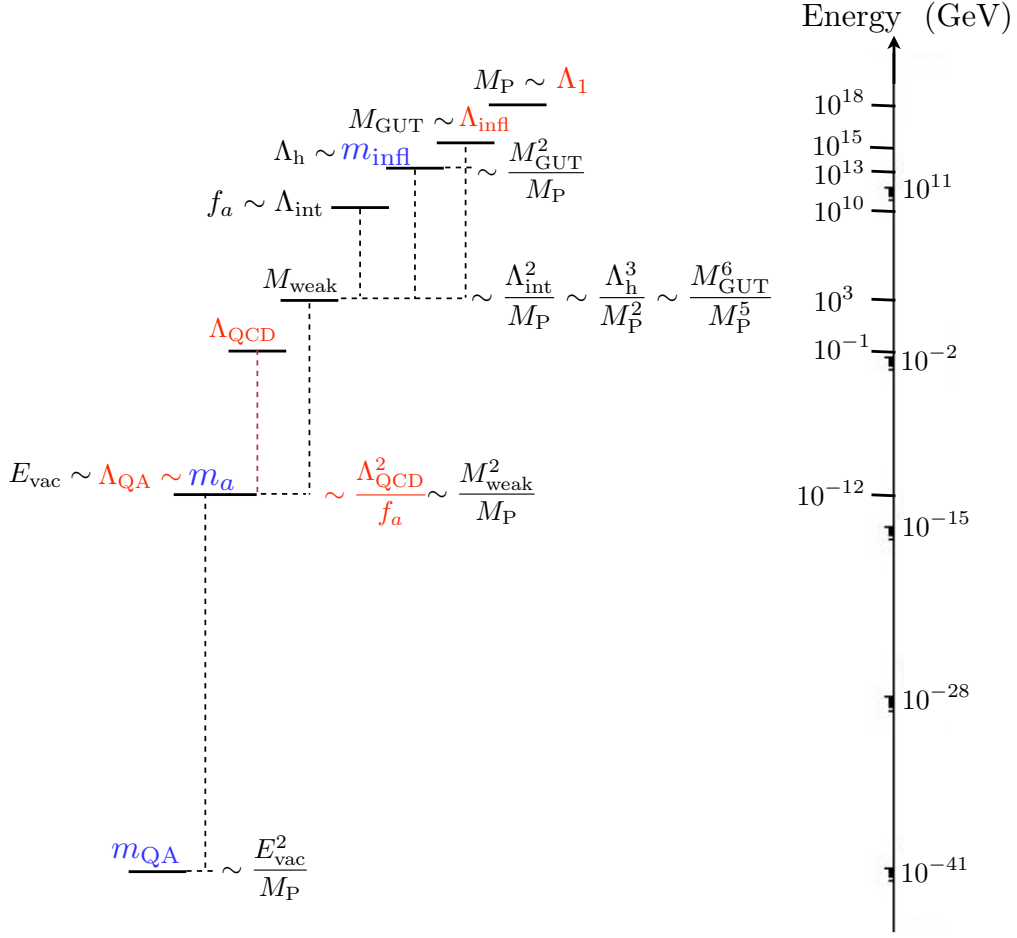


Figure 1: Scales, masses and the axionic see-saw.

in mind the relation (5.2) from the gaugino condensation, we may write $M_{\text{weak}} \sim \Lambda_{\text{int}}^2 / M_{\text{P}}$, which entangles the scales $M_{\text{P}}, \Lambda_{\text{h}}$ and Λ_{int} as $\Lambda_{\text{h}}^3 \sim M_{\text{P}} \Lambda_{\text{int}}^2$.

Fig. 1 summarizes what we call the axionic see-saw. Given one ratio $R \sim \Lambda_{\text{h}} / M_{\text{P}} \sim 10^{-5}$ we can parameterize all relevant axionic scales with this one parameter. This may be illustrated as the following chain:

$$M_{\text{P}} \sim m_{\phi} \xrightarrow{R} \Lambda_{\text{h}} \sim m_{\text{infl}} \xrightarrow{R^2} M_{\text{weak}} \xrightarrow{R^3} E_{\text{vac}} \sim m_a \xrightarrow{R^6} m_{\text{QA}}.$$

5.2 The role of supersymmetry

Previous discussions usually assumed the presence of supersymmetry at the weak scale as a solution of the hierarchy problem. But as we have said, axions do not really need susy, although both schemes are compatible with each other. One of the strong arguments for weak scale susy, the existence of a WIMP dark matter candidate is no longer valid in the axion framework. So we do

not need to be prejudiced about the value of the susy breakdown scale. Fine tuning questions could be relevant towards low values of M_{susy} . They might be relevant for the stability of the weak scale but the smallness of the vacuum energy cannot be explained in that way.

Given this situation we would now have to see how the susy scales like the gravitino mass, the soft breaking terms of the supersymmetric partners in the MSSM and the μ -parameter fit into the axionic see-saw scheme of Fig. 1. As we have explained above, in previous works [12, 35] some identifications have been made, but they were usually done with the prejudice for weak scale susy. So let us here be more general and discuss the framework from a top-down approach in string theory. As we have seen, string theory provides its intrinsic mass scale M_{string} that is relevant for the axion scheme. Typically all axion decay constants are of order of M_{string} and the small size of the scale of the QCD axion is a challenge in string model building [31, 32].

Could there be a relation between the axion scale and the scale of susy breakdown? As we already remarked, consistent string theories in $D = 10$ require supersymmetry. Of course supersymmetry is broken at some scale, but that could be a scale sufficiently low compared to M_{string} . We could argue that this scale could be identified with the scale $f_a \sim 10^9 - 10^{12}$ GeV of the QCD axion. This leaves open the two especially interesting possibilities: weak scale susy or as an alternative susy at the scale of the invisible axion.

5.3 The susy scale and hints from the LHC

LHC is the only present experiment that can provide hints about the fate of supersymmetry. So let us now have a look at the preliminary LHC data and see whether we can learn something.

- No signs of susy have been seen yet. This implies at least (multi) TeV range for the supersymmetric partners in the MSSM.
- There is now strong evidence for a Higgs boson with a mass around 125 GeV. This is rather high for the MSSM (heavy susy particles are needed), but rather low for the SM. The quadratic Higgs self coupling λ runs to zero at a scale smaller than the Planck scale (for a recent calculation see ref. [23]). This is the scale where new physics is needed to complete the SM.
- Interestingly enough, this scale coincides with the axion scale of the QCD axion.
- We could now speculate that the susy scale is identified with the scale of the axion decay constant, describing a remote supersymmetry (tele-susy).

So we could thus consider the SM without (weak scale) susy and extrapolate to higher energies till we reach the region where λ turns negative. We do not need a WIMP candidate for CDM matter as we will use the axion and we postulate the axion scale as the scale of broken susy [24, 36]. Of course, this “tele-susy” will no longer be able to solve the hierarchy problem of the weak scale. In addition we have to consider the fine tuning problem of the quintessential axion (that would not be solved by weak scale susy either).

Now we can ask the question how this speculation about tele-susy fits in the axionic see-saw scheme of Fig. 1 and how it differs from previous considerations that assumed weak scale susy. First of all, this question partially regards the value of the gravitino mass and of the μ term in this

scheme. As for the μ term, note that it is a supersymmetric mass term that could decouple from the susy scale. Even in the case of tele-susy we could have a μ term of the size of the weak scale and provide a small tree-level mass for the Higgs boson, even if the gravitino mass is identified with the axion decay constant.

In an alternative scenario the vacuum energy might be somehow related to the gravitino mass, maybe via a volume suppression or a suppression with the mechanism of a doubly suppressed gravitino mass [35]. Further relevant questions concern the preferred schemes of susy breakdown and mediation, as well as the relation between soft mass terms, gravitino mass and vacuum energy. Of course, one of the central points of tele-susy regards the origin of the susy breakdown scale and its coincidence with the scale of the QCD axion. It might find a solution within the axionic see-saw in very similar way as the μ term in the case of weak scale susy [37].

6. Conclusions

The announcement of the Higgs particle exhibited in the most explicit way that we are in the middle of a discovery era in particle physics. Furthermore, promising experiments and observations in the cosmological front are underway. However, particle physics and cosmology involve a number of well separated scales which often make it difficult to discuss their phenomena simultaneously. In the present paper we presented a pattern of scales which brings them closer: the axionic see-saw.

The axionic see-saw is a pattern of energy scales which emerges out of the impact between certain aspects of particle physics beyond the standard model and cosmology. On the one hand it is related to an effective field theory of “useful” axions. The latter are axions which play an important role in discussions of three key ingredients of cosmology, namely inflation, dark matter and dark energy. In the present paper, we examined to what extent it is possible to obtain axion candidates for the inflaton and quintessence as well as for cold dark matter via axions. The result is that starting with four axions and a potential arising from the breaking of their corresponding shift symmetries, it is indeed possible to achieve the above picture. On the other hand, it is meaningful to ask how supersymmetry and its relevant scales fit into this axionic see-saw pattern. We discussed this question under the light of the implications of the current experimental data and without prejudice on the value of the supersymmetry breaking scale. One possibility is of course weak scale supersymmetry, which has been discussed before. A novel proposal is motivated by the observation that a Higgs boson mass of 125 GeV implies the vanishing of its self coupling at a scale which approximately coincides with the scale of the QCD axion. This observation allows us to speculate that the new physics which should arise at that intermediate scale in order to complete the standard model could be a remote supersymmetry which we called tele-susy.

Tele-susy is further motivated by the fact that the most promising candidate for an ultraviolet completion of the standard model, string theory, requires supersymmetry for its internal consistency. Therefore, even if the LHC disfavors weak scale supersymmetry in the future, a tele-susy has to operate at an intermediate scale between the weak and the Planck scale. The existence of a dark matter candidate, a strong motivation for weak scale supersymmetry, is not lost in this scheme, this role being played by the QCD axion.

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