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Aspects of String Cosmology

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We review recent progress in string cosmology, where string dualities are applied so as to obtain complete cosmological evolutions, free of any essential singularities. Two classes of models are analyzed. The first class consists of string gas cosmologies associated to certain thermal configurations of type II $\mathcal{N} = (4,0)$ models. Finite temperature is introduced along with non-trivial "gravito-magnetic" fluxes that lift the Hagedorn instabilities of the canonical ensemble and restore thermal T-duality symmetry. At a critical maximal temperature additional thermal states become massless sourcing stringy S-branes, which facilitate a bounce between the two dual, asymptotically cold phases. Unlike previous incarnations of pre-Big Bang cosmologies, the models remain perturbative throughout the cosmological evolution. The second class consists of exact solutions to classical string theory that admit a Euclidean description in terms of compact parafermionic worldsheet systems. The Euclidean target space corresponds to a non-singular, compact T-fold, which can be used to construct a normalizable Hartle-Hawking wavefunction for the cosmology.

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1. Introduction

The hot Big-Bang model gives a robust description of the evolution of the Universe, from the onset of Nucleosynthesis until present [1]. Key cosmological puzzles concerning the observed large-scale smoothness, the flatness and horizon problems have inspired inflationary cosmology, where a phase of rapid accelerated expansion occurs in the very early cosmological history, eventually settling into radiation-dominated evolution [2]. The model can be adapted so as to take into account the current accelerating expansion of the Universe, as well as gravitational effects observed in large scale structures, by introducing dark energy and dark matter [3]. Thus the resulting cosmological scenario involves a very rich phenomenological model, called the ΛCDM model, based on classical general relativity and quantum field theory, with high temperature eras, symmetry breaking phase transitions, and proportionally large amounts of dark matter and dark energy dominating the very late time evolution.

This standard cosmological model presents some of the greatest challenges to fundamental physics. Two of these have proved to be particularly acute over the years. Firstly, if we extrapolate the cosmological evolution back in time, using the equations of general relativity and quantum field theory, we are driven to an initial singularity, where the Universe collapses to zero volume and the description breaks down [4]. The second concerns the nature of dark energy. The simplest explanation for it is a positive, however unnaturally small, cosmological constant, $\Lambda \sim 10^{-120} M_p^4$, many orders of magnitude smaller than the Planck and elementary particle physics scales. To date no symmetry principle or mechanism is known to explain its value. (For a recent review concerning the cosmological constant problem, see [5].) Moreover, if dark energy persists arbitrarily long, it would imply that the Universe approaches de Sitter space in the far future, with a cosmic event horizon, and so portions of space will remain unobservable, forever. The observable part of the Universe is in a highly mixed state. Therefore, within the context of general relativity and the Standard Model, we lack a coherent framework to analyze the cosmology of our Universe, from beginning to end.

If string theory is a complete theory of quantum gravity, it should eventually provide a consistent cosmological framework. The hope is that by incorporating fundamental duality symmetries and stringy degrees of freedom in time-dependent settings, we will be able to obtain complete cosmological histories, free of any essential singularities, and new tools for model building.

Indeed, string dualities have given us profound insights into the nature of *Space* over the years. New phenomena arise at short distances of order the string scale, $l_s = \sqrt{\alpha'}$, or the Planck length, l_p, which do not admit a conventional field theory description, with Riemannian concepts breaking down. Unlike a field theoretic incarnation, string theory is a UV finite theory of quantum gravity. T-duality, or small/large volume duality, implies that shrinking radii past the string scale does not produce a lower dimensional theory. The stringy spacetime uncertainty principles, $\Delta x \Delta t \sim l_s^2$, $\Delta x \Delta t \sim l_p^2$, point to a minimal length and an intrinsic non-commutative geometry, leading to the UV/IR connection [6]. There are examples of singularity resolution, such as orbifold [7] and conifold singularities [8], and topology change [9, 10], where the appearance of extra massless states, localized at the singularity, make it fuzzy and smooth. Non-conventional thermodynamics, with Hagedorn and black hole phases, signal a maximal temperature and non-trivial phase transitions [11–18]. Finally, there are robust examples of non-perturbative strong/weak coupling dualities – see e.g. [19] and references therein – and holographic gauge theory/gravity dualities [20,21], illustrating how string theory can provide concrete answers to many of the puzzles one has to face in trying to quantize Einstein's theory of general relativity.

Some of the important lessons, relevant to the discussion below, are the following. Locality, geometry and even topology are approximate concepts, acquiring a more precise definition at low enough energies: $E \leq 1/l_s$ [10,22]. From holography, we learn that gravity and *Space* can emerge in special, quantum mechanical systems. E.g. a large N maximally supersymmetric gauge theory in 4-dimensions gives rise to a 10-dimensional gravitational theory, a string theory on $AdS_5 \times S_5$ [21]. Finally, apparently singular regimes and/or geometries can be mapped via string dualities into non-singular ones, with well defined effective descriptions [10, 22]. Much of this insight has been obtained from studies of static, equilibrium configurations of superstrings, but given the principle of relativity, it seems inevitable that similar results hold for the time-dependent cases.

Thus progress in String Cosmology can be achieved if we manage to extend the web of string dualities to time-dependent, cosmological settings. See e.g. [23–28] [10] for work towards this end. This endeavor is both technically and conceptually challenging. Indeed, in phases with (spontaneous) supersymmetry breaking and geometric variation the string equations can become very difficult to solve, and in many cases even hard to formulate. With the moduli acquiring time-dependence, these may wander through cross-over regions of moduli space, where we have no control over the quantum corrections, and the effective field theory approach breaks down. At a more fundamental level, it is hard to identify and compute the correct, precise observables, and we lack a second quantized version of the theory to probe it directly off-shell.

It is also challenging to extend the web of holographic dualities to cosmological backgrounds. The construction of the holographic theory is very sensitive to the global structure of spacetime, with the dual variables living at the boundary of spacetime. For asymptotically de Sitter cosmologies (like our own), the natural boundaries lie to the infinite future, suggesting a form of spacelike holography: E.g. the dS_4/Euclidean CFT_3 correspondence [29]. The dual holographic theory is conjectured to be a 3-dimensional CFT living on the future (spacelike) boundary of de Sitter space. Symmetry considerations fix the central charge of the CFT to be inversely proportional to the square of the asymptotic value of the Hubble parameter – the Hubble parameter $H(t)$ decreases monotonically with time and asymptotes to a constant for asymptotically de Sitter cosmologies. Thus, $c \sim 1/[H(t \to \infty)]^2 G$.

Such a boundary CFT, if it exists, may be non-unitary, and in fact it was argued that the finiteness of de Sitter entropy implies the existence of Poincare recurrences at very late time scales, which in turn prevent the realization of local observables in the infinite time limit [30]. There is no explicit, microscopic construction in string theory. If realized however, it would be a holographic example where *Time* and the cosmological history emerge, perhaps as RG flow in the CFT. The relation between the central charge and the Hubble parameter suggests that the field theory RG flow gets mapped to the time-reversed cosmological evolution. Therefore, reconstructing the very early Universe would amount to the difficult task of decoding the hologram in the deep infrared of the boundary CFT. Another conceptual difficulty is that no single observer can measure the boundary CFT correlators.

In this lecture we will revisit the possibility of realizing eternal string cosmologies where an initially contacting phase bounces/emerges into an expanding thermal phase. We will argue that string theory contains the ingredients which could resolve strong curvature regimes, via duality transformations that lead to a well-defined, effective description [23–28] [10]. Two classes of string cosmological solutions will be discussed to illustrate this pattern. The first class consists of string gas cosmologies associated to certain special, thermal configurations of type II $\mathcal{N}=(4,0)$ models [31–33]. Finite temperature is introduced along with non-trivial "gravito-magnetic" fluxes that lift the Hagedorn instabilities of the canonical ensemble and restore thermal T-duality symmetry [34–36] [1]. The cosmological evolutions describe bouncing Universes, with the bounce occurring at a stringy extended symmetry point. The second class consists of exact solutions to classical string theory that admit a Euclidean description in terms of compact parafermionic worldsheet systems [38, 39]. The Euclidean target space corresponds to a non-singular, compact T-fold, which can be used to construct a normalizable Hartle-Hawking wavefunction for the cosmology [39].

2. Bouncing string cosmologies

Before focusing on stringy examples, we review the situation in classical general relativity. The singularity theorems of Penrose and Hawking show that a smooth reversal from contraction to expansion is impossible unless an energy condition of the form

$$T_{\mu\nu}v^{\mu}v^{\nu} \geq 0 \tag{2.1}$$

is violated [4]. For the null energy condition (NEC), v^{μ} stands for any null future pointing vector. Let us see how these theorems apply for homogeneous and isotropic Friedmann-Robertson-Walker (FRW) cosmologies:

$$ds^2 = -dt^2 + [a(t)]^2 d\Omega_k^2. \tag{2.2}$$

As usual a stands for the scale factor and the Hubble parameter is given by $H = \dot{a}/a$. We also assume that the cosmological evolution is supported by various sources with total energy density ρ and pressure P, which comprise together a perfect fluid.

The relevant equations are the Friedmann-Hubble equation

$$H^2 = \frac{8\pi}{3}G\rho - \frac{k}{a^2} \tag{2.3}$$

and the 1st law of thermodynamics for adiabatic evolution:

$$\dot{\rho} + 3H(\rho + P) = 0. \tag{2.4}$$

These two equations imply that

$$\dot{H} = -4\pi G(\rho + P) + \frac{k}{a^2}. \tag{2.5}$$

Therefore, if the NEC is satisfied,

$$\rho + P \geq 0, \tag{2.6}$$

for flat ($k = 0$) and open ($k = -1$) Universes, the Hubble parameter decreases monotonically with time, $\dot{H} \leq 0$, and reversal from contraction ($H < 0$) to expansion ($H > 0$) is not possible. The two

[1] Additional work on thermal duality includes [37].

phases are separated by a singularity, or the expanding phase is past geodesically incomplete. All known (macroscopic) sources of energy and matter in our Universe satisfy the NEC.

For closed Universes ($k = 1$), we get

$$\ddot{a} = -\frac{4\pi}{3} G a (\rho + 3P), \tag{2.7}$$

and so for the bounce to occur at a singularity the strong energy condition (SEC),

$$\rho \geq -3P, \tag{2.8}$$

must be satisfied. There are field theoretic sources for which the NEC holds but the SEC is violated, e.g. positive vacuum energy or a positive cosmological constant, and global de Sitter space is an example of a closed FRW cosmology, where (at the classical level) a contracting phase smoothly reverses to an expanding inflationary phase. The problem with such an initially exponentially contracting phase is that it requires the Universe to be sufficiently empty for an infinite amount of time. During exponential contraction perturbations grow large, and so the Universe is likely to thermalize before expansion sets in, and within the field theoretic context, collapse to a singularity. See [40] for further discussions.

Another possibility is to consider a Universe which is eternally inflating, or expanding sufficiently fast forever (requiring that $H_{av} > 0$ throughout the cosmological history). Even without requiring an energy condition, the authors of [41] show that such a Universe cannot be past geodesically complete and must have a beginning, presumably an initial singularity. Scalar field driven inflation cannot be the ultimate theory of the very early Universe.

There are various notable attempts to overcome the reversal problem within the string theoretic set-up. Let us summarize the main ideas of some of these.

- There are various incarnations of the pre Big Bang scenario [26]. Typically in the pre Big Bang phase the dilaton runs from weak to strong coupling. As we approach strong coupling, new terms in the effective action such as higher derivative interactions and potentials can become relevant, invalidating some of the assumptions of the singularity theorems and thus facilitating the bounce. The difficulty of these models is in maintaining analytical control over the strongly coupled dynamics at the bounce.

- Various (weakly coupled) null/spacelike orbifold models of the singularities, where the orbifold is obtained by modding out with a boost [42, 43]. Adding a particle in such a spacetime amounts to also adding in the covering space an infinite number of boosted images, causing strong backreaction, and possibly gravitational collapse [44]. See also [45] for a counterexample.

- String gas cosmology [23, 24]. Here the idea is that the Universe starts as a compact space, e.g. a 9-dimensional torus with all radii close to the string scale and temperature close to (but below) the Hagedorn temperature: $T \sim T_H$. There is a competition between the thermally excited momentum modes, which keep the spatial torus from shrinking, and thermally excited winding modes which prevent the Universe from expanding. The system is presumed to be in a quasi-static phase, until thermal fluctuations cause the winding states to annihilate and some dimensions to expand. At the level of the two derivative dilaton/gravity effective action, there is a singularity a finite time in the past, where also the dilaton grows to strong coupling.

- Modular cosmology [27, 46], cosmological billiards [47], brane collisions and the Ekpyrotic scenario [48] and matrix cosmology [49].

Further work on string cosmology includes [50].

We shall focus on another class of bouncing thermal string cosmologies, exploiting stringy phase transitions and phenomena that can occur at temperatures close to the Hagedorn temperature. The non-singular cosmological solutions are based on a mechanism that resolves the Hagedorn instabilities of finite temperature strings, realizable in a large class of initially $\mathcal{N}=(4,0)$ type II superstring models [31–33]. We also review some aspects of strings at finite temperature and (partial) spontaneous supersymmetry breaking via geometrical fluxes.

2.1 Thermal configurations of type II $\mathcal{N}=(4,0)$ models

We consider weakly coupled type II $(4,0)$ models on initially flat backgrounds:

$$R^d \times T^{10-d}, \tag{2.9}$$

where the internal toroidal radii are taken close to the string scale. There are 16 real spacetime supersymmetries arising from the left-moving sector of the worldsheet. The right-moving susy is broken spontaneously by twisting some of the internal radii with F_R, the right-moving fermion number. Under the Z_2 symmetry $(-1)^{F_R}$ the right-moving R sector changes sign.

This pattern of asymmetric susy breaking leads to extended symmetry points, when the internal radii are at the fermionic point [31–36]. At finite temperature, such points in moduli space are preferred, with the moduli participating in the breaking of the right-moving supersymmetries being stabilized at the extended symmetry point values [51]. As a result the odd-F_R sector is heavy, with masses being bounded from below by the string scale: $m^2 \geq 1/(2\alpha')$.

As illustrative examples, consider the two dimensional Hybrid vacua, on $R^2 \times T^8$, where all the internal radii are taken at the fermionic point $R = 1/\sqrt{2}$ [31, 36] – we work in string units where $\alpha' = 1$. At this point the eight compact supercoordinates can be replaced with 24 left-moving and 24 right-moving worldsheet fermions. The 24 left-moving fermions are split into two groups of 8 and 16. The one-loop partition function is given by

$$Z_{\rm Hyb} = \frac{V_2}{(2\pi)^2} \int_{\mathscr{F}} \frac{d^2\tau}{4({\rm Im}\tau)^2} \, \frac{1}{\eta^8} \, \Gamma_{E_8}(\tau) \, (V_8 - S_8) \, \left(\bar{V}_{24} - \bar{S}_{24}\right), \tag{2.10}$$

and exhibits holomorphic/anti-holomorphic factorization.

In the left-moving sector, the group of 8 fermions are described in terms of the $SO(8)$ characters,

$$O_8 = \frac{\theta_3^4 + \theta_4^4}{2\,\eta^4}\,, \qquad V_8 = \frac{\theta_3^4 - \theta_4^4}{2\,\eta^4}\,,$$

$$S_8 = \frac{\theta_2^4 - \theta_1^4}{2\,\eta^4}\,, \qquad C_8 = \frac{\theta_2^4 + \theta_1^4}{2\,\eta^4}\,, \tag{2.11}$$

as in the conventional superstring models, and the other 16 fermions are described by the chiral E_8 lattice: Γ_{E_8}. In the right-moving sector, the fermions are described in terms of the $SO(24)$ characters:

$$\bar{V}_{24} - \bar{S}_{24} = \frac{1}{2\bar{\eta}^{12}} \left(\bar{\theta}_3^{12} - \bar{\theta}_4^{12} - \bar{\theta}_2^{12} \right) = 24. \tag{2.12}$$

Despite the breaking of the right-moving supersymmetries, this sector exhibits *Massive Spectrum Degeneracy Symmetry* (MSDS) [35]. This degeneracy is broken at the right-moving massless sector, a fact that leads to the breaking of the right-moving supersymmetries. The left-moving supersymmetry remains unbroken. The massless sector of the physical, level-matched, spectrum consists of 24×8 bosons and 24×8 fermions arising in the $V_8\bar{V}_{24}$ and $S_8\bar{V}_{24}$ sectors respectively. Notice in particular that the right moving R sector is massive.

The model can be also exhibited as a freely acting, asymmetric orbifold compactification of the type II superstring to 2 dimensions. The relevant half-shifted $(8,8)$ lattice is given by

$$\Gamma_{(8,8)} \left[{}^{\bar{a}}_{\bar{b}} \right] = \Gamma_{E_8} \times \bar{\theta} \left[{}^{\bar{a}}_{\bar{b}} \right]^8 \rightarrow$$

$$\frac{\sqrt{\det G_{IJ}}}{(\sqrt{\tau_2})^8} \sum_{\tilde{m}^I, n^J} e^{-\frac{\pi}{\tau_2}(G+B)_{IJ}(\tilde{m}+\tau n)^I(\tilde{m}+\bar{\tau} n)^J} \times e^{i\pi(\tilde{m}^1\bar{a}+n^1\bar{b}+\tilde{m}^1 n^1)}. \tag{2.13}$$

The modular covariant cocycle describes the coupling of the lattice to the right-moving fermion number F_R. In particular, only one internal cycle is twisted by F_R. At the MSDS point, the metric and antisymmetric B-field tensors, G_{IJ}, B_{IJ} take special values, leading to holomorphic/anti-holomorphic factorization and enhanced gauge symmetry with the local gauge group given by [36]

$$U(1)_L^8 \times [SU(2)_R]_{k=2}^8. \tag{2.14}$$

We have focused on the highly symmetric Hybrid vacua, where the presence of exact right-moving MSDS symmetry leads to exact computations, as we will see below, but a large class of $(4,0)$ models can be constructed in various dimensions [32–34].

Next we consider the models at finite temperature. To avoid strong Jeans instabilities and gravitational collapse into black holes, we compactify R^{d-1} on a large torus with each cycle having radius $R \gg 1$, and take the string coupling to be sufficiently weak. Backreaction can be ignored, if the size of the thermal system is much larger than its Schwarzschild radius: $R_S \sim GM = G\rho R^3$ in 4 dimensions, where the energy density is set by the temperature. This allows for the following range for the sizes of the radii R:

$$1 \ll R \ll \frac{1}{\sqrt{G\rho}} \sim \frac{1}{g_s}, \tag{2.15}$$

where the last equality follows for temperatures close to the string scale. Both inequalities can be satisfied at sufficiently weak coupling. At larger values of the coupling constant, we cannot ignore backreaction and we must take into account the induced cosmological evolution. The string coupling must still be kept small so as to be able to maintain conditions of quasi-static thermal equilibrium and trust the perturbative computations of various thermodynamical quantities.

In string theory new instabilities set in at temperatures close to the string scale, $T \sim 1/l_s$, the Hagedorn instabilities, which signal non-trivial phase transitions [12–18]. The origin of these instabilities is due to the exponential rise in the density of (single-particle) string states at large mass [11]:

$$n(m) \sim e^{\beta_H m}. \tag{2.16}$$

Because of the exponential growth, the single string partition function

$$Z \sim \int^{\infty} dm n(m) e^{-\beta m} = \int^{\infty} dm e^{-(\beta-\beta_H)m} \tag{2.17}$$

diverges for temperatures above Hagedorn: $T > T_H = 1/\beta_H$. The Hagedorn temperature is set by the coefficient of the exponent in the asymptotic formula for the density of states and it is close to the string scale:

$$\beta_H = 2\pi\sqrt{2\alpha'} \tag{2.18}$$

in Type II superstrings.

Therefore the critical behavior as $\beta \to \beta_H$ is governed by string states of large mass $m \gg 1/l_s$, or high level N – recall that $m^2 \sim N/\alpha'$. At weak coupling, the typical size of such a string state is large, of order $l \sim N^{1/4} l_s$ [52, 53]. We can think of it as a random walk of $N^{1/2}$ bits. Now the entropy carried by an excited long string of mass m is greater than the entropy of n smaller strings, each having mass m/n. So close to the Hagedorn temperature, percolation phenomena take place with multiple strings coalescing into fluctuations of a single long, tangled string.

The critical point can be also described by an effective field theory of a massless complex scalar field, manifesting a UV/IR connection [12–14, 16, 17]. In quantum field theory, the thermal effective theory is obtained by compactifying Euclidean time on a circle with period set by the inverse temperature, $2\pi R_0 = 1/T$, and imposing periodic boundary conditions for bosonic fields and anti-periodic boundary conditions for fermions. In string theory, the thermal system can be described in terms of a freely acting orbifold, obtained by twisting the Euclidean time circle with the spacetime fermion number F. For type II superstrings this amounts to coupling the Euclidean time $\Gamma_{1,1}(R_0)$ lattice with the following co-cycle [54]:

$$e^{i\pi(\tilde{m}^0(a+\bar{a})+n^0(b+\bar{b}))}. \tag{2.19}$$

In this picture, the instabilities appear at a critical compactification radius set by the Hagedorn temperature. Certain string winding modes, with $(n_0 \neq 0)$, become massless precisely when $R_0 = R_H = 1/(2\pi T_H)$. They become tachyonic at smaller radii, when $R_0 < R_H$. More precisely, two winding modes pair up to form a complex scalar field, whose thermal mass is given by

$$m^2(R_0) = R_0^2 - R_H^2. \tag{2.20}$$

As a result, near the critical point the behavior of the partition function is captured by the thermal scalar path integral:

$$\mathscr{Z} \sim \int [d\varphi] e^{-S[\varphi]}$$

$$S[\varphi] \sim \int d^{d-1}x (\partial_i \varphi^* \partial^i \varphi + m^2(R_0)\varphi^*\varphi). \tag{2.21}$$

We can illustrate an aspect of this correspondence, by recovering the asymptotic formula for the density of states at large mass [23, 53, 55]. With all spatial dimensions being compact and close to the critical point, the logarithm of the partition function is dominated by the lowest eigenvalue of the Klein-Gordon operator $-\nabla^2 + m^2(R_0)$, given by the square of the mass: $\lambda_0 = m^2(R_0)$. Therefore

$$Z_c = \ln \mathscr{Z} \sim -\ln \lambda_0 \sim -\ln(R - R_H) \sim \int^{\infty} dm e^{-\beta m} n(m). \tag{2.22}$$

The logarithmic behavior as $R \to R_H$ can be reproduced for

$$n(m) = e^{\beta_H m}/m. \tag{2.23}$$

When $d-1$ spatial dimensions are non-compact, a similar computation yields [23, 53, 55]

$$n(m) \sim V_{d-1} e^{\beta_H m}/m^{(d+1)/2}. \tag{2.24}$$

So the Hagedorn divergence for $R_0 < R_H$ can be interpreted as an IR instability of the underlying Euclidean thermal background. Tachyon condensation gives a genus-zero contribution to the free energy, $F \sim 1/g_s^2$, leading to large backreaction, which, presumably, brings the thermal ensemble to a speedy end [14]. In type II $(4,0)$ models, perturbatively stable configurations can be produced, if in addition to temperature, we turn on vacuum potentials associated to the graviphoton G_{I0} and B_{I0} fields, where the index I is along an internal direction twisted by F_R [34, 36]. See also [56]. In particular, we turn on the $U(1)_L$ combination, $G_{0I} + 2B_{0I}$, of these fields. At finite temperature, such vacuum potentials cannot be gauged away, as they correspond to topological vacuum parameters. These *gravito-magnetic* fluxes modify the thermal masses of all states charged under the graviphoton fields, and for large enough values, the tachyonic instabilities can be lifted. Equivalently the contribution to the free energy of the massive oscillator states gets regulated (refined), reducing the effective density of thermally excited states, and restoring asymptotic supersymmetry [57].

The Hagedorn free models can be described in terms of freely acting asymmetric orbifolds of the form $(-1)^{F_L}\delta_0$, where δ_0 is a Z_2-shift along the Euclidean time circle [31, 32, 34, 36]. In the Hybrid example, the partition function is given explicitly by

$$\frac{Z_{\text{Hyb}}}{V_1} = \int_{\mathscr{F}} \frac{d^2\tau}{8\pi(\text{Im}\,\tau)^{3/2}} \, (\bar{V}_{24} - \bar{S}_{24}) \, \frac{\Gamma_{E_8}(\tau)}{\eta^8}$$
$$\times \sum_{m,n} \Big(V_8 \, \Gamma_{m,2n}(R_0) + O_8 \, \Gamma_{m+\frac{1}{2},2n+1}(R_0) - S_8 \, \Gamma_{m+\frac{1}{2},2n}(R_0) - C_8 \, \Gamma_{m,2n+1}(R_0) \Big), \tag{2.25}$$

and it is finite for all values of the thermal modulus R_0. In fact, the model remains tachyon-free under all deformations of the dynamical moduli associated with the compact, internal eight-manifold [36].

All such models exhibit a number of universal properties, irrespectively of spacetime dimension [32]. The gravito-magnetic fluxes lead to a restoration of the stringy T-duality symmetry along the thermal circle:

$$R_0 \to R_c^2/R_0, \quad S_8 \leftrightarrow C_8, \tag{2.26}$$

where in all models, the self-dual point occurs at the fermionic point $R_c = 1/\sqrt{2}$. The partition function is finite and duality invariant, but it is not a smooth function of R_0. At the self-dual point

R_c additional thermal states become massless enhancing the gauge symmetry associated to the Euclidean time circle,

$$U(1)_L \times U(1)_R \to [SU(2)_L]_{k=2} \times U(1)_R, \tag{2.27}$$

and inducing a conical structure in Z (as a function of R_0), signaling a stringy phase transition.

For example in the Hybrid model, 2×24 states in the $O_8\overline{V}_{24}$ sector become massless, precisely at the self-dual point:

$$m^2 = \left(\frac{1}{2R_0} - R_0\right)^2. \tag{2.28}$$

Away from the critical point the mass square is strictly positive, and the states do not become tachyonic at smaller values of the radius R_0 [2]. The corresponding left-moving and right-moving momentum charges and vertex operators are given by

$$p_L = \pm 1, \quad p_R = 0, \quad O_\pm = \psi_L^0\, e^{\pm i X_L^0}\, \mathscr{O}_R. \tag{2.29}$$

So the additional massless states carry both non-trivial momentum and winding charges.

In the Hybrid model the thermal partition function can be computed exactly thanks to right-moving MSDS symmetry [31, 36]:

$$\frac{Z_{\text{Hyb}}}{V_1} = 24 \times \left(R_0 + \frac{1}{2R_0}\right) - 24 \times \left|R_0 - \frac{1}{2R_0}\right|. \tag{2.30}$$

There is complete suppression of the massive oscillator contributions away from the critical point. However, stringy behavior survives at the critical point giving rise to the conical structure. From the thermal effective field theory point of view, such non-analytic behavior is induced after integrating out the additional massless states. With one spatial dimension non-compact, each complex boson becoming massless contributes a factor given by the absolute value of the mass: $-|m|$. Since in the Hybrid model there are 24 such states, with masses given by equation (2.28), the non-analytic term in the partition function is accounted for. Thermal configurations of non-critical heterotic strings in two dimensions enjoy very similar properties [58].

With $d-1$ spatial dimensions non-compact, the partition function acquires a higher order conical structure:

$$\sim \left|R_0 - \frac{1}{2R_0}\right|^{d-1}, \tag{2.31}$$

implying a milder transition as a function of R_0. Recall however that to avoid non-perturbative Jeans instabilities, we must keep all but at least one of the large spatial dimensions compact, and so the infinite volume and $R_0 \to R_c$ limits may not commute. Therefore, we can take a number of spatial dimensions to be arbitrarily large (but compact), and still the conical structure be linear at the critical point.

Thermal duality implies the existence of two dual asymptotic regimes dominated by the light thermal momenta, $R_0 \gg R_c$, and the light thermal windings, $R_0 \ll R_c$, respectively. In the regime of light thermal momenta the partition function is given by

$$\frac{Z}{V_{d-1}} = \frac{n^* \Sigma_d}{(2\pi R_c)^{d-1}} \left(\frac{R_c}{R_0}\right)^{d-1} + \mathscr{O}\left(e^{-R_0/R_c}\right), \tag{2.32}$$

[2] This is to be contrasted with heterotic strings at finite temperature, where the two dual phases, at small and large values of the thermal modulus R_0, are separated by an intermediate tachyonic region.

giving rise to the characteristic behavior of massless thermal radiation in d dimensions. The temperature is given by the inverse period of the Euclidean time circle, $T = 1/2\pi R_0$; n^* is the number of effectively massless degrees of freedom and Σ_d stands for the Stefan-Boltzmann constant of massless thermal radiation in d dimensions.

By duality, we get that in the regime of light thermal windings, $R_0 \ll R_c$, the partition function is given by

$$\frac{Z}{V_{d-1}} = \frac{n^*\Sigma_d}{(2\pi R_c)^{d-1}} \left(\frac{R_0}{R_c}\right)^{d-1} + \mathscr{O}\left(e^{-R_c/R_0}\right). \tag{2.33}$$

Notice in particular that $Z \to 0$ as $R_0 \to 0$. Now in standard thermodynamics the thermal partition function decreases monotonically as the temperature decreases. So the correct definition of temperature cannot be $T = 1/2\pi R_0$ in this regime. That is, the temperature in this regime is not set by the inverse period of the Euclidean time circle. The light winding excitations are non-local in X^0, but are local in the T-dual of X^0. In fact by T-duality, we can interpret them as ordinary thermal excitations associated with the large T-dual circle, whose radius is given by $\tilde{R}_0 = R_c^2/R_0$. So the temperature in this regime is given by $T = 1/2\pi\tilde{R}_0 = R_0/(2\pi R_c^2)$, and the system at small radius R_0 is again effectively cold.

Thus the thermal system has two dual, asymptotically cold phases as the value of the thermal modulus R_0 varies. Each thermal phase arises via spontaneous symmetry breaking, as we deform away from the intermediate extended symmetry point. The two phases are distinguished by the light thermally excited spinors. At large radii, $R_0 > R_c$, these transform under the S_8-Spinor of the $SO(8)$ symmetry group, while at small radii, $R_0 < R_c$, the light thermally excited spinors transform in terms of the conjugate C_8-Spinor. The extended symmetry point is purely stringy; it has no precise thermal interpretation but instead is T-duality invariant.

Operators associated with the extra massless states induce transitions between the purely momentum and winding modes. The operators O_+ and O_-, given in equation (2.29), raise and lower p_L by one unit but leave p_R unchanged. In particular since they transform in the vector representation of the symmetry group $SO(8)$, they induce transitions between the purely momentum S_8 and purely winding C_8 spinors, which become light in the two asymptotically cold regimes respectively:

$$< C_8|O_-|\, S_8 > \neq 0. \tag{2.34}$$

As a result the stringy phase transition at R_c can be resolved in the presence of genus-zero condensates of the additional massless thermal states, which can mediate transitions between purely winding and momentum states, *"gluing together" the asymptotic regimes* [31–33]. These condensates can be described in terms of non-trivial textures, which define embeddings of the spatial manifold into the field configuration space via non-zero spatial gradients of the fields associated to the extra massless states: $\nabla_\perp \varphi^I \neq 0$ [32]. As we will argue, in the Lorenztian description, where the temperature becomes time dependent, the condensates give rise to a spacelike brane (S-brane) configuration with negative pressure contributions localized at a time slice at which the temperature reaches its critical value.

Therefore thermal duality implies the existence of a maximal critical temperature $T \leq T_c$. The stringy system conceals its short distance behavior [22]. Defining the thermal modulus σ via $R_0/R_c = e^\sigma$, T-duality acts by reversing its sign: $\sigma \to -\sigma$. In terms of σ and the critical temperature T_c, the physical temperature can be written in a duality invariant way as follows:

$$T = T_c\, e^{-|\sigma|}, \quad T_c = \frac{1}{2\pi R_c} = \frac{1}{\sqrt{2}\pi}. \tag{2.35}$$

The expression is valid in both asymptotically cold regimes. Consequently the energy density and pressure are bounded $\rho \leq \rho_c$, $P \leq P_c$, never exceeding certain maximal values. This is a crucial difference from thermal field theory models.

In the Hybrid model, the thermal partition function can be written as

$$\frac{Z}{V_1} = (24\sqrt{2})\, e^{-|\sigma|} = \Lambda\, T, \quad \Lambda = \frac{24\sqrt{2}}{T_c}. \tag{2.36}$$

and so the pressure, energy density and maximal energy density are given by

$$P = \rho = \Lambda\, T^2, \quad \rho_c = \Lambda\, T_c^2 = \frac{24}{\pi}. \tag{2.37}$$

So in each thermal phase the equation of state is effectively that of thermal massless radiation in two dimensions thanks to right moving MSDS symmetry. In the higher dimensional cases, the exact right-moving MSDS structure gets replaced by *right moving asymptotic supersymmetry*, ensuring that well up to the critical point, the partition function is dominated by the contributions of the thermally excited massless states: $Z \sim T^{d-1}$ [32]. In order to maintain semi-quantitative control in the remaining part of this work, we will ignore stringy corrections to the one-loop string partition function in the higher d cases close to the critical point, and approximate the thermal system with that of massless thermal radiation up to the critical temperature. We will incorporate however the crucial localized pressure contributions induced by the additional massless thermal states at the critical point.

2.2 Cosmological evolutions

The backreaction on the initially flat metric and dilaton background will induce a cosmological evolution. We consider first the case where the underlying thermal modulus σ is a monotonic function of time scanning all three regimes of the string thermal system [31, 32]. Correspondingly the temperature grows from small values, reaching its maximal value T_c, and then drops again to zero. Each regime admits a local effective theory description, associated with a *distinct* α'-*expansion:*

- $R_c/R_0 \gg 1$, $(\sigma \ll 0)$; the regime of light thermal windings: $\{\mathscr{W}(\sigma < 0)\}$.
- $|R_0/R_c - R_c/R_0| \ll 1$, $(\sigma \sim 0)$; the intermediate $SU(2)$ extended symmetry point, where additional thermal states become massless: $\{\mathscr{B}(\sigma = 0)\}$.
- $R_0/R_c \gg 1$, $(\sigma \gg 0)$; the regime of light thermal momenta: $\{\mathscr{M}(\sigma > 0)\}$.

So a stringy transition occurs at T_c $(\sigma = 0)$, connecting two asymptotically cold phases. In each of these phases the source comprises a thermal gas of strings coupled to the dilaton/gravity system. Near T_c condensates associated with the extra massless thermal scalars can form and decay giving rise to an S-brane configuration [3] – see [59] for discussions concerning stringy S-branes in various contexts. Precisely at the critical point, the equations of motion of the thermal effective theory allow for non-trivial (genus-0) backgrounds, in which the gradients of the fields associated to the extra massless thermal states satisfy [32]

$$G_{IJ}\nabla_{\hat{\mu}}\varphi^I\nabla_{\hat{\nu}}\varphi^J = \frac{\kappa}{(d-1)}g_{\hat{\mu}\hat{\nu}}, \tag{2.38}$$

where G_{IJ} is the metric on the field configuration space; $g_{\hat{\mu}\hat{\nu}}$ is the metric on the spatial manifold and κ is a positive constant that sets the strength of the condensates. The resulting stress tensor is compatible with the symmetries of the spatial metric, which we require to be homogeneous and isotropic. Effectively, the condensates amount to *negative pressure contributions, proportional to* $-\kappa$*, which are well localized around the transition surface* $T = T_c$ *in the Lorentzian*.

At finite string coupling, we expect the thickness or duration of the S-brane in time to be very short, set by the string scale. So in the Lorentzian effective action we treat the S-brane as a δ-function source (thin-brane limit). (Later on we will explore the possibility of spreading the intermediate regime, with the temperature being constant at its critical value, for a long period in time.) Integrating out the extra massless thermal scalars via equation (2.38), we obtain the brane contribution to the effective action [31, 32]:

$$\begin{aligned} S_{\text{brane}} &= -\kappa\int d\sigma d^{d-1}x\sqrt{g_\perp}e^{-2\phi}\delta(\sigma) \rightarrow \\ &-\kappa\int d\tau d^{d-1}x\sqrt{g_\perp}e^{-2\phi}\delta(\tau-\tau_c), \end{aligned} \tag{2.39}$$

where τ_c is the time at which the temperature reaches its critical value, $\sigma(\tau_c) = 0$, and $g_\perp$ stands for the determinant of the induced metric on the constant time slice $\tau = \tau_c$. The tension of the brane is set by κ. The energy density of such a spacelike brane vanishes (consistently with reflecting boundary conditions on the first time derivative of the dilaton field [31]), but it gives negative pressure contributions in the spatial directions. Thus it provides the violation of the null energy condition (NEC) which can lead to a transition from a contracting phase to an expanding phase (when the cosmology is viewed in the Einstein frame). Henceforth we will refer to the intermediate regime as the "Brane Regime".

The first class of non-singular string cosmologies we discuss consists of transitions from a "Winding regime" to a "Momentum regime" via such a thin S-brane:

$$\mathscr{C}(\tau) \equiv \{\mathscr{W}(\tau<\tau_c)\}\oplus\{\mathscr{B}(\tau=\tau_c)\}\oplus\{\mathscr{M}(\tau>\tau_c)\}. \tag{2.40}$$

Without loss of generality we may choose $\tau_c = 0$. Since we take the string coupling to be weak and the size of the spatial manifold to be much larger than the string scale, our discussions above lead to the following effective action for the cosmological dynamics, which is valid in both the asymptotic regimes and also close to the critical point:

$$S = \int d^dx\, e^{-2\phi}\,\sqrt{-g}\left(\frac{1}{2}R + 2(\nabla\phi)^2\right) + \int d^dx\,\sqrt{-g}\,P - \kappa\int d^dx\sqrt{g_\perp}e^{-2\phi}\delta(\tau). \tag{2.41}$$

[3] It would be interesting to associate this configuration with formation and decay of long, tangled strings spread in space.

The first term is the genus-0 dilaton-gravity action written in string frame, while the second is the contribution of the thermal effective potential $-P$. In terms of the genus-1 thermal partition function, the energy density and pressure are given by

$$P = T\,\frac{Z}{V_{d-1}}, \qquad \rho = -P + T\,\frac{\partial P}{\partial T} \sim (d-1)P. \tag{2.42}$$

The equation of state $\rho = P$ is exact in the 2d Hybrid models.

We concentrate on spatially flat homogeneous and isotropic solutions

$$ds^2 = -N(\tau)^2 d\tau^2 + a(\tau)^2 dx_i dx^i, \tag{2.43}$$

where a is the string frame scale factor and N is the lapse function. Homogeneous and isotropic solutions with negative spatial curvature can be also constructed. Since in both thermal phases the Universe is asymptotically cold, this must be a bouncing cosmology. Irrespectively of the running dilaton, the thermal entropy (per co-moving cell of unit coordinate volume, physical volume a^{d-1})

$$S = \frac{a^{d-1}}{T}(\rho + P) \sim (aT)^{d-1} \tag{2.44}$$

is conserved, implying that in each of the two asymptotic thermal phases the scale factor and temperature satisfy:

$$aT = \text{constant}. \tag{2.45}$$

Therefore, during the initial “Winding Regime” where the temperature increases, the Universe is in a contracting phase, which reverses to expansion once the Universe enters into the “Momentum Regime” with decreasing temperature. Moreover, since the temperature and all thermodynamical quantities are bounded from above by critical values, which they attain at the brane, the scale factor must be bounded from below by its value at the brane: $a \geq a_c$. This critical value of the scale factor is given in terms of the entropy and the maximal critical temperature:

$$a_c = \left(\frac{S T_c}{\rho_c + P_c}\right)^{1/d-1} \sim \frac{S^{1/d-1}}{T_c}. \tag{2.46}$$

In particular a_c can be kept large in string units if the entropy S is large. The bounce is facilitated by the extra negative pressure provided by the S-brane at the transition surface, inducing also a bounce on the dilaton. The singular regime $a \to 0$ of classical general relativity is absent.

Notice that continuity of the scale factor across the brane, ensures the continuity of the thermal entropy across the transition surface, as in second order phase transitions. The reversal of contraction to expansion via the S-brane is what makes the transition between the two asymptotically cold phases possible, avoiding a “heat death” and maintaining adiabaticity throughout and across the brane.

From the action (2.41), we can obtain the equations of motion [32]. Varying with the lapse function N, we get the following first order equation

$$\frac{1}{2}(d-1)(d-2)H^2 = 2(d-1)H\dot{\phi} - 2\dot{\phi}^2 + e^{2\phi}N^2\rho, \tag{2.47}$$

consistently with the vanishing of the brane contribution to the energy density. For the scale factor a, we get

$$(d-2)\left(\frac{\ddot{a}}{a}-H\frac{\dot{N}}{N}\right)+\frac{1}{2}(d-2)(d-3)H^2$$
$$=2\ddot{\phi}+2(d-2)H\dot{\phi}-2\dot{\phi}^2-2\frac{\dot{N}}{N}\dot{\phi}-e^{2\phi}N^2P+\kappa N\delta(\tau), \tag{2.48}$$

including the localized negative pressure contributions from the S-brane. Finally the equation of motion of the dilaton is

$$\ddot{\phi}+(d-1)H\dot{\phi}-\dot{\phi}^2-\frac{\dot{N}}{N}\dot{\phi}-\frac{d-1}{2}\left(\frac{\ddot{a}}{a}-H\frac{\dot{N}}{N}\right)-\frac{1}{4}(d-1)(d-2)H^2=-\frac{1}{2}\kappa N\delta(\tau). \tag{2.49}$$

Irrespectively of spacetime dimension, the structure of these equations is such that the dilaton experiences an impulsive force at the brane, $2\ddot{\phi}=-\kappa N_c\delta(\tau)+\ldots$, while $\ddot{a}$ is smooth. So the brane induces a discontinuity in the first time derivative of the dilaton field

$$2(\dot{\phi}_+-\dot{\phi}_-)=-N_c\kappa, \tag{2.50}$$

while the first time derivative of the scale factor $\dot{a}$ remains continuous. Criticality of the temperature at the brane – this is a crucial string theory input – leads to the vanishing of the first time derivative of the scale factor: $\dot{a}=0$. Continuity of the dilaton field and the metric across the brane, and the first order equation (2.47) then imply that $\dot{\phi}_+=-\dot{\phi}_-$. In other words, the dilaton undergoes an *elastic* bounce across the brane:

$$\dot{\phi}_+=-\dot{\phi}_-=-N_c\kappa/4. \tag{2.51}$$

Since the brane tension κ is positive, the dilaton must be initially increasing. It crosses the brane and then decreases. As a result, the dilaton is bounded from above by its value at the brane: $\phi\leq\phi_c$. The critical value of the dilaton and the maximal energy density set the slope of the dilaton just before the transition, the brane tension κ, and hence the strength of the condensates associated to the extra massless thermal scalars, via equation (2.47):

$$\kappa=2e^{\phi_c}\sqrt{2\rho_c}. \tag{2.52}$$

These boundary conditions are in accordance with entropy conservation throughout and across the brane:

$$\dot{\rho}+(d-1)H(\rho+P)=0. \tag{2.53}$$

Assuming massless thermal radiation up to the critical point, from both sides of the transition, the pressure is given by $P=n^*\Sigma_d T^d$ and the conserved thermal entropy by $S=d(aT)^{d-1}n^*\Sigma_d=$ constant. Then we obtain the following expressions

$$a_cT_c=aT=\left(\frac{S}{dn^*\Sigma_d}\right)^{1/(d-1)}, \quad \kappa=2\sqrt{2(d-1)}\,\sqrt{n^*\Sigma_d}\;\;T_c^{d/2}\,e^{\phi_c} \tag{2.54}$$

for the critical value of the scale factor and the relation between the brane tension and the critical value of the dilaton.

With massless thermal radiation up to the critical point, we can obtain exact cosmological solutions to the equations of motion. In the conformal gauge, $N = a$, the string frame scale factor and dilaton are given by [32]

$$\ln\left(\frac{a}{a_c}\right) = \frac{1}{d-2}\left[\eta_+ \ln\left(1+\frac{\omega a_c|\tau|}{\eta_+}\right) - \eta_- \ln\left(1+\frac{\omega a_c|\tau|}{\eta_-}\right)\right],$$

$$\phi = \phi_c + \frac{\sqrt{d-1}}{2}\left[\ln\left(1+\frac{\omega a_c|\tau|}{\eta_+}\right) - \ln\left(1+\frac{\omega a_c|\tau|}{\eta_-}\right)\right], \tag{2.55}$$

where

$$4\omega = \kappa\frac{d-2}{\sqrt{d-1}}, \qquad \eta_\pm = \sqrt{d-1} \pm 1. \tag{2.56}$$

The solutions are invariant under time reversal, $\tau \to -\tau$, in accordance with the gluing conditions across the brane discussed above. In the neighborhood of the brane, $|\kappa a_c \tau| \ll 1$, the metric is regular while the dilaton exhibits a conical structure:

$$\ln\left(\frac{a}{a_c}\right) = \frac{1}{16(d-1)}(\kappa a_c \tau)^2 + \mathscr{O}(|\kappa a_c \tau|^3)$$

$$\phi = \phi_c - \frac{|\kappa a_c \tau|}{4} + \mathscr{O}((\kappa a_c \tau)^2). \tag{2.57}$$

In the far past and future, $|\kappa a_c \tau| \gg 1$, the dilaton asymptotes to a constant, the temperature drops and the scale factor tends to infinity. Asymptotically we get

$$a \sim |\tau|^{2/d-2}, \tag{2.58}$$

recovering the characteristic relation between the scale factor and conformal time in a radiation dominated Universe.

The string coupling is bounded by its critical value at the brane: $g_s \leq g_c = e^{\phi_c}$. Hence our perturbative approach is valid when this critical coupling, g_c, is sufficiently small. The Ricci scalar curvature also attains its maximal value at the brane, which is set by the brane tension:

$$\mathscr{R}_c = \kappa^2/4 = \mathscr{O}(g_c^2). \tag{2.59}$$

In particular the curvature is finite throughout the cosmological evolution; there is *no essential singularity*. Since the critical coupling sets the value of the brane tension in string units, we conclude that *both g_s and α' corrections remain under control*, provided that g_c is small enough.

We may transform to the Einstein frame via

$$(N_E, a_E, 1/T_E) = e^{-\frac{2\phi}{d-2}}\,(N, a, 1/T). \tag{2.60}$$

Thus the geometrical quantities in the Einstein frame inherit the conical structure of the dilaton field. The discontinuities of the Hubble parameter in the Einstein frame and the first time derivative of the dilaton (across the transition surface at T_c) are resolved by the brane pressure, in accordance with the Israel junction conditions [60], which require that the induced metric on the transition surface be continuous and the extrinsic curvature to jump by a factor determined by the localized pressure [61].

Before we conclude this section, we make a comparison between the string cosmologies we obtained, which incorporate the transition across the stringy S-brane, with the corresponding solutions in the classical dilaton-gravity theory, coupled to massless thermal radiation, where such a phase transition is not possible. The latter are given by the solutions in equation (2.55) *without the absolute value on the time variable,* and coincide with the stringy solutions in the "Momentum Regime". Removing the absolute value on the time variable, we see that the dilaton becomes a monotonic function of time, and grows without a bound in the past. So the dilaton-gravity system is strongly coupled in the past, and perturbation theory breaks down. The string frame scale factor still undergoes a bounce at $\tau = 0$, but as we evolve backwards in time, its first time derivative develops a singularity a finite time in the past, leading to a curvature singularity. In the Einstein frame, we recover the singular regime $a_E \to 0$ of classical general relativity.

The situation is drastically different in the stringy cases. Starting in the contracting phase, the Einstein frame temperature increases monotonically with time. The analysis of [17] reveals that additional thermal states become massless at a critical temperature determined by the string coupling, "protecting" the system from entering the regime $a_E \to 0$ of classical general relativity. If at this critical point the coupling is weak, then the additional thermal states are the perturbative winding modes discussed above, and the analysis of this section applies on how to follow the transition via the stringy S-branes. Large enough entropy is needed in order to keep the string frame scale factor parametrically larger than the string scale, in accordance with equation (2.54). If on the other hand the coupling at the critical point is large, then the extra massless thermal states are non-perturbative in nature, but still these can source the required localized pressure needed for a reversal to expansion. Non-perturbative string dualities can be applied in order to map to a weakly coupled description. It would be interesting to apply such dualities so as to complete the cosmological history in these cases as well.

2.2.1 Spreading the "Brane Regime"

We observe that at the brane, the first time derivative of the temperature vanishes, $\dot{T} = 0$, with the temperature reaching its maximal, critical value T_c. We explore in this section the possibility of "spreading" the "Brane regime" for an arbitrarily long period of time in the past [33]. During such a long "Brane regime," the string frame temperature remains constant, frozen at its critical value. It follows via the entropy conservation law, equations (2.44) and (2.53), that the string frame scale factor must also be constant: $a = a_c$. Modulo the running dilaton, such a model shares features with the "emergent cosmological scenario" [62] where the Universe is in a long quasi-static phase, with $H = 0$, before it exits into an expanding phase.

At the critical point, various condensates associated with the extra massless fields may give rise to a dilaton effective potential $V(\phi)$. In order for the temperature and string frame scale factor to remain constant during the backreacted cosmological evolution, this potential must take the following form [33]

$$V(\phi) = B + Ce^{-2\phi}, \quad B = P_c; \tag{2.61}$$

that is, the constant part has to be equal to the value of the thermal pressure at the critical temperature, while the coefficient of the exponential term can be arbitrary. It turns out that the parameters B

and C can be obtained in terms of fluxes in the underlying effective gauged supergravity description of the thermal system at the extended symmetry point ($\sigma = 0$).

Recall that at the critical point, the $U(1)_L$ gauge symmetry associated to the Euclidean time circle gets enhanced to an $SU(2)_L$ gauge symmetry. Also, there is at least one spatial circle with radius fixed at the fermionic point, which couples to the right-moving fermion number F_R and contributes to the spontaneous breaking of the right-moving supersymmetries. The $U(1)_R$ gauge symmetry associated to this circle gets enhanced to an $SU(2)_R$ gauge symmetry. So at the fermionic extended symmetry point, symmetries reorganize in such a way so that an alternative description is possible, where the diagonal S^1 cycle blows up to an $SU(2)$-manifold with string-scale volume and flux. As a result, at the critical point, the target space can be described in terms of a $d+2$-dimensional space.

Now consider the underlying $d+2$-dimensional gauged supergravity. Once fluxes and gradients along the 2 compact directions are turned on, the kinetic terms of the internal massless scalars, graviphotons and matter gauge bosons give rise respectively to the following dilaton effective potential [33, 63]:

$$Ae^{-2\phi} + B(\mathscr{U}) + \tilde{C}e^{-2\phi}, \tag{2.62}$$

where $\mathscr{U}$ stands for the volume of the $10-d$ internal, toroidal manifold. This form for the effective potential persists when the theory is dimensionally reduced to d-dimensions, since the extra compact $SU(2)_{k=2}/U(1)$ manifold has fixed volume at the string scale. So to support a long "Brane Regime" requires to turn on non-trivial fluxes and gradients so that

- $B(\mathscr{U}) = P_c$
- $A + \tilde{C} = C$, which can be arbitrary.

Thus during a "Brane Regime" the only non-trivial evolution is that of the dilaton, which depends on the flux parameter C of the effective potential. The only equation that remains to be satisfied is the Friedmann equation, which, for constant scale factor and dilaton potential given by equation (2.61), takes the form:

$$2\dot{\phi}^2 = a_c^2\left[(\rho_c + P_c)e^{2\phi} + C\right]. \tag{2.63}$$

The solutions corresponding to $C = 0$ and $C > 0$ are given as follows [33]

$$C = 0: \qquad e^{-\phi} = a_c\sqrt{\frac{\rho_c + P_c}{2}}(-\tau), \qquad \forall\, \tau \leq \tau_+ < 0,$$

$$C > 0: \quad e^{-\phi} = \sqrt{\frac{\rho_c + P_c}{C}}\sinh\left[a_c\sqrt{\frac{C}{2}}(-\tau)\right], \quad \forall\, \tau \leq \tau_+ < 0. \tag{2.64}$$

- In both cases the "Brane regime" starts at $\tau = -\infty$, with super-weak string coupling.
- At time $\tau_+ < 0$, this regime can exit via a thin S-brane to the "Momentum, radiation regime," described in the previous section. The reason for choosing the upper end of the "Brane Regime" to be negative is to maintain the validity of perturbation theory throughout the cosmological evolution. Notice that throughout the "Brane Regime" the dilaton is monotonically

increasing. When it crosses the thin S-brane, it undergoes a bounce and starts to decrease. The junction conditions on the first time derivative of the dilaton just before and after the transition surface can thus be met, as required by a positive tension brane.

- The choice of τ_+ determines the critical string coupling at the transition towards the "Momentum dominated phase": $\phi_+ = \phi(\tau = \tau_+)$.

In summary, this cosmology can be represented pictorially as follows

$$\mathscr{C}(\tau) \equiv \{\mathscr{B}(\tau \leq \tau_c)\} \oplus \{\mathscr{M}(\tau > \tau_c)\}. \tag{2.65}$$

Initially the Universe has constant σ-model temperature and scale factor, $T = T_c$ and $a = a_c$. The string coupling grows from very weak values in the very early past reaching a maximal value g_c at τ_c, at which point the Universe exits into the radiation dominated "Momentum regime". Both g_s and α' corrections are under control provided that g_c is small enough.

Finally let us note that the case $C < 0$ allows us to obtain a "Brane regime" of finite time duration [33]:

$$C < 0: \; e^{-\phi} = \sqrt{\frac{\rho_c + P_c}{|C|}} \sin\left[a_c \sqrt{\frac{|C|}{2}}(-\tau) \right] \quad \tau_- \leq \tau \leq \tau_+ < 0, \tag{2.66}$$

opening up the possibility of constructing string cosmological solutions, where an initial "Winding Regime" undergoes a transition to a "Momentum Regime" via a thick S-brane. One possibility of realizing this case is by introducing positive spatial curvature [33], and so this would be an example of a closed stringy cosmology, which avoids both the Big-Bang and Big-Crunch singularities of classical general relativity. Of course we would still need to investigate the stability of such a Universe against collapse in the presence of extra matter and fluctuations.

3. Parafermionic cosmologies

In this section we consider a class of exact cosmological solutions to classical string theory, which are described at the worldsheet level by superconformal field theories (SCFT) of the form [38, 39]

$$SL(2,R)_{-|k|}/U(1) \; \times \; K. \tag{3.1}$$

The first factor corresponds to a gauged Wess-Zumino-Witten model based on the $SL(2,R)$ group manifold, with the level k taken to be negative. The second factor stands for a suitable internal, compact CFT. In global coordinates, the sigma-model metric and dilaton field are given by

$$(\alpha')^{-1} ds^2 = (|k|+2)\frac{-dT^2 + dX^2}{1 + T^2 - X^2}$$

$$e^{2\Phi} = \frac{e^{2\Phi_0}}{1 + T^2 - X^2}. \tag{3.2}$$

In the superstring case, this sigma-model metric is exact to all orders in the α' expansion [64]. Additional work on these types of models includes [65].

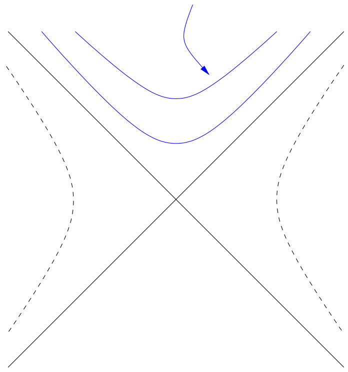

Figure 1: The $SL(2,R)/U(1)$ cosmology.

The geometry consists of a singularity-free light-cone region, and there are time-like curvature singularities in the regions outside the light-cone horizons, where the dilaton field is also singular. See figure 1. The singularities occur at the hyperbolas

$$X = \pm\sqrt{1+T^2}. \tag{3.3}$$

Despite the fact that the singularities follow accelerated trajectories, their proper distance (as measured by the length of the slices T =constant) remains finite in string units:

$$L = \sqrt{(|k|+2)\alpha'}\int_{-\sqrt{1+T^2}}^{\sqrt{1+T^2}} \frac{dX}{\sqrt{1+T^2-X^2}} = \pi\sqrt{(|k|+2)\alpha'}. \tag{3.4}$$

It can be kept parametrically larger than the string scale for large level $|k|$. So in a sense we can think of the cosmology as being spatially closed [39]. Observe also that if we perform a double analytic continuation on the coordinates, we obtain Witten's 2-dimensional black hole solution [66]. This double analytic continuation is equivalent to changing the sign of the level k and rotating the Penrose diagram of the cosmology by 90 degrees.

At the singularities the sigma-model geometrical description breaks down, but as we will see, the underlying worldsheet CFT gives a prescription to describe them. The cosmological region of interest is the future part of the lightcone region. Parametrizing this region with a new set of coordinates (t,x), defined via

$$T = t\cosh x, \quad X = t\sinh x, \tag{3.5}$$

the metric and dilaton are given by

$$(\alpha')^{-1}ds^2 = (|k|+2)\frac{-dt^2+t^2dx^2}{1+t^2}, \quad e^{2\Phi} = \frac{e^{2\Phi_0}}{1+t^2}. \tag{3.6}$$

Therefore we obtain an expanding, asymptotically flat geometry, with the string coupling vanishing at late times. Asymptotically we get a timelike linear dilaton background. The surfaces of constant time t are shown in figure 1.

The cosmological observer never encounters the singularities, as these lie behind the visible horizons at $T = \pm X$. However, signals from the singularities can propagate into the lightcone region, through the surface $t = 0$, and influence its future evolution. In this sense, the cosmology is similar to a Big-Bang cosmology.

The central charge of the superconformal $SL(2,R)/U(1)$ model at negative level k is given by

$$c = 3 - \frac{6}{|k|+2}, \quad \hat{c} = 2 - \frac{4}{|k|+2}. \tag{3.7}$$

In superstring theory we must tensor it with other conformal field theories so as to satisfy the condition $\hat{c}_{\text{tot}} = 10$ for worldsheet gravitational anomalies to cancel. To obtain an effectively four dimensional cosmological model, we add two large (however compact) supercoordinates (y, z), with radii $R_{y,z} = R \gg l_s$, together with a compact SCFT system of central charge

$$\delta\hat{c} = 6 + 4/(|k|+2). \tag{3.8}$$

Since the cosmological region of interest is non-compact (and asymptotically flat), the model admits the desired four dimensional interpretation [39]. *This is so irrespectively of how small the level $|k|$ is.* As an example consider the case $|k| = 2$. Then we may set the internal CFT factor to be

$$K \equiv T^2_{y,z} \times T^7, \tag{3.9}$$

where the volume of the 7-dimensional toroidal manifold is chosen to be at the string scale. Since the central charge of the $SL(2,R)/U(1)$ factor is smaller than the central charge corresponding to two flat macroscopic directions, the models are in fact super-critical.

The metric in the Einstein frame is given by

$$ds^2_E = (|k|+2)\alpha'(-dt^2 + t^2dx^2) + R^2\ (1+t^2)(dy^2 + dz^2), \tag{3.10}$$

thus describing an anisotropic cosmology. At late times however, and for large R, it asymptotes to an isotropic flat Freedmann model with scale factor growing linearly with time: $a \sim t$. The effective energy density and pressure supporting this kind of cosmological evolution must satisfy the following equation of state:

$$\rho_{\text{eff}} = -3P_{\text{eff}}. \tag{3.11}$$

Therefore the $SL(2,R)/U(1)$ cosmology is at the cross-point between accelerating and decelerating Universes.

Upon rotation to Euclidean signature, we obtain a disk parametrized by a radial coordinate $\rho \leq 1$ and an angular variable $\phi \in [0, 2\pi)$. The Euclidean metric and dilaton are given by

$$(\alpha')^{-1}ds^2 = (|k|+2)\frac{d\rho^2 + \rho^2 d\phi^2}{1-\rho^2}, \qquad e^{2\Phi} = \frac{e^{2\Phi_0}}{1-\rho^2}, \tag{3.12}$$

with the singularity occurring at the boundary circle $\rho = 1$. The characteristic feature of this geometry is that the radial distance of the center to the boundary of the disk is finite, but the circumference of the boundary circle at $\rho = 1$ is infinite. Geometrically the space looks like a bell.

The Euclidean background admits a superconformal field theory description based on the $SU(2)/U(1)$ coset model at level $|k|$. See e.g. [67] for a review. The interesting feature is that the Euclidean CFT is *compact* and unitary. In fact since it corresponds to an $N = 2$ minimal model, the level $|k|$ must be quantized. So in order for the Euclidean theory to be well defined, we set $|k|$ to be a positive integer.

This worldsheet CFT is perfectly well behaved around $\rho = 1$, where the geometrical sigma-model description breaks down. Near this region the gauged WZW action is given by

$$S_{WZW} = -\frac{|k|+2}{2\pi} \int d^2 z \phi F_{z\bar{z}} + \dots, \tag{3.13}$$

with the leading term in the expansion corresponding to a simple topological theory. From the form of the action near $\rho = 1$, we also learn that worldsheet instantons for which, $\int F_{z\bar{z}} = 2\pi i n$, break the $U(1)$ symmetry shifting the angle ϕ to a discrete symmetry $Z_{|k|+2}$, in accordance with the algebraic description of the worldsheet system in terms of $Z_{|k|+2}$ parafermionic currents [68]. It is clear that in the presence of such instantons, the Euclidean path integral is invariant only under discrete shifts of the angle ϕ: $\delta\phi = 2\pi m/(|k|+2)$.

We argue now that the non-singular description of the theory is an almost-geometrical one [39], in terms of a compact T-fold [70]. In order to construct this, we first perform a T-duality transformation along the angular direction ϕ. The resulting sigma model is based on the following metric and dilaton

$$(\alpha')^{-1} ds'^2 = \frac{(|k|+2)}{1-\rho'^2} \left(d\rho'^2 + \frac{\rho'^2}{(|k|+2)^2} d\phi'^2 \right)$$
$$e^{2\Phi'} = \frac{e^{2\Phi_0}}{(|k|+2)(1-\rho'^2)}, \quad \rho' = (1-\rho^2)^{\frac{1}{2}}. \tag{3.14}$$

The transformation on the radial coordinate ρ exchanges the boundary of the disk and its center. The T-dual description is *weakly coupled* near $\rho = 1$ or $\rho' = 0$, where the original metric and dilaton field were singular. The only singularity there is a benign orbifold singularity. In fact the T-dual description is equivalent to the $Z_{|k|+2}$ orbifold of the original model [67, 69]. Despite the presence of a conical singularity in the geometry, the string theory amplitudes are finite and well behaved.

The T-fold can now be constructed as follows [39]. We take the original disk and cut-off the region past a non-singular circle, e.g. the region past the circle $\rho = \rho' = 1/\sqrt{2}$, $\rho > 1/\sqrt{2}$, containing the apparent singularity at $\rho = 1$. The cut-off region is replaced with the interior of the T-dual geometry, $\rho' < 1/\sqrt{2}$, with a well behaved geometrical description (in terms of the T-dual variables). The two patches are glued together along this non-singular circle via a T-duality transformation. In particular the Euclidean T-fold is compact, and it does not have any boundaries or singularities. We emphasize that the gluing of the T-dual patches is *non-geometrical*, as it involves a T-duality transformation on the metric and other fields.

For the cosmology as well, we can obtain a regular T-fold description as the target space of the CFT. Here we observe that T-duality interchanges the light-cone and the singularities [38]. We must glue the T-duals along a hyperbola in between the lightcone and the singularities. The gluings are shown in figure 2. The resulting almost-geometrical description is very similar to 2-dimensional de-Sitter space, which we can think of as a hyperboloid embedded in three-dimensional flat space.

We may think of the Euclidean T-fold (or the corresponding compact CFT) as describing a string field theory instanton, similar in fact to the case of the round sphere which is an instanton solution to Einstein's classical theory of general relativity in the presence of a positive cosmological

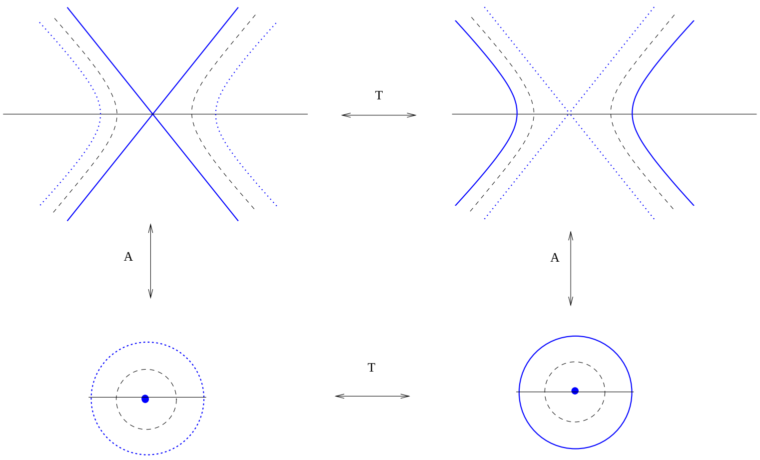

Figure 2: The cosmological T-fold. As we cross the hyperbola (striped black line) in the original patch moving towards the apparent singularity (dotted blue line), we pass to the T-dual patch, and continue motion towards the non-singular lightcone region.

constant. In the latter case, the radius of curvature of the sphere is set by the cosmological constant, while in the string theoretic case the effective radius of curvature is set by the quantized level k.

The T-fold construction allows us to *formally* define a Hartle-Hawking wavefunction [71] for the cosmology, which we can think of as a vector in the Hilbert space of the 2nd quantized string field theory. To this extend, we need to perform a "half T-fold" Euclidean path integral over *fluctuations of all* target space fields with specified values on the boundary [39]:

$$\Psi[h_\partial, \phi_\partial, \dots] = \int [dg][d\phi]\dots e^{-S(g,\phi,\dots)}. \tag{3.15}$$

More explicitly, we cut the cosmological T-fold in two along a slice of time reversal symmetry, the $T = 0$ slice in each patch, and replace the past geometry with its Euclidean counterpart. Values for the target space fields must be specified on this $T = 0$ slice. See figure 3. No other condition needs to be specified since the full Euclidean T-fold has no boundaries or singularities, thus generalizing the Hartle-Hawking no boundary proposal to string field theory.

The wavefunction is hard to compute, but we can understand some of its global properties by computing its norm. The norm of the wavefunction is given in terms of the full Euclidean path integral, over fluctuations around the on-shell closed string background. The full path integral can be computed in a 1st quantized formalism by summing over all closed worldsheet topologies, including a sum over disconnected diagrams. In fact it is equal to the exponential of the *total, connected* string partition function Z_{string},

$$||\Psi||^2 = e^{Z_{\text{string}}}, \tag{3.16}$$

calculable perturbatively as a sum over all (connected) closed Riemann surfaces of genus $0, 1, 2, \dots$, if the string coupling $e^{2\Phi_0}$ is small enough. Two crucial conditions for the norm to be finite are that the underlying SCFT be compact and to lead to a *tachyon free* Euclidean model [39]. In the superstring case the latter condition can be satisfied by imposing a suitable GSO projection [72].

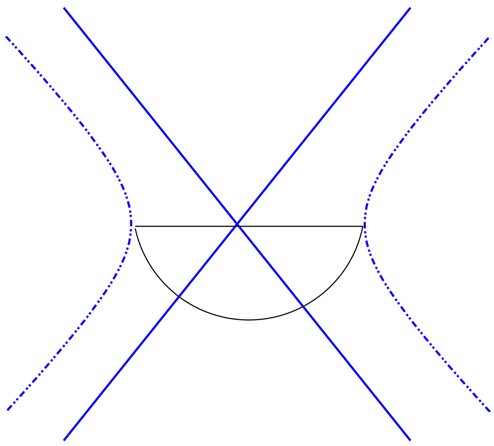

Figure 3: The gluing of the half-disk to the cosmology.

As in the case of de Sitter space, the norm of the wavefunction can be interpreted as a thermal ensemble. In the de Sitter case, the norm is given in terms of the cosmological constant Λ by

$$||\Psi_{\rm dS}||^2 = e^{3/(8G^2\Lambda)}, \tag{3.17}$$

with $3/8G^2\Lambda$ corresponding to the finite de Sitter entropy [71]. Likewise in the case of the stringy cosmology at hand, $Z_{\rm string}$ corresponds to a thermal string amplitude [39]. But unlike the de Sitter case, the genus-zero contribution to $Z_{\rm string}$ vanishes. This is because when the underlying CFT is compact, the spherical CFT partition function is finite and we must divide it with the infinite volume of the conformal Killing group in order to get the string theory result. Also since the on-shell background satisfies the sigma-model β-function equations, its tree-level action vanishes. As a result in perturbative string theory, the leading contribution arises at the genus-1 level.

The effective physical temperature of the $SL(2,R)/U(1)$ cosmological model is set by the inverse of a radius, which depends on the level k as follows [39, 73]:

$$T = 1/2\pi R, \qquad R = \sqrt{(|k|+2)\alpha'}. \tag{3.18}$$

So it is below the Hagedorn temperature, $R > R_H = \sqrt{2\alpha'}$, for all $|k| > 0$. As a result, the corresponding genus-1 thermal string amplitude is finite and Ψ is normalizable. For example, when $|k| = 2$, the parafermionic factor of the cosmology has central charge $\hat{c} = 1$, and it is equivalent to a fermion together with a compact boson at radius $2\sqrt{\alpha'} > R_H$. One way to see that this is the correct value for the radius is that for this radius, the T-dual is equivalent to the Z_4 orbifold of the original model. Explicit computations of the one-loop thermal string partition function in this case can be found in [39]. Observe that the effective temperature becomes equal to Hagedorn, $R = R_H$, at $|k| = 0$, precisely when the cosmology disappears from the target space.

Hence the norm of the wavefunction $||\Psi||^2$ is finite, and in fact it is a function of the moduli associated to the internal CFT K [39]. It would be very interesting to use it in order to define relative probabilities for different string compactifications, related to these parafermionic cosmologies, as well as precise stringy observables associated to the cosmology. To this end, notice that in the asymptotically flat region of the cosmology, we can define scattering states, and so an "S-vector" can be in turn defined, in terms of the overlaps of these scattering states with the Hartle-Hawking state associated to the cosmology.

4. Conclusions

In this lecture we have uncovered cosmological implications of certain *stringy gluing mechanisms*, realizable in a large class of thermal type II $\mathcal{N} = (4,0)$ models, which connect distinct string effective theories. The mechanisms are triggered by additional thermal states, which become massless at a critical, maximal temperature T_c. The region around the critical temperature admits a "brane interpretation," with the brane tension sourced by non-trivial spatial gradients associated to the extra massless thermal scalars. In the Einstein frame, the cosmological solutions describe bouncing Universes, connecting in some of the examples two asymptotically cold thermal phases. Unlike many versions of pre Big Bang models in the existing literature, these cosmological solutions remain perturbative throughout the evolution, provided that the critical value of the string coupling at the brane is sufficiently small. Indeed this class of bouncing cosmologies provides the first examples, where *both the Hagedorn instabilities as well as the classical Big Bang singularity are successfully resolved*, remaining in a perturbative regime throughout the evolution.

Eternal, bouncing cosmologies open new perspectives, to address the cosmological puzzles of standard hot Big Bang cosmology. Most of these problems are based on the assumption that the Universe starts out very small and hot, with Planckian size and temperature. In our set up, however, the minimal size of the Universe can be parametrically larger than the string or Planck scale. The horizon problem in particular is essentially nullified. Causal contact over large scales is assured given the fact that the Universe was in a contracting phase for an arbitrarily long period of time. The large entropy problem does not arise either. If the Universe begins cold and large (larger than the present-day Universe), it will by dimensional analysis be likely to contain a sufficient amount of entropy. The stringy cosmologies we obtained open up the possibility to study the homogeneity problem within a concrete set-up. Indeed the study of the growth and propagation of cosmological perturbations from the contracting to the expanding phase via the stringy S-brane is currently underway [61], opening up the possibility of realizing the "Matter Bounce" scenario in string theory that produces a scale invariant spectrum of primordial cosmological fluctuations [74]. If successful these stringy bouncing cosmologies can form a basis, alternative to inflation, of realizing phenomenologically viable models with complete, singularity free cosmological histories.

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