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A natural solution to the gravitino overabundance problem

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We revisit the gravitino generation from the MSSM thermal plasma taking into account the global $U(1)$ R-symmetry that generically characterizes the supersymmetry breaking sectors. In the reheated early universe the thermal effects tend to restore the R-symmetry. An R-symmetric phase suppresses the gaugino masses and hence the production of the helicity $\pm 1/2$ gravitino component from the MSSM sector. We show that the gravitino yield from the MSSM is dominated by the temperature that the R-symmetry breaking takes place and not by the reheating temperature of the universe.

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1. Introduction

The gravitino is a well motivated dark matter candidate predicted in the local supersymmetric extensions of the Standard Model (SM) of the particle physics. It is the lightest sparticle (LSP) in the scenarios of gauge mediation of supersymmetry breakdown (GMSB) and it can be cosmologically stable if R-parity is sufficiently conserved, for reviews and references therein [1, 2, 3].

However the stable gravitino relic energy density is temperature and model dependent. The gravitinos acquire a thermal abundance of the correct order of magnitude only for mass $m_{3/2} \sim$ keV. If $m_{3/2}$ is larger then the gravitino production from scatterings off the thermal plasma impose an upper bound on the reheating temperature. Violation of these bounds results in the notorious overclosure of the universe problem. For a decoupled stable gravitino its relic abundance is given by [4, 5, 6, 7, 8]

$$\Omega_{3/2}h^2 \simeq 0.2 \left(\frac{T_{\rm rh}}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{100\,\mathrm{GeV}}{m_{3/2}}\right)\left(\frac{m_{\tilde{g}}(\mu)}{1\,\mathrm{TeV}}\right)^2 \tag{1.1}$$

where $m_{\tilde{g}}$ the gluino mass at the electroweak scale and $T_{\rm rh}$ the reheating temperature that has to be tuned in order that $\Omega_{3/2}h^2 \sim 0.11$ [9].

The gravitino is tightly related with the supersymmetry breaking sector. Firstly, the gravitino is massive because in spontaneously broken local supersymmetric theories there is a spin $1/2$ goldstino fermion that provides the longitudinal components for the gravitino. Secondly, the scalar superpartners of the goldstino are a complex scalar and an auxiliary F-component whose non-zero vacuum expectation value (vev) breaks supersymmetry. The value of the F-term determines the mass of the gravitino: $m_{3/2} = F/(\sqrt{3}M_{\rm Pl})$, where $M_{\rm Pl} = 2.4\times 10^{18}$GeV the reduced Planck mass. We denote here the goldstino chiral supermultiplet and the dynamical scalar field component with X.

Further details of the hidden (referred also as secluded sector in the context of gauge mediation) sector affect the Standard Model sparticle spectrum. A generic feature is the presence of an exact or approximate continuous global $U(1)$ symmetry called R-*symmetry.* Such a symmetry is allowed in $N=1$ supersymmetry and its generator does not commute with the generators of supersymmetry. The R-symmetry, as pointed out at [10] is deeply connected with the supersymmetry breaking. For example in a Wess-Zumino model the presence of the R-symmetry is a necessary condition for supersymmetry breaking and a spontaneously broken R-symmetry a sufficient condition if the superpotential is a generic function of the fields. Hence, if there is a supersymmetry breaking vacuum an R-symmetry, exact or approximate, must be generally expected.

In addition, R-symmetry has an important effect on the sparticle mass spectrum: in case it is an exact symmetry of the Lagrangian and preserved in the non-supersymmetric vacuum then Majorana gaugino masses are not allowed [11, 12, 13] -for a different scenario with Dirac gauginos see ref. [14]. This fact considered together with the previous arguments suggest that the R-symmetry has to be an approximate symmetry of the low energy effective theory. However, the R-breaking effects may be negligible at a higher energy scale. In the early universe highly energetic environment the R-symmetry can have been actually restored.

Here we present the thermal effects on the supersymmetry breaking sector and we demonstrate that the R-symmetry, although broken at low energies, it is restored at high temperatures. We put forward that an R-symmetric thermal phase has dramatic effects on the thermal production of the

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gravitinos from the MSSM sector [15]. The gaugino masses being R-suppressed will not contribute to the goldstino production rate thereby, suppressing the resulting gravitino abundance. We study minimal and standard examples of supersymmetry breaking that exhibit spontaneous or explicit breaking of the R-symmetry. Our result is that the production of the helicity $\pm 1/2$ component of the stable gravitino from the MSSM effectively shuts off at high temperatures. The gravitino relic density can easily satisfy the dark matter constraint $\Omega_{3/2}h^2 \lesssim 0.11$ for temperatures much higher than those suggested by the relation (1.1). The gravitino can account for the dominant dark matter component for reasonable values of the parameters and for a remarkably wide range of reheating temperatures given that the MSSM is the only sector that contributes to the gravitino production. The suppression of the gaugino masses is argued to be of the form

$$\frac{m_\lambda(T)}{m_\lambda} = \mathscr{O}\left(\frac{X(T)}{X_0}\right), \tag{1.2}$$

where $X(T)$ is the thermal average value of the R-charged spurion scalar field that breaks the R-symmetry and $X(T=0) \equiv X_0$ its zero temperature vev. The $m_\lambda(T)$ is the gaugino mass at finite temperature that differs from the zero temperature m_λ due to the different R-symmetry breaking scale that is controlled by the $X(T)$ value.

2. Gravitino relic density review

Gravitinos can be produced in the early universe via several different ways. Here we focus on the thermal production of gravitinos from the thermalized MSSM plasma, for some reviews see [16, 17]. In the thermal bath the MSSM particles occasionally interact to produce a gravitino through the interactions like $A_\rho A_\sigma \to \lambda \psi_\mu$. The produced gravitinos, ψ_μ, then propagate through the universe essentially without interacting. If they are stable they contribute to the present dark matter density. The gravitino production cross section, for a gravitino with momentum p, taking into account both the helicity $\pm 3/2$ and helicity $\pm 1/2$ contributions to the self energy reads

$$\sigma_{3/2}(p) \propto \frac{g^2}{M_{\rm Pl}^2}\left(1+\frac{m_{\tilde g}^2}{3m_{3/2}^2}\right) \tag{2.1}$$

where we considered only processes involving gluons and gluinos, with mass $m_{\tilde g}$, that dominate the cross section and g the QCD strong coupling [6]. The gravitino source term $n_{\rm rad}^2 = n_{eq}^2$ arises from MSSM interactions while the sinking term originating from the inverse interaction, i.e. from gravitino annihilation processes, is non-negligible only for light gravitinos. The gravitino yield $Y_{3/2} \equiv n_{3/2}/n_{\rm rad}$, is found by solving the Boltzmann equation describing the gravitino production

$$\frac{dY_{3/2}}{dT} = -\frac{\langle \sigma_{3/2} v \rangle n_{\rm rad}}{HT}. \tag{2.2}$$

For *temperature independent* $\langle \sigma_{3/2} v \rangle$ the right hand side is independent of the temperature since $n_{\rm rad} \propto T^3$ and $H \propto T^2$. The result is that the gravitino relic number density is linearly proportional to the reheating temperature $T_{\rm rh}$. The constant of proportionality is the gravitino production cross section with the leading $2 \to 2$ processes are the QCD interactions. The only temperature dependent effect as we integrate from $T_{\rm rh}$ to T, with $T_{\rm rh} \gg T$, is the dilution factor $g_*(T < 1\,{\rm MeV})/g_*(T_{\rm rh}) = (43/11)/(915/4)$ for the MSSM.

For a light gravitino ($m_{3/2} < m_{\tilde{g},\tilde{f}}$) a freezing out temperature can be defined at which the production of gravitino from the MSSM thermal plasma has effectively ceased

$$T^f_{3/2} \sim 2\times 10^{10}{\rm GeV}\left(\frac{m_{3/2}}{10{\rm MeV}}\right)^2\left(\frac{1{\rm TeV}}{m_{\tilde{g}}}\right)^2. \tag{2.3}$$

Also, the relic abundance is given by

$$\Omega_{3/2}h^2 \simeq 0.2\left(\frac{T_{\rm min}}{10^{10}\,{\rm GeV}}\right)\left(\frac{100\,{\rm GeV}}{m_{3/2}}\right)\left(\frac{m_{\tilde{g}}(\mu)}{1\,{\rm TeV}}\right)^2 \tag{2.4}$$

where $T_{\rm min} \equiv \min(T_{\rm rh}, T^f_{3/2})$ i.e. the gravitino relic density is determined as a function of reheating temperature. Therefore, for relatively heavy gravitino, e.g. of the order of GeV, decreasing the reheating temperature the gravitino abundance is truncated. Asking for $\Omega_{3/2}h^2 \lesssim 0.1$ a bound is applied on the reheating temperature

$$T_{\rm rh} \lesssim 10^6{\rm GeV}\left(\frac{m_{3/2}}{10{\rm MeV}}\right) \tag{2.5}$$

when the gluino has mass $m_{\tilde{g}} \sim 1$ TeV. For gravitino mass $m_{3/2} \sim 100$ GeV the constraint on the $\Omega_{\rm DM}h^2$ constrains the reheating temperature at $T_{\rm rh} \lesssim 10^{10}$ GeV. Whether or not these bounds are saturated specifies the percentage of gravitino at the dark matter abundance.

The relic density of the helicity $\pm 3/2$ gravitino component is given by the expression

$$\Omega_{3/2}h^2 \simeq 0.1\frac{m_{3/2}}{100\,{\rm GeV}}\frac{T_{\rm rh}}{10^{10}\,{\rm GeV}} \tag{2.6}$$

and requiring $\Omega_{3/2}h^2 \lesssim 0.1$ one obtains

$$T_{\rm rh} \lesssim 10^{10}\,{\rm GeV}\times\frac{100\,{\rm GeV}}{m_{3/2}}. \tag{2.7}$$

3. Thermal restoration of the $U(1)_R$ symmetry

3.1 Exact R-symmetry

The $U(1)_R$ symmetry can be an exact symmetry of the theory that breaks spontaneously in the vacuum. A basic model is the following

$$W = FX + \lambda X\phi\bar{\phi} \tag{3.1}$$

$$K = |X|^2 + \varepsilon_4\frac{|X|^4}{\Lambda_*^2} + \varepsilon_6\frac{|X|^4}{\Lambda_*^4} \tag{3.2}$$

The superpotential, W, is the ordinary gauge mediation superpotential with ϕ, $\bar{\phi}$ the messenger fields. The Kähler potential, K, for $\varepsilon_4 = 1$ and $\varepsilon_6 < 0$ induces spontaneous R-symmetry breaking that can be attributed to the integrated out fields that account for the Kähler corrections [18]. An example is the O'Raifeartaigh-like sector proposed in ref. [19].

In a thermalized universe it is the messenger fields ϕ, $\bar{\phi}$ that control the thermal average value of the R-charged spurion X-field and thus, the thermal restoration or breaking of the R-symmetry. The R-symmetry is a symmetry of the vacuum when $X=0$ and breaks spontaneously when $X\neq 0$. For the above theory (3.1), (3.2) the R-symmetry is restored due to thermal effects [20] at the temperature [21, 22, 15]

$$T_{\not R} \equiv \frac{4}{\sqrt{N}}\frac{F}{\lambda\Lambda_*}. \tag{3.3}$$

For the minimal case of a $\mathbf{5}+\bar{\mathbf{5}}$ messenger sector i.e. the ϕ, $\bar{\phi}$ the messenger quarks and leptons form a single complete $SU(5)$ representation and $N=5$. The R-symmetry breaks via a second order phase transition from $X=0$ to $|X|\sim \Lambda_*/\sqrt{|\varepsilon_6|}$. The critical temperature (3.3) can be quickly estimated: the spurion has negative squared mass $4F^2/\Lambda_*^2$ at the origin and receives thermal corrections $N\lambda^2T^2/4$ from the messenger fields.

3.2 Approximate R-symmetry

The theory (3.1), (3.2) exhibits an exact thermal restoration of the $U(1)_R$ symmetry. Theories that include explicit $U(1)_R$ violating terms like the following paradigms [23, 24]

$$W = FX+\lambda X\phi\bar{\phi}+c, \qquad W = FX+\lambda X\phi\bar{\phi}-M\phi\bar{\phi} \tag{3.4}$$

are approximately R-symmetric at high temperatures. Indeed, for high enough temperatures the R-violating terms c and $M\phi\bar{\phi}$ are negligible and an approximate R-symmetry restoration takes place. This can be seen from the evolution of the thermal average value for the R-charged X field coupled with N messenger fields ϕ and $\bar{\phi}$ in the fundamental representation. For $\varepsilon_4=-1$ and negligible sixth order Kähler corrections we find

$$X^{(c)}_{min}(T) = \frac{4\frac{c}{M^2_{\rm Pl}}F - \frac{2Fc}{3\Lambda_*^2M^2_{\rm Pl}}T^2}{8\frac{F^2}{\Lambda_*^2}+\frac{N}{2}\lambda^2T^2}, \qquad X^{(M)}_{min}(T) = \frac{\frac{1}{2}M\lambda T^2}{8\frac{F^2}{\Lambda_*^2}+\frac{N}{2}\lambda^2T^2} \tag{3.5}$$

The R-symmetry breaking scale is the vev $\langle X\rangle \equiv X_0$. For the case of gravitational stabilization the vev is the $X_0^{(c)} = c\Lambda_*^2/(2FM^2_{\rm Pl})$. For the messenger mass case, after the translation $X\to M/\lambda - X$ it is $X_0^{(M)}=M/\lambda$.

According to (3.5) the thermal average value tends to restore the R-symmetry. We can parametrize the degree of the R-symmetry breaking by defining the parameter b_R:

$$b_R(T)\equiv \frac{X(T)}{X_0}. \tag{3.6}$$

Temperatures higher than the cut-off scale are not expected (for $\Lambda_*\gtrsim 10^{-4}M_{\rm Pl}$) since a thermal equilibrium cannot be achieved. Thus, for the case of gravitational stabilization the second term at the numerator (3.5) is negligible. The parameter b_R takes the universal (for both cases) form

$$b_R(T) = \frac{\left(4\frac{F}{\Lambda_*}\right)^2}{\left(4\frac{F}{\Lambda_*}\right)^2+N\lambda^2T^2} \tag{3.7}$$

From the definition of $T_{\not{R}}$, eq. (3.3), the (3.7) is recast to the simpler form

$$b_R(T) = \frac{1}{1+\left(\frac{T}{T_{\not{R}}}\right)^2}\,. \tag{3.8}$$

Obviously, when $T \to 0$ the R-symmetry breaking scale takes its maximum value, i.e. the zero temperature one, and when $T \to \infty$ the R-symmetry is restored. In other words, the $b_R(T)$ parametrizes the R-symmetry breaking scale at finite temperature with respect to the zero temperature scale. We note that $T_{\not{R}} = T(b_R = 0.5)$.

4. Gravitino thermal production revisited

The supersymmetric Lagrangian contains the gauge interaction terms

$$\mathscr{L}_{gauge} = \int d^4x d^2\theta \left\{ \frac{1}{4} W_1^{\alpha} W_{1\,\alpha} + \frac{1}{2}\mathrm{tr}(W_2^{\alpha} W_{2\,\alpha}) + \frac{1}{2}\mathrm{tr}(W_3^{\alpha} W_{3\,\alpha}) \right\} + \mathrm{h.c}$$

$$+ \int d^4x d^2\theta d^2\bar{\theta} \left(\sum_{ij} \Phi_i^{\dagger} e^{g_j V_j} \Phi_i + \mathrm{h.c} \right) \tag{4.1}$$

where V_j are the vector and $W_{j\alpha}$ the corresponding field strength superfields associated to $U(1)_Y$, $SU(2)$ and $SU(3)$ for $j = 1,2,3$ respectively. The complete supersymmetric Lagrangian (without the soft terms) includes also the superpotential Yukawa interactions and the Higgs sector. The (4.1) apart from being gauge invariant it is also R-invariant. For instance, the $W^{\alpha}W_{\alpha}$ has R charge $+2$ and by a $U(1)_R$ rotation it transforms to $e^{2i\alpha}W^{\alpha}W_{\alpha}$ which cancels out with the corresponding rotation at the Grassmannian variable $d^2\theta \to e^{-2i\alpha}d^2\theta$. Introducing an explicit *Majorana* gaugino mass in a supersymmertic manner extends the Lagrangian (4.1) with the part

$$\mathscr{L}_{\mathrm{gaugino}} = \int d^4x d^2\theta \left\{ \frac{1}{4} 2\theta\theta\, m_{\lambda_1} W_1^{\alpha} W_{1\,\alpha} + \frac{1}{2} 2\theta\theta\, m_{\lambda_2} \mathrm{tr}(W_2^{\alpha} W_{2\,\alpha}) + \frac{1}{2} 2\theta\theta\, m_{\lambda_3} \mathrm{tr}(W_3^{\alpha} W_{3\,\alpha}) \right\} \tag{4.2}$$

which breaks the R-symmetry. Thereby the radiatively generated at 1-loop gaugino masses by the operator $\int d^2\theta \ln X W^{\alpha} W_{\alpha} + \mathrm{h.c.}$

$$m_{\lambda} = \frac{\alpha}{4\pi}\frac{\lambda F}{M_{\mathrm{mess}}} = \frac{\alpha}{4\pi}\frac{F}{X_0} \tag{4.3}$$

should *not* be expected if the *vacuum* of the theory is R-symmetric.

The crucial consequence of the thermal tendency to restore the R-symmetry is that the gravitino cross section from the scattering processes off MSSM thermal radiations, $\langle \sigma_{3/2} v \rangle$, becomes thermal dependent. Since the gaugino masses, m_{λ}, require an insertion of both the scalar and auxiliary components of X, while the scalars require only auxiliary components, the gauginos become lighter than the sfermions as $X(T)$ decreases by $\mathscr{O}(X(T)/X_0)$. In the models of ordinary gauge mediation, that we investigate here, there is no hierarchy between the sfermions and the gauginos at zero temperature, i.e. $m_{\lambda} \sim m_{\tilde{f}}$. Hence, due to the R-symmetry as the temperature increases and the $X(T)$ is driven to the origin the gauginos masses minimize compared to the zero temperature gaugino masses. We claim a scaling:

$$\frac{m_\lambda(T)}{m_\lambda(T=0)} = \mathscr{O}\left(\frac{X(T)}{X_0}\right) = \mathscr{O}(b_R)\,. \tag{4.4}$$

Here we will consider that: $m_\lambda(T)/m_\lambda(T=0) = b_R(T)$.

The above arguments suggest a thermally sensitive cross section for the production of gravitinos from scattering processes off the thermal radiations. Considering the dominant QCD $2 \to 2$ processes the (2.1) cross section becomes temperature dependent

$$\sigma_{3/2}(p,T) \propto \frac{g^2}{M_{\rm Pl}^2}\left(1 + b_R^2(T)\frac{m_{\tilde{g}}^2}{3m_{3/2}^2}\right). \tag{4.5}$$

We note that the gluino mass has already a mild dependence on the temperature due to the renormalization of the gauge coupling constants that has been taken into account.

Another subtle point is whether the finite temperature effects enhance the gravitino mass or the goldstino coupling. This was discussed at ref. [25] and [26] where it was shown that thermal effects do *not* give contribution to the gravitino production rate.

Here, we are considering an opposite effect: at high temperatures the goldstino generation rate may be suppressed due to the R-symmetry restoration. The formula that gives the gravitino abundance $Y_{3/2}$ is given by (2.2). According to our arguments the right hand side of the equation is not any more temperature independent but it reads:

$$\frac{dY_{3/2}}{dT} = -\frac{\langle \sigma_{3/2}(T) v\rangle n_{\rm rad}}{HT}\,. \tag{4.6}$$

For gravitino of mass $m_{3/2} < 100$ GeV we can neglect the yield of the helicity $\pm 3/2$ component for temperatures less than $10^{12} m_{3/2}^{-1}$ GeV^2, see (2.7). Focusing on the interactions of the helicity $\pm 1/2$ modes then the yield variable $Y_{3/2}$ is given by

$$Y_{3/2}(T) - Y_{3/2}(T_{\rm rh}) = -\frac{g_*(T)}{g_*(T_{\rm rh})}\left\{\frac{n_{\rm rad}\left\langle \sigma_{3/2}^{(1/2)} v\right\rangle}{HT}\right\}\int_{T_{\rm rh}}^{T} dT\, b_R^2(T) \tag{4.7}$$

where

$$\left\langle \sigma_{3/2}^{(1/2)}(T) v\right\rangle = \left\langle \sigma_{3/2}^{(1/2)} v\right\rangle b_R^2(T) \tag{4.8}$$

as we can see from (4.5). We remind the reader that the quantity in the brackets is temperature independent.

Another point that should be taken into consideration is whether the processes that involve chiral supermultiplets, i.e. quark and squarks, contribute to the production of the helicity $\pm 1/2$ component. If their contribution is non negligible then the suppression, due to the R-symmetry, of the goldstino production may not be significant if the massive (not-suppressed by the R-symmetry) sfermions take over the gluino-trancated production process. However, the goldstino production rate from chiral supermultiplets is proportional to $m_{\tilde{f}}^4$ which is suppressed at high energies compared to the gluino contribution for dimensional reasons: the interactions of the gravitino to gauge supermultiplets are described by dimension-five operators and those to chiral supermultiplets by dimension-four ones. Hence, at high temperatures, $m_{\tilde{q}}, m_{\tilde{g}} \ll T$, contributions involving the cubic goldstino-quark-squark coupling are suppressed by $m^2_{\tilde{q}}/T^2$ relative to the gluino contributions. Hence, we can focus only at those $2 \to 2$ reactions where at least one of the three other external particles is a member of a gauge supermultiplet and ignore those with chiral supermultiplets altogether.

4.1 Exact R-symmetry thermal restoration

For models that exhibit exact thermal restoration of the $U(1)_R$ symmetry the $b_R(T)$ can be approximated by a step function[1]: $b_R(T > T_{\not{R}}) = 0$ and $b_R(T < T_{\not{R}}) = 1$. Hence, the gravitino yield (4.7) reads in this case:

$$Y_{3/2}(T) - Y_{3/2}(T_{\rm rh}) = -\frac{g_*(T)}{g_*(T_{\rm rh})} \left\{ \frac{n_{\rm rad} \left\langle \sigma^{(1/2)}_{3/2} v \right\rangle}{HT} \right\} \int^T_{T_{\not{R}}} dT\, b^2_R(T) \tag{4.9}$$

$$\simeq \frac{g_*(T)}{g_*(T_{\not{R}})} \left\{ \frac{n_{\rm rad} \left\langle \sigma^{(1/2)}_{3/2} v \right\rangle}{HT} \right\} \Bigg|_{T_{\not{R}}} T_{\not{R}} \tag{4.10}$$

where we took into account that $T \ll T_{\not{R}}$. Also, $Y_{3/2}(T_{\rm rh}) \sim 0$ since we consider that the dominant source of gravitino production are the scatterings in the thermal plasma and any pre-inflationary abundance was diluted by inflation. Therefore, for $T < 1$ MeV, i.e. after nucleosynthesis, for a decoupled gravitino and for $T_{\rm rh} > T_{\not{R}}$ the gravitino yield is

$$Y_{3/2} \simeq 1.1 \times 10^{-10} \left(\frac{T_{\not{R}}}{10^{10}\,{\rm GeV}} \right) \left(\frac{100\,{\rm GeV}}{m_{3/2}} \right) \left(\frac{m_{\tilde{g}}(\mu)}{1\,{\rm TeV}} \right)^2 \tag{4.11}$$

and the relic density,

$$\Omega_{3/2} h^2 \simeq 0.2 \left(\frac{T_{\not{R}}}{10^{10}\,{\rm GeV}} \right) \left(\frac{100\,{\rm GeV}}{m_{3/2}} \right) \left(\frac{m_{\tilde{g}}(\mu)}{1\,{\rm TeV}} \right)^2 . \tag{4.12}$$

The $T_{\not{R}}$ temperature, $T_{\not{R}} = 4F/(\lambda \Lambda_* \sqrt{N})$, can be written it in terms of the gravitino mass

$$T_{\not{R}} = \frac{4}{\sqrt{N}} \frac{\sqrt{3} m_{3/2} M_{\rm Pl}}{\lambda \Lambda_*} = 4 \frac{\sqrt{3} m_{3/2}}{\lambda \Lambda_*} 2.4 \times 10^{18}\,{\rm GeV} \tag{4.13}$$

and the gravitino relic density is recast into

$$\Omega_{3/2} h^2 = 0.2 \times \frac{16.6}{\sqrt{N}} \left(\frac{10^{10}\,{\rm GeV}}{\lambda \Lambda_*} \right) \left(\frac{m_{\tilde{g}}(\mu)}{1\,{\rm TeV}} \right)^2 . \tag{4.14}$$

This is a remarkable result. Firstly, the gravitino abundance does *neither* depend on the gravitino mass *nor* on the reheating temperature. Secondly, the quantities which control the yield are the

[1] This is indeed an approximation since a second order phase transition is not a discontinuous process; instead at the critical temperature T_X there is no barrier and the transition occurs smoothly.

supersymmetry breaking fundamental parameters λ and Λ_*. For $\sqrt{N} = \mathscr{O}(1-7)$, the gravitino does not overclose the universe when

$$\lambda \Lambda_* \gtrsim 10^{11}\,\text{GeV}. \tag{4.15}$$

It can account for the dominant dark matter component when $\lambda\Lambda_* \sim 10^{11}$; for example, when $\lambda = 10^{-5}$ and $\Lambda_* = 10^{16}$ GeV = $\mathscr{O}$(GUT) scale, the gravitino is the dark matter of the universe given that the MSSM scatterings are the leading source of gravitino production.

4.2 Approximate *R*-symmetry thermal restoration

For models that break explicitly the R-symmetry the $b_R(T)$ is given by $b_R(T) = [1+(T/T_{\not R})^2]^{-1}$ and the relevant part of the integral (4.7) is

$$\int_{T_{\rm rh}}^{T} dT\, b_R^2(T) = \frac{1}{2} T_{\not R} \left\{ \frac{T_{\not R} T}{T_{\not R}^2 + T^2} + \text{Arctan}\left(\frac{T}{T_{\not R}}\right) \right\} \Bigg|_{T_{\rm rh}}^{T}. \tag{4.16}$$

For, $T < 1$ MeV, i.e. after nucleosynthesis we expect $T \ll T_{\not R}, T_{\rm rh}$. Hence,

$$\int_{T_{\rm rh}}^{T} dT\, b_R^2(T) \cong -\frac{1}{2} T_{\not R} \left\{ \frac{T_{\not R} T_{\rm rh}}{T_{\not R}^2 + T_{\rm rh}^2} + \text{Arctan}\left(\frac{T_{\rm rh}}{T_{\not R}}\right) \right\} \tag{4.17}$$

For reheating temperatures higher than the $T_{\not R}$, which are the cases that we are interested in, and especially for $T_{\rm rh} \gg T_{\not R}$ the integral approximates to

$$\int_{T_{\rm rh}}^{T} dT\, b_R^2(T) \simeq -\frac{1}{2} T_{\not R} \text{Arctan}\left(\frac{T_{\rm rh}}{T_{\not R}}\right) - \frac{1}{2} T_{\not R} \left(\frac{T_{\not R}}{T_{\rm rh}}\right) \tag{4.18}$$

$$\simeq -\frac{1}{2} T_{\not R} \text{Arctan}\left(\frac{T_{\rm rh}}{T_{\not R}}\right) \equiv -\frac{\theta_{\rm rh}}{2} T_{\not R} \tag{4.19}$$

where, $\theta_{\rm rh} \equiv \text{Arctan}(T_{\rm rh}/T_{\not R})$ a coefficient that is roughly of order one: $\pi/4 < \theta_{\rm rh} < \pi/2$.

For $T_{\rm rh} = T_{\not R}$ the integral takes the value

$$\int_{T_{\rm rh}}^{T} dT\, b_R^2(T) = -\frac{T_{\not R}}{4} - \frac{T_{\not R}}{2}\frac{\pi}{4} \tag{4.20}$$

while for reheating temperatures lower than $T_{\not R}$, i.e. $T_{\rm rh} < T_{\not R}$ the value is

$$\int_{T_{\rm rh}}^{T} dT\, b_R^2(T) \simeq -\frac{1}{2} T_{\rm rh} - \frac{T_{\not R}}{2} \text{Arctan}\left(\frac{T_{\rm rh}}{T_{\not R}}\right). \tag{4.21}$$

Taking into account that, $\text{Arctan}x = x - x^3/3 + x^5/5 - x^7/7 + \ldots$, the integral for temperatures for $T_{\rm rh} < T_{\not R}$ converges to the $(-)T_{\rm rh}$ value as expected.

The conclusion is that for the high reheating temperatures $T_{\rm rh} > T_{\not R}$ the integral is $-T_{\not R}\,\theta_{\rm rh}/2$ or roughly $-T_{\not R}/2$. Therefore the (4.7) reads

$$Y_{3/2}(T) - Y_{3/2}(T_{\rm rh}) = -\frac{g_*(T)}{g_*(T_{\rm rh})} \left\{ \frac{n_{\rm rad} \left\langle \sigma_{3/2}^{(1/2)} v \right\rangle}{HT} \right\} \int_{T_{\rm rh}}^{T} dT\, b_R^2(T) \Rightarrow \tag{4.22}$$

$$Y_{3/2}(T) \simeq \frac{g_*(T)}{g_*(T_0)} \left\{ \frac{n_{\rm rad} \left\langle \sigma_{3/2}^{(1/2)} v \right\rangle}{HT} \right\} \Bigg|_{T_0} \frac{T_{\not R}}{2} \tag{4.23}$$

where we considered the coefficients g_* to be dominated by the temperature $T_{\not R}$. The contribution of the gravitino abundance to the energy density of the universe is half times that of (4.12)

$$\Omega_{3/2} h^2 \simeq 0.2 \left(\frac{T_0/2}{10^{10}\,\text{GeV}} \right) \left(\frac{100\,\text{GeV}}{m_{3/2}} \right) \left(\frac{m_{\tilde g}(\mu)}{1\,\text{TeV}} \right)^2 . \tag{4.24}$$

and substituting $T_{\not R} = 4F/(\sqrt{N}\lambda\Lambda_*)$ it is recast to

$$\Omega_{3/2} h^2 \simeq 0.1 \times \frac{16.6}{\sqrt{N}} \left(\frac{10^{10}\,\text{GeV}}{\lambda\Lambda_*} \right) \left(\frac{m_{\tilde g}(\mu)}{1\,\text{TeV}} \right)^2 . \tag{4.25}$$

We see that when the superpotential has an approximate $U(1)_R$ symmetry the result is basically the same with that of an exact R-symmetric superpotential. Hence, in terms of the gravitino relic abundance the (softly) R-violating theories behave much like the exact R-symmetric theories.

It is interesting to note that the all important temperature $T_{\not R}$ has *no* dependence on the R-symmetry breaking parameters c and M. Although they define the supersymmetry breaking vev X_0 they cancel out at the ratio $b_R(T) = X(T)/X_0$.

We mention, that in the case of approximate R-symmetric models the gravitino abundance is suppressed by a factor of two compared with the case of exact restoration of the R-symmetry. At first sight this seems a paradox for, the expectation may have been that the gravitino production is stronger suppressed when an exact restoration of the R-symmetry takes place than when the restoration is approximate. However, we believe that this small discrepancy originates from the fact that we simplified the thermal evolution of the $X(T)$ by assuming a step-like behaviour while the symmetry breaking occurs smoothly via a graduate increase of the mean value of the scalar field. The $T_{\not R}$ value of the integral (4.9) should be considered as the maximal and thus, the $Y_{3/2}$ bound in this case is a conservative one.

Finally, we would like to comment on the production of gravitinos from the thermal plasma due to the top Yukawa coupling, an effect considered at [8]. The production rate is enhanced by the additional term

$$\gamma_{\rm top} = 1.30 \frac{9\lambda_t T^6}{2M_{\rm Pl}^2 \pi^5} \left(1 + \frac{A_t^2}{3m_{3/2}^2} \right) \tag{4.26}$$

apart from the processes involving gauge supermultiplets, i.e. these that we have already analysed:

$$\gamma_V = \frac{T^6}{2(2\pi)^3 M_{\rm Pl}^2} \sum_{N=1}^{3} n_N \left(1 + \frac{m_{\lambda_N}^2}{3m_{3/2}^2} \right) f_N \tag{4.27}$$

where f_N a factor which includes the gauge couplings. According to [8] the production processes that include the top quark Yukawa coupling enhance the gravitino production rate by almost 10% or more if the A_t is bigger than the gaugino masses. This effect can become very important when the production of the helicity $\pm 1/2$ component is suppressed by the vanishing gaugino masses. However, much like the gaugino masses, the A-terms require interactions which violate the $U(1)_R$ symmetry and therefore, we expect to be suppressed as well at high temperatures.

4.3 Conclusions and discussion

The standard paradigm in cosmology is that the dark matter is a weak interacting massive particle (WIMP). The weak interactions can maintain the dark matter in thermal equilibrium with the observable particles in the early universe until $T_f \sim M_{\rm WIMP}/20$. When the WIMP stops annihilating efficiently it freezes out and its relic abundance is given by the approximate formula

$$\Omega_{WIMP}h^2 \simeq 0.1 \left(\frac{x_f}{10}\right)\left(\frac{1\times 10^{-26}{\rm cm\,s^{-1}}}{\langle\sigma v\rangle}\right). \tag{4.28}$$

where $x_f \equiv M_{\rm WIMP}/T_f$ at the time of the freeze out. The fact that WIMP cross section value gives roughly $\Omega_{\rm WIMP} \sim 0.1$ makes the neutralino dark matter a compelling candidate. The very attractive feature of the (4.28) is that it is independent of the reheating temperature given that $T_{\rm rh} \gtrsim M_{\rm DM}/20$, in order the WIMP to equilibrate.

A similar behaviour is suggested by the (4.24) for the gravitino:

$$\Omega_{3/2}h^2 \simeq 0.1 \times \frac{16.6}{\sqrt{N}}\left(\frac{10^{10}\,{\rm GeV}}{\lambda\Lambda_*}\right)\left(\frac{m_{\tilde{g}}(\mu)}{1\,{\rm TeV}}\right)^2 . \tag{4.29}$$

The abundance depends on the product $\lambda\Lambda_*$ i.e. on fundamental quantities of supersymmetry breaking. A theory with a physical cut off Λ_* related to the GUT scale and a coupling $\lambda \sim 10^{-5}$ which is naturally small in the sense that small Yukawa coupling values are expected in the IR macroscopic theory. In addition, considering that the supersymmetry breaking local minimum must be thermally preferred [22] we conclude that small values of the coupling λ are favourable. Decreasing the cut off and increasing the value of the coupling renders the supersymmetry preserving vacuum more attractive. Hence, we find an interesting window of values where the supersymmetry breaking vacuum is selected and the gravitino does not overclose the universe, or even accounts for the dominant dark matter component. This parameter space also specifies the gravitino mass range.

Furthermore, the gravitino yield from the MSSM is independent of the reheating temperature given that the reheating temperature is larger than $T_{\not R}$. According to (2.6), for example a gravitino with mass 1 GeV constrains the reheating temperature to be $T_{\rm rh} \lesssim 10^{12}$ GeV which is 10^4 times relaxed relatively to the conventional bound (2.5). If there are no other sources that generate gravitinos then the reheating temperature is bounded from above only by helicity $\pm 3/2$ gravitino component yield. In fact, the allowed window of the reheating temperatures is remarkably wide.

In this letter we have focused on the gravitino production solely by the MSSM sector. Generically the restoration of the R-symmetry implies that the messengers fields are thermalized. Due to the direct superpotential coupling between the messengers and the goldstino superfields the thermalized messengers can efficiently generate goldstinos imposing strict bounds on the reheating temperature and the GMSB parameters. Furthermore, during an R-symmetric phase the spurion X is displaced from the zero temperature minimum and under certain circumstances its oscillations dominate the energy density of the universe. We mention that there is a parameter space that the gravitino yield from messenger thermal scatterings and decays is subleading and the spurion does not produce late entropy. If the values of the GMSB models parameters lie in this region of the parameter space then the gravitino production is dominated by the MSSM sector and the results of this work are valid for the total gravitino abundance. More details about the gravitino production from the MSSM and GMSB sector in the thermalized early universe can be found in the ref. [27].

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We emphasize that the all important temperature is the $T_{\not R} = 4F/(\lambda\Lambda_* \sqrt{N})$. Its order of magnitude can be understood easily by recalling that the mass of the spurion X (which at the tree-level is a flat direction) is $m_X \sim F/\Lambda_*$ and its thermal mass is $\delta m_X^T \sim \sqrt{N}\lambda T$, hence

$$T_{\not R} \sim \frac{m_X}{\sqrt{N}\lambda}. \tag{4.30}$$

The $m_X \sim F/\Lambda_*$ is of the order of the TeV scale thereby, it is the smallness of the Yukawa coupling to the messenger fields that makes the $T_{\not R}$ large suggesting a GeV mass range gravitino for dark matter.

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