PROCEEDINGS OF SCIENCE

A geometric constraint on supergravity inflation

Andrea Borghese*
Centre for Theoretical Physics,
University of Groningen,
Nijenborgh 4, 9747 AG Groningen, The Netherlands.
E-mail: a.borghese@rug.nl

Diederik Roest
Centre for Theoretical Physics,
University of Groningen,
Nijenborgh 4, 9747 AG Groningen, The Netherlands.
E-mail: d.roest@rug.nl

Ivonne Zavala
Centre for Theoretical Physics,
University of Groningen,
Nijenborgh 4, 9747 AG Groningen, The Netherlands.
E-mail: e.i.zavala@rug.nl

We discuss a bound on the possibility to realise inflation in any $\mathcal{N} = 1$ supergravity coupled to chiral multiplets. The bound is derived from the study of particular directions on the scalar manifold called sGoldstino's. These scalar fields are singled out by spontaneous supersymmetry breaking. We point out some implications of this bound and in particular to what extent the requirement of effective single-field and slow-roll can both be realised in the models of interest. Finally, we discuss the possible extension of the bound to other four dimensional supergravity theories.

Proceedings of the Corfu Summer Institute 2012 "School and Workshops on Elementary Particle Physics and Gravity"
September 8-27, 2012
Corfu, Greece

*Speaker.

PoS(Corfu2012)110

1. Introduction

More than thirty years after its discovery, inflation remains our best theoretical candidate in order to describe the early universe [1, 2]. It naturally explains the high degree of homogeneity at large scales in the present Universe, thus solving classical problems associated with e.g. the horizon of CMB beyond patches of 1 degree and the nearly flat spatial geometry. In addition to this homogeneity, it also provides a compelling explanation for the inhomogeneities in both the CMB and the LSS.

Focussing on CMB observations [3], data show fluctuations in the temperature power spectrum which are almost scale invariant and to a large extent Gaussian. In particular, the spectral index is measured to be

$$n_s = 0.968 \pm 0.012\,, \tag{1.1}$$

while the $f_{\rm NL}^{\rm local}, f_{\rm NL}^{\rm equil}, f_{\rm NL}^{\rm ortho}$ parameters, related to non-gaussianity at the bispectrum level are measured to be

$$\begin{aligned} f_{\rm NL}^{\rm local} &= 32 \pm 21\,, \\ f_{\rm NL}^{\rm equil} &= 26 \pm 140\,, \\ f_{\rm NL}^{\rm ortho} &= -202 \pm 104\,. \end{aligned} \tag{1.2}$$

All observations are, so far, perfectly consistent with the simplest class of inflationary models, namely the single-field, slow-roll ones. For this class of models many cosmological data are related to the so called slow-roll parameters[1]

$$\varepsilon \equiv \frac{G^{IJ}\, D_I V\, D_J V}{2\,\ell_p^2\, V}\,, \qquad \eta \equiv \text{min. eigenvalue}\,\Big(\frac{G^{IK}\, D_K D_J V}{\ell_p^2\, V}\Big)\,. \tag{1.3}$$

In particular, the spectral index (1.1) is given by

$$n_s = 1 - 6\,\varepsilon + 2\,\eta\,, \tag{1.4}$$

and one finds $f_{\rm NL} = \mathscr{O}(\varepsilon)$, in perfect agreement with observational constraints.

In view of the phenomenological success of the inflationary paradigm, it is natural to look for a possible embedding into a more fundamental theory of quantum gravity, such as string theory. Indeed in recent years a large research effort has been devoted to finding realisations of inflation in string theory. Despite a number of interesting and influential examples, this search is somewhat hampered by our limited knowledge of the contours of the playground: it remains unclear to this day which string theory compactifications are admissible, and what their resulting features are.

One example is D-brane inflation (see e.g. [4] for a review with several references), which has seen a lot of progress in the last years in the context of type IIB flux compactifications, where moduli stabilisation is under good theoretical control. Much effort has been put in computing the scalar potential of the D3-brane position, which is responsible to drive inflation. Another approach

[1] Here we give the multi-field generalisation of the slow-roll parameters in order to make contact with the supergravity models. G_{IJ} is the field dependent metric on the manifold described by the scalar fields. Furthermore we use the definition $\ell_p^2 = 1/M_p^2 = 8\pi G_N$.

within type IIB flux compactifications has been to study modular inflation in the Large Volume scenario [5]. Here the scalar potential responsible to drive inflation can be explicitely computed and inflation realised [6].

Another example is provided by the analysis of the inflationary properties of flux compactifications of type IIA string theory. Restricting oneself to Calabi-Yau compactifications with only standard NS-NS 3-form flux, R-R fluxes, D6-branes and O6-planes at large volume and small string coupling, one can stabilise the moduli at the classical level [7]. However, such constructions always satisfy a very simple and nevertheless strong lower bound on the first slow-roll parameter [8], $\varepsilon \geq 27/13$, violating the slow-roll assumption. Surprisingly, in order to derive this lower bound, only two of the total set of moduli fields had to be taken into account: one finds violation of the slow-roll condition already in the projection onto the two-dimensional plane spanned by the dilaton and the volume modulus. Based solely on the dynamics of these two fields, it has been argued that cosmological observations have ruled out geometric IIA compactifications. A possible way to circumvent this no-go theorem would be to replace the six-torus by negatively curved internal manifolds.

In this contribution we want to extend the above analyses in a different direction, namely that of four dimensional supergravity models. These belong to a very general class of models in which it is natural to embed inflation. Moreover, due to the presence of many scalar fields, there is still room for all the subtleties of multi-field inflation, curvatons, isocurvature perturbations and non-Gaussianities, to name a few. Finally these models are very interesting in wiev of the relationship with string theory. Indeed some supergravity theories can be obtained as a low energy limit of string theory. Thus we could look at the attempt of embedding inflation in supergravity as a first step towards the search of a UV complete theory of inflation.

We start with the simplest supergravity model, namely $\mathcal{N} = 1$ coupled to n chiral multiplets. Instead of taking this problem head-on, or statistically sample a large number of possibilities as in [9], we derive an analytic bound by employing a simplification analogous to the two-dimensional projection of [8]. However, in this case the only directions out of all scalar fields that are singled out are the so-called sGoldstino directions. These are the scalar partners of the would-be Goldstino that is eaten up by the gravitino in the process of spontaneous supersymmetry breaking. Therefore, it is supersymmetry breaking that dynamically determines a number of preferred directions in moduli space.

It has been shown in various supergravity contexts that the sGoldstino directions are very efficient in tracing possible scalar instabilities [10, 11, 12]. For this reason, one can use the sGoldstino directions to derive an upper bound on the second slow-roll parameter η. In the context of minimal, F-term supergravity this slow-roll condition was discussed in [13]. Note that it does not require the sGoldstino's and inflaton directions to coincide. We demonstrate that additionally imposing the condition of effective single-field inflation[2] leads to a much stronger bound. We discuss the inflationary implications of this bound and see to what extent it allows for e.g. single-field and slow-roll inflation. Intriguingly, we also find the necessity to introduce a negatively curved manifold as in [8], but now as the scalar manifold spanned by the Kähler fields instead of the internal compactification manifold. In principle, the same line of thought could be followed in the context of $\mathcal{N} > 1$ super-

[2]In the presence of many scalar fields one can talk about effective single-field inflation whenever one scalar excitation is responsible for the inflationary dynamics while the other fields are stabilised with square mass above the Hubble constant (which is given by the value of the scalar potential along the inflationary trajectory).

gravity theories. We briefly outline how the bound obtained in the context of minimal supergravity could be extended to such theories and then we draw some conclusions.

2. Genesis of the bound

The Lagrangian

The field content of our model is given by a graviton $e_\mu{}^a$ and a gravitino ψ_μ coupled to n chiral supermultiplets. Each of these is composed by a chiral spin-1/2 field χ and a complex field ϕ. It has been shown that the ϕ^i, $i = 1, \dots n$ fields organise themselves in a Kähler-Hodge manifold. This geometric structure is a fundamental ingredient in building the theory.

The Lagrangian is given by (modulo four fermion terms)

$$e^{-1}\mathscr{L} = \mathscr{L}_{\text{KIN}} + \mathscr{L}_{\text{F-M}} - V\,, \tag{2.1}$$

where

$$\begin{aligned}\mathscr{L}_{\text{KIN}} = &+\frac{1}{2\ell_p^2}R - \frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\mu\psi_\rho - G_{i\bar{\jmath}}\,\partial_\mu\phi^i\,\partial^\mu\bar{\phi}^{\bar{\jmath}} + \\ &+\frac{\ell_p^2}{2}G_{i\bar{\jmath}}\left(\bar{\chi}^i\gamma^\mu D_\mu\bar{\chi}^{\bar{\jmath}} + \bar{\chi}^{\bar{\jmath}}\gamma^\mu D_\mu\chi^i\right)\,.\end{aligned} \tag{2.2}$$

The gravitino ψ_μ is a Majorana spinor while χ^i is a left-handed spinor

$$\mathbb{P}_L\chi^i = \frac{1}{2}(\mathbb{1}+\gamma_5)\chi^i = \chi^i\,. \tag{2.3}$$

The fermionic mass terms are given by

$$\begin{aligned}\mathscr{L}_{\text{F-M}} = &+\frac{\ell_p^2}{2}e^{\ell_p^2\frac{K}{2}}W\,\bar{\psi}_\mu\mathbb{P}_R\gamma^{\mu\nu}\psi_\nu - \frac{\ell_p^2}{2}e^{\ell_p^2\frac{K}{2}}D_iD_jW\,\bar{\chi}^i\chi^j + \text{h.c.} + \\ &+\frac{\ell_p^2}{\sqrt{2}}e^{\ell_p^2\frac{K}{2}}D_iW\,\bar{\psi}_\mu\gamma^\mu\chi^i + \frac{\ell_p^2}{\sqrt{2}}G_{i\bar{\jmath}}\,\bar{\psi}_\mu\gamma^\nu\left(\partial_\nu\bar{\phi}^{\bar{\jmath}}\right)\gamma^\mu\chi^i + \text{h.c.}\,,\end{aligned} \tag{2.4}$$

and the scalar potential is given by

$$V = -3\,\ell_p^2 e^{\ell_p^2 K}\,W\,\overline{W} + e^{\ell_p^2 K}\,G^{i\bar{\jmath}}\,D_iW\,D_{\bar{\jmath}}\overline{W}\,. \tag{2.5}$$

Every derivative is covariantised w.r.t. Kähler transformations besides local Lorentz transformations. Whenever a derivative acts on a quantity with Kähler indices $(i,\bar{\imath})$ it needs to be further covariantised w.r.t. diffeomorphisms on the Kähler manifold[3]. The Lagrangian is therefore fully specified by the following two quantities:

- $K = K(\phi^i,\bar{\phi}^{\bar{\imath}})$ is the Kähler potential and, by definition, the metric on the Kähler manifold is given by $\partial_i\partial_{\bar{\jmath}}K \equiv G_{i\bar{\jmath}}$. It has mass dimension two while the scalar fields ϕ^i are normalised to the Planck mass.

- $W = W(\phi^i)$ is the holomorphic superpotential, which has mass dimension three.

[3]More on our conventions and a detailed derivation of (2.2)-(2.5) can be found in [14].

The scalar potential (2.5) of any F-term supergravity is made up of two opposing contributions. The negative definite term, related to the superpotential itself, sets the AdS scale. In contrast, the positive definite term is related to the first covariant derivatives of the superpotential D_iW. The latter quantities are referred to as F-terms and play an essential role as the order parameter for supersymmetry breaking.

Scalar mass matrix

A way of carrying out the analysis of this class of supergravity theories is by considering the following real combination

$$G = K + \ell_p^{-2} \ln\left|\ell_p^3 W\right|^2 . \tag{2.6}$$

This function is by construction Kähler invariant and hence one does not have to worry about covariantising derivatives w.r.t. this kind of transformations. In terms of this function the scalar potential reads

$$V = \ell_p^{-4} e^{\ell_p^2 G}\left(\ell_p^2 G^{i\bar{j}} G_i G_{\bar{j}} - 3\right), \tag{2.7}$$

where G_i denotes the simple partial derivative of G w.r.t. ϕ^i. The first derivative is given by

$$\partial_i V = \ell_p^2 G_i V + \ell_p^{-2} e^{\ell_p^2 G}\left(G_i + G^j D_i G_j\right). \tag{2.8}$$

The second derivatives are thus

$$\begin{aligned} D_i D_j V &= \ell_p^2\left(G_i D_j V + G_j D_i V\right) + \ell_p^2\left(D_i G_j + \ell_p^2 G_i G_j\right) V + \\ &\quad + \ell_p^{-2} e^{\ell_p^2 G}\left(2 D_{(i} G_{j)} + G^k D_i D_j G_k\right), \\ D_{\bar{i}} D_j V &= \ell_p^2\left(G_{\bar{i}} D_j V + G_j D_{\bar{i}} V\right) + \ell_p^2\left(G_{\bar{i}j} - \ell_p^2 G_{\bar{i}} G_j\right) V + \\ &\quad + \ell_p^{-2} e^{\ell_p^2 G}\left[G_{\bar{i}j} + \left(D_{\bar{i}} G^k\right)\left(D_j G_k\right) - \mathscr{R}_{j\bar{i}k\bar{l}} G^k G^{\bar{l}}\right]. \end{aligned} \tag{2.9}$$

Using these derivatives we are able to construct the squared mass matrix for the scalar fields at any point in field space. It is given by

$$m^{2I}{}_J = \begin{bmatrix} m^{2i}{}_j & m^{2i}{}_{\bar{j}} \\ m^{2\bar{i}}{}_j & m^{2\bar{i}}{}_{\bar{j}} \end{bmatrix} = \begin{bmatrix} G^{i\bar{k}} D_{\bar{k}} D_j V & G^{i\bar{k}} D_{\bar{k}} D_{\bar{j}} V \\ G^{\bar{i}k} D_k D_j V & G^{\bar{i}k} D_k D_{\bar{j}} V \end{bmatrix}. \tag{2.10}$$

where we have used the collective index $I = (i, \bar{\imath})$.

sGoldstino directions

Spontaneous supersymmetry breaking is induced by D_iW. We consider a configuration of the theory in which supersymmetry is broken, i.e. $D_iW \neq 0$. We see from (2.4) that the mixing between the gravitino and the chiral spin-$1/2$ fields is sourced exactly by the order parameter of supersymmetry breaking and is encoded in the term

$$\frac{\ell_p^2}{\sqrt{2}} e^{\ell_p^2 \frac{K}{2}} D_i W\, \bar{\psi}_\mu \gamma^\mu \chi^i = -\frac{1}{\ell_p} \bar{\psi}_\mu \gamma^\mu \left(\mathbb{P}_L \zeta\right), \tag{2.11}$$

where we have defined a linear combination of spin-$1/2$ fields

$$\mathbb{P}_L \zeta = -\frac{\ell_p^3}{\sqrt{2}} e^{\ell_p^2 \frac{K}{2}} D_i W\, \chi^i . \tag{2.12}$$

This field is usually called the Goldstino. Indeed, it is possible to show that the dynamics of the gravitino can be disentangled from that of the spin-$1/2$ fields, by performing a supersymmetry transformation in which the supersymmetry parameter ε is proportional to the Goldstino. Going to the so-called unitary gauge it is possible to eliminate from the spectrum the Goldstino. This is the analogue of the Higgs mechanism for spontaneous gauge symmetry breaking, often called super-Higgs mechanism. The missing degrees of freedom are absorbed by the gravitino.

We now consider the supersymmetry variation of the Goldstino field. Apart from terms involving fermions, it is given by

$$\begin{aligned}\delta(\mathbb{P}_L\zeta) &= -\frac{\ell_p^3}{2}\,e^{\ell_p^2\frac{K}{2}}\,D_iW\,\frac{1}{\ell_p}\gamma^\mu\,\partial_\mu\phi^i(\mathbb{P}_R\varepsilon)+\frac{\ell_p^2}{2}\,e^{\ell_p^2K}\,G^{i\bar{\jmath}}\,D_iW\,D_{\bar{\jmath}}\overline{W}\,(\mathbb{P}_L\varepsilon)\\ &= -\frac{\ell_p^3}{2}\,e^{\ell_p^2\frac{K}{2}}\,D_iW\,\frac{1}{\ell_p}\gamma^\mu\,\partial_\mu\phi^i\,(\mathbb{P}_R\varepsilon)+\frac{\ell_p^2}{2}\,V_+\,(\mathbb{P}_L\varepsilon)\,, \end{aligned} \tag{2.13}$$

where in the second term we recognise the positive definite part of the scalar potential, denoted by V_+. In the first term the complex quantity

$$\frac{\ell_p^3}{2}\,e^{\ell_p^2\frac{K}{2}}\,D_iW \tag{2.14}$$

defines, for a fixed value of ϕ^i, a direction in the scalar manifold. After a Kähler transformation, it can be written as

$$\frac{\ell_p^2}{2}\,e^{\ell_p^2\frac{G}{2}}\,G_i\,. \tag{2.15}$$

We normalise the direction to a unit vector taking

$$g_i=\frac{G_i}{\sqrt{G_jG^j}}\,. \tag{2.16}$$

At this point we would like to point out a slight subtlety concerning the terminology of the Goldstino and sGoldstino's. For cosmological purposes, in which one usually considers time-dependent scalar fields, the definition of the linear combination of spin-$1/2$ fields which gives the Goldstino is slightly different from what is discussed above. This is mainly due to the presence of couplings of the schematic form $\bar{\psi}(\partial\phi)\chi$ in (2.4). In that case a more careful analysis applies which can be found for instance in [14]. Therefore, referring to the g_i directions as the sGoldstino's is a small abuse of notation in the time-dependent case. Nevertheless, these directions can be defined on the scalar manifold as long as supersymmetry is broken and we will use this in what follows.

A geometric bound

In this section we follow the steps of [13] and consider the projection of the mass matrix on the direction specified by g_i. For any complex quantity U_i with $U_i\bar{U}^i=1$ we could define two dinstinct real orthonormal directions $(U_i,\bar{U}_{\bar{\imath}})/\sqrt{2}$ and $(iU_i,-i\bar{U}_{\bar{\imath}})/\sqrt{2}$. The same could be done with the sGoldstino direction g_i. Consider now the projection of the mass matrix along these directions

$$\frac{1}{2}\begin{bmatrix}g_i & g_{\bar{\imath}}\end{bmatrix}\begin{bmatrix}m^{2\,i}{}_j & m^{2\,i}{}_{\bar{\jmath}}\\ m^{2\,\bar{\imath}}{}_j & m^{2\,\bar{\imath}}{}_{\bar{\jmath}}\end{bmatrix}\begin{bmatrix}g^j\\ g^{\bar{\jmath}}\end{bmatrix}\quad,\qquad \frac{1}{2}\begin{bmatrix}-g_i & g_{\bar{\imath}}\end{bmatrix}\begin{bmatrix}m^{2\,i}{}_j & m^{2\,i}{}_{\bar{\jmath}}\\ m^{2\,\bar{\imath}}{}_j & m^{2\,\bar{\imath}}{}_{\bar{\jmath}}\end{bmatrix}\begin{bmatrix}-g^j\\ g^{\bar{\jmath}}\end{bmatrix}, \tag{2.17}$$

If we take the averaged sum of these two quantities and normalise it w.r.t. the potential we are left with

$$\eta_{\rm sG} \equiv \frac{g^{\bar{\imath}} g^j D_{\bar{\imath}} D_j V}{\ell_p^2 V} = \frac{2}{3\gamma} + \frac{4}{\sqrt{3}} \frac{1}{\sqrt{1+\gamma}} \Re \left\{ g^i \frac{D_i V}{V} \right\} + \frac{\gamma}{1+\gamma} \frac{G^{\bar{\imath} j} D_{\bar{\imath}} V \, D_j V}{\ell_p^2 V^2} - \frac{1+\gamma}{\gamma} \tilde{\mathscr{R}} , \quad (2.18)$$

where we have defined

$$\gamma = \frac{\ell_p^4 V}{3\, e^{\ell_p^2 G}} = \frac{\ell_p^2 V}{3\, |m_{3/2}|^2} , \quad (2.19)$$

$\Re$ denotes the real part and $\tilde{\mathscr{R}}$ is the sectional curvature related to the plane defined by g_i on the scalar manifold

$$\tilde{\mathscr{R}} \equiv \frac{\mathscr{R}_{\bar{\imath} j \bar{k} l}\, g^{\bar{\imath}} g^j g^{\bar{k}} g^l}{\ell_p^2} . \quad (2.20)$$

Notice that $\eta_{\rm sG}$ is obtained from the averaged sum of two masses. We will come back to this point in the next section. In [13] $\eta_{\rm sG}$ is used to obtain a bound on the second slow-roll parameter η depending on the first slow-roll parameter ε, γ and the sectional curvature $\tilde{\mathscr{R}}$. In order to get the bound we first notice that, for any unit vector $U_I = (U_i, \bar{U}_{\bar{\imath}})/\sqrt{2}$ with $U_i \bar{U}^i = 1$, we have

$$\eta \leq \frac{U_I\, m^{2I}{}_J\, U^J}{V} \quad , \quad \left| \bar{U}^i \frac{D_i V}{V} \right| \leq \sqrt{\varepsilon} . \quad (2.21)$$

Combining this information and pluging it into (2.18), we obtain

$$\eta \leq \eta_{\rm sG} \leq \frac{2}{3\gamma} + \frac{4}{\sqrt{3}} \frac{1}{\sqrt{1+\gamma}} \sqrt{\varepsilon} + \frac{\gamma}{1+\gamma} \varepsilon - \frac{1+\gamma}{\gamma} \tilde{\mathscr{R}} . \quad (2.22)$$

We will be interested in the last inequality of the chain (2.22), namely the one which relates $\eta_{\rm sG}$ to ε and $\tilde{\mathscr{R}}$. Before going through the discussion, consider the limit $\gamma \gg 1$ which corresponds to the case in which the mass of the gravitino is well below the scalar potential during inflation and the F-term dominates. In this case we get the simple bound

$$\eta_{\rm sG} \leq \varepsilon - \tilde{\mathscr{R}} . \quad (2.23)$$

Both bounds are very interesting as they relate the slow-roll parameters to the geometry of the scalar manifold. In the next section, after a small summary regarding all the quantities appearing in (2.22), we will analyse their inflationary implications.

3. Implications of the bound

In this section we discuss the implications of the geometric bounds derived above, (2.22) and (2.23). In order to do that, we first recap the information contained in this bound and its physical meaning:

- γ is the ratio between the scalar potential and the gravitino mass (2.19). It tells one which is the relative importance between the two contributions to the scalar potential. If $\gamma \to -1$ the scalar potential is dominated by the negative definite contribution. When $\gamma \sim 0$ the two terms are of the same order. Finally when $\gamma \gg 1$ the supersymmetry breaking F-terms dominate over the AdS scale.

During inflation $\ell_p^2 V \sim H^2$ and the limit $\gamma \gg 1$ means the gravitino mass is well below the Hubble scale since the F-term dominates, as is the case in F-term inflation. Unless other mechanism enters the game, it is reasonable to assume today's gravitino mass being of the same order of magnitude as the one during inflation. If that is the case and today's gravitino mass being at the TeV scale, then $\gamma \gg 1$ is a very sensible limit to take.

- η_{sG} is the averaged sum of two scalar masses normalised to the value of the potential. This is a key point. If we want to embed effective single-field inflation in F-term supergravity, there are two possible scenarios. In the first one, the inflaton is not one of the sGoldstino directions. In this case, if one wants the sGoldstino fields to be spectators during inflation, their masses should be of order H or above and hence $\eta_{\text{sG}} \gtrsim 1/2$. In the second scenario the inflaton is one of the sGoldstino directions: this is referred to as sGoldstino inflation (for recent analyses, see e.g. [15, 16, 17, 18]). Even in this case η_{sG} should be of order $1/2$ or larger, because the orthogonal sGoldstino field needs to be stabilised along the inflationary trajectory. Thus in any case, *single-field inflation* requires $\eta_{\text{sG}} \gtrsim 1/2$.

- ε is the generalisation of the first slow-roll parameter to the case of many scalar fields. It is a measure of the sum of the squared velocity of all the fields. Despite the multi-field generalisation, *slow-roll inflation* requires $\varepsilon \ll 1$.

- $\tilde{\mathscr{R}}$ is the sectional curvature related to the plane identified by the sGoldstino directions. In general the Riemann tensor of a manifold is completely specified once all the sectional curvatures are given. For our purposes it is sufficient to say that, if $\tilde{\mathscr{R}} \sim 1$, there are some components of the Riemann tensor which are of order ℓ_p^{-2} and as a consequence we are dealing with a strongly curved scalar manifold. In other words, the scalar kinetic terms in (2.2) cannot be simply given by

$$-G_{i\bar{\jmath}}\,\partial_\mu\phi^i\,\partial^\mu\bar{\phi}^{\bar{\jmath}} \simeq -\sum_{i=1}^{n}\partial_\mu\phi^i\,\partial^\mu\bar{\phi}^{\bar{\imath}}\,, \tag{3.1}$$

but one needs to take into account the presence of the Kähler metric. Therefore *canonical kinetic terms* require $\tilde{\mathscr{R}} = 0$.

Let us now discuss the inflationary implications of the bound derived above. For concreteness we will focus on inflationary scenarios for which $\gamma \gg 1$, which is the case in F-term inflation. For other scenarios with finite values of γ, a similar analysis can be equally done. From the form of the bound (2.23) and the above discussion it can be seen that one can only impose consistently two of the conditions {single-field, slow-roll, canonical kinetic terms} together. Let us discuss the three possible consistent combinations.

Slow-roll single-field inflation

The first possibility consists of imposing the first two conditions: slow-roll and effective single-field inflation. From the geometric bound (2.23), we see that the sectional curvature of the scalar manifold must be strictly negative. In other words, slow-roll and single-field inflation require having non-canonical kinetic terms for the inflaton and all the scalar fields present. Moreover, the non-canonical kinetic terms should correspond to a metric whose Riemann curvature has a number of components which are negative and of order order one in Planck units. Note that this rules out a number of examples discussed in [13].

The fact that non-canonical terms are required at any point in field space implies that the full inflationary trajectory should extend to the point where these terms become relevant – if this were not the case then inflation should proceed independent of these terms, which is inconsistent with the bound (2.23). Therefore the requirement of non-canonical kinetic terms, with corrections to the metric of order one in Planck units, implies that one must have large field inflation. As a consequence, effectively single-field and slow-roll cannot be realised in small field F-term inflation. Note that this statement on the full inflationary trajectory follows from an analysis of the bound (2.23) for a single point in field space.

There is a small caveat to this statement. Indeed $\tilde{\mathscr{R}}$ is a specific sectional curvature associated to the plane defined by the sGoldstino fields. By carefully constructing the Kähler- and superpotential it is possible to obtain an inflationary trajectory along which the inflaton is completely orthogonal to the sGoldstino fields (see e.g. [19, 20]). The latter are stabilised and, even being $\tilde{\mathscr{R}} \lesssim -\frac{1}{2}$, still one can obtain canonical kinetic terms for the inflaton allowing for small field inflation. The special features of this model provide an escape from my conclusions. On the other hand, as long as there is a non-negligible overlap between the inflaton and the sGoldstino fields along the inflationary path, my analysis applies.

Observationally, the consequence of having large field inflation is the prediction that tensor modes can be detectable. The argument proceeds via the Lyth bound [21], which relates inflationary trajectories of order one in Planck units to a ratio r between tensor to scalar perturbations of percent level. The latter corresponds to observable tensor modes, which are therefore a prediction of F-term inflation.

Furthermore, the implications for the curvature perturbations in inflation with non-standard kinetic terms have been studied largely in the literature, starting with the work of Garriga-Mukhanov [22]. Writing the scalar part of the lagrangian as a general function $P(X,\phi)$, with $X = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, we see that the kinetic term for the inflaton gives rise to a linear function of X in the present case. Thus, using the results of [22], one sees that the resulting perturbations coincide with the canonical case. In particular, the “speed of sound" of the perturbations c_s, equals the speed of light. In this case, possible departures from the Gaussian spectrum in the equilateral configuration, parameterised by $f_{\rm NL}^{\rm eq} \propto 1/c_s^2$ are negligible [23, 24]. Moreover, as has been shown in [25], non-Gaussianities of the local form $f_{\rm NL}^{\rm loc}$, are suppressed by $1-n_s$ for single field inflation. Thus in this case, one obtains standard single field predictions for the scalar perturbations.

Finally, in the more general case with arbitrary values of γ, the slow-roll and single-field conditions imply

$$\tilde{\mathscr{R}} \lesssim \frac{4-3\gamma}{6(1+\gamma)}, \tag{3.2}$$

and therefore they still impose a geometric bound on the Kähler manifold of F-term supergravity.

Slow-roll with canonical kinetic terms

The next possibility is to impose slow-roll inflation and canonically normalised fields. Thus we see that the bound (2.23) implies that we have to consider multifield inflation with canonical terms for all the fields. In this case, large non-Gaussianity can be generated dynamically by inflation due to the interplay of all fields and large isocurvature perturbations.

Large non-Gaussianity of the local form $f_{\rm NL}^{\rm loc}$ generated during inflation has been shown to be generically hard to achieve (for a review with several references see [26]) and is very much model dependent. Therefore, without knowledge on the form of the potential, one can only conclude that potentially large non-Gaussianities due to multifield dynamics could be generated in these type of models.

Single-field with canonical kinetic terms

The last possible combination is to impose effective single field inflation with canonical kinetic terms. In this case, the geometric bound (2.23) implies that $\varepsilon \gtrsim 1/2$ and thus slow-roll inflation cannot be realised.

4. Extension of the bound

In this section we want to argue how an analysis similar to that we have just showed could be carried out for different supergravity theories. The idea is that, when we focus on the sGoldstino directions, it is always possible to write down a bound of the form

$$\eta_{\rm sG} \leq c_0 \left(f(\gamma) + g(\gamma)\tilde{\mathscr{R}} \right) + c_{1/2}\, \frac{1}{\sqrt{1+\gamma}}\, \sqrt{\varepsilon} + c_1\, \frac{\gamma}{1+\gamma}\, \varepsilon\,. \tag{4.1}$$

Clearly the details for each supergravity theory are different. nevertheless it is always possible to single out the sGoldstino's and compute their average mass. We show here how the story goes through in the $\mathcal{N}=2$ theory coupled to n hypermultiplets and then draw some conclusions.

Fixing conventions

Consider 4 dimensional $\mathcal{N}=2$ supergravity with n hypermultiplets coupled to the supergravity multiplet. The spectrum of the theory is given by $(e_\mu^a,\, \psi_\mu^i,\, A_\mu)$ plus n times $(\xi^\alpha,\, q^X)$, where we adopt the following conventions for indices in the theory[4]

In what follows we stick to Louis et al. treatment [12]. First we list some interesting facts about quaternionic-Kähler geometry. On the scalar manifold we consider a Killing vector k_X such that

$$D_{(X}k_{Y)}(q) = 0\,, \tag{4.2}$$

where D is a derivative covariant with respect to diffeomorphisms on the scalar manifold and $SU(2)$. We indicate the metric on the scalar manifold with G_{XY} and the triplet of $SU(2)$ complex structures with ${J_{XY}}^r$. These complex structures satisfy several properties.

$$\begin{aligned} D_X {J_{YW}}^r &= 0 \\ {J_{WX}}^r\, {J^W}_Y{}^s &= G_{XY}\,\delta^{rs} + \varepsilon^{rst}\, J_{XY\,t}\,, \\ {J_{XY}}^r\, J_{WZ\,r} &= -\mathscr{R}_{XYWZ} + W_{XYWZ} - G_{X[W}\, G_{|Y|Z]} - {J_{X[W}}^r\, J_{|Y|Z]\,r}\,. \end{aligned} \tag{4.3}$$

[4] In what follows I also set $\ell_p^2 = 1$.

Table 1: Conventions on $\mathcal{N} = 2$ indices

$\mu = 0,\ldots,3$	space-time world indices
$a = 0,\ldots,3$	space-time flat indices
$i = 1,2$	number of susy generators
$\alpha = 1,\ldots,2n$	$Sp(2n)$ indices labeling hyperspinors
$X = 1,\ldots,4n$	indices on the scalar quaternionic-Kähler manifold
$r = 1,2,3$	adjoint $SU(2)$ indices

The Killing vector is derived from a triplet of Killing prepotential

$$D_X \mathscr{P}^r = 2\, J_{XY}{}^r\, k^Y\,. \tag{4.4}$$

The previous relation could be, in some sense, inverted obtaining the Killing vector in terms of the Killing prepotential

$$k_X = -\frac{1}{6}\, J_{XY}{}^r\, D^Y \mathscr{P}_r\,. \tag{4.5}$$

In the $\mathcal{N} = 2$ case we have four sGoldstino directions. We will focus on those defined by

$$U^r_X = -J_X{}^{Y\,r}\, k_Y = -\frac{1}{2}\, D_X \mathscr{P}^r \tag{4.6}$$

Scalar sector of the theory

The scalar potential is given by

$$V(q) = 4\, k^X\, k_X - 3\, \mathscr{P}^r\, \mathscr{P}_r\,. \tag{4.7}$$

The first derivative is given by

$$D_X V = 8\, k^Y\, D_X k_Y - 6\, \mathscr{P}^r\, D_X \mathscr{P}_r\,, \tag{4.8}$$

and the second one is given by

$$D_X D_Y V = 8\, (D_X k^W)\, (D_Y k_W) - 8\, \mathscr{R}_{XWYZ}\, k^W k^Z - 6\, (D_X \mathscr{P}^r)\, (D_Y \mathscr{P}_r) - 6\, \mathscr{P}^r\, D_{(X} D_{Y)} \mathscr{P}_r\,. \tag{4.9}$$

This gives directly the mass matrix $2\, m^2_{XY} = D_X D_Y V$.

sGoldstino projection

If we sandwich the mass matrix with the symmetric sGoldstino directions we obtain four terms

$$\begin{aligned} U^{Xr}\, U^Y_r\, m^2_{XY} \; = \; & +4\, (U^{X\,r}\, D_X k^W)\, (U^Y_r\, D_Y k_W) - 4\, \mathscr{R}_{XWYZ}\, U^{X\,r} k^W\, U^Y_r\, k^Z + \\ & -3\, (U^{X\,r}\, D_X \mathscr{P}^s)\, (U^Y_r\, D_Y \mathscr{P}_s) - 3\, \mathscr{P}^s\, U^{Xr}\, U^Y_r\, D_{(X} D_{Y)} \mathscr{P}_s\,. \end{aligned} \tag{4.10}$$

The norm of the sGoldstino projection is given by

$$U^r_X\, U^X_r = 3\, k_X k^X\,. \tag{4.11}$$

The quantity $\eta_{\rm sG}$ is defined as

$$\eta_{\rm sG} \equiv \frac{U^{Xr}\, U^Y_r\, m^2_{XY}}{(U^s_W U^W_s)\, V}\,. \tag{4.12}$$

The γ parameter

We can again define the γ-parameter

$$\gamma \equiv \frac{V}{3\,|m_{3/2}|^2} = \frac{4\,k^X k_X - 3\,\mathscr{P}^r \mathscr{P}_r}{3\,\mathscr{P}^s \mathscr{P}_s}\,, \tag{4.13}$$

from which we derive

$$1+\gamma = \frac{4\,k^X k_X}{3\,\mathscr{P}^r \mathscr{P}_r} \quad , \qquad \frac{1+\gamma}{\gamma} = \frac{4\,k^X k_X}{V}\,. \tag{4.14}$$

The bound

After several tedious algebraic manipulations it is possible to write the following inequality

$$\eta_{\rm sG} \leq -\frac{19+27\,\gamma}{9\,\gamma} - \frac{13\sqrt{2}}{6\sqrt{3}\,\sqrt{1+\gamma}}\,\sqrt{\varepsilon} + \frac{\gamma}{2\,(1+\gamma)}\,\varepsilon - \frac{1+\gamma}{\gamma}\,\tilde{\mathscr{R}}\,. \tag{4.15}$$

Indeed, this bound is exactely of the form (4.1) and, furthermore, an analysis completely similar to that performed in the $\mathscr{N}=1$ case could be carried out.

The analysis for the $\mathscr{N}=2$ case is still preliminary and so are similar analyses for $\mathscr{N}=4,8$ theories. Nevertheless the result (4.15) and other results hint at the possibility of obtaining a full set of bounds of the form (4.1).

5. Conclusions

In this contribution we have shown how spontaneous supersymmetry breaking can constrain the possibility of embedding inflation in supergravity theories. We have first done that by analysing the simplest $\mathscr{N}=1$ model. In there, remerkably, the combination of slow-roll and single-field imposes a very strong constraint on the theory. The curvature of the Kähler manifold spanned by the chiral scalars necessarily includes negative components which are order one in Planck units. Only Kähler manifolds with this property satisfy the necessary but not sufficient condition for slow-roll, single field inflation. This rules out many of the examples considered in the literature, see e.g. [13]. Moreover, as discussed in section 3, this automatically implies that the full inflationary trajectory will be in the large field class. A consequence is the generation of observable tensor modes in the polarisation of the CMB.

We have also shown how the same analysis could be extended to other four dimensional supergravity theories. Despite the fact that some result are still preliminary there seems to be the possibility of obtaining a full set of bounds of the form (4.1).

Acknowledgements

The author wants to thank Diederik Roest and Ivonne Zavala for their collaboration in the research on these topics. Furthermore the author thanks the organizers of the “School and Workshop on Elementary Particle Physics and Gravity" for the possibility to participate and present this contribution. The author’s resesearch is supported by a VIDI grant from the Netherlands Organisation for Scientific Research (NWO).

References

[1] A. H. Guth, *The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*, Phys.Rev. **D23** (1981) 347–356.

[2] A. D. Linde, *A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems*, Phys.Lett. **B108** (1982) 389–393.

[3] **WMAP Collaboration** Collaboration, E. Komatsu *et al.*, *Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation*, Astrophys.J.Suppl. **192** (2011) 18 [1001.4538].

[4] L. McAllister and E. Silverstein, *String Cosmology: A Review*, Gen.Rel.Grav. **40** (2008) 565–605 [0710.2951].

[5] V. Balasubramanian, P. Berglund, J. P. Conlon and F. Quevedo, *Systematics of moduli stabilisation in Calabi-Yau flux compactifications*, JHEP **0503** (2005) 007 [hep-th/0502058].

[6] J. P. Conlon and F. Quevedo, *Kahler moduli inflation*, JHEP **0601** (2006) 146 [hep-th/0509012].

[7] O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, *Type IIA moduli stabilization*, JHEP **07** (2005) 066 [hep-th/0505160].

[8] M. P. Hertzberg, S. Kachru, W. Taylor and M. Tegmark, *Inflationary Constraints on Type IIA String Theory*, JHEP **12** (2007) 095 [0711.2512].

[9] N. Agarwal, R. Bean, L. McAllister and G. Xu, *Universality in D-brane Inflation*, JCAP **1109** (2011) 002 [1103.2775].

[10] M. Gomez-Reino and C. A. Scrucca, *Locally stable non-supersymmetric Minkowski vacua in supergravity*, JHEP **05** (2006) 015 [hep-th/0602246].

[11] M. Gomez-Reino and C. A. Scrucca, *Constraints for the existence of flat and stable non-supersymmetric vacua in supergravity*, JHEP **09** (2006) 008 [hep-th/0606273].

[12] M. Gomez-Reino, J. Louis and C. A. Scrucca, *No metastable de Sitter vacua in N=2 supergravity with only hypermultiplets*, JHEP **02** (2009) 003 [0812.0884].

[13] L. Covi, M. Gomez-Reino, C. Gross, J. Louis, G. A. Palma *et al.*, *Constraints on modular inflation in supergravity and string theory*, JHEP **0808** (2008) 055 [0805.3290].

[14] D. Freedman and A. Van Proeyen, *Supergravity*, Cambridge University Press (2012) 607 pp.

[15] L. Alvarez-Gaume, C. Gomez and R. Jimenez, *Minimal Inflation*, Phys.Lett. **B690** (2010) 68–72 [1001.0010].

[16] L. Alvarez-Gaume, C. Gomez and R. Jimenez, *A Minimal Inflation Scenario*, JCAP **1103** (2011) 027 [1101.4948].

[17] L. Alvarez-Gaume, C. Gomez and R. Jimenez, *Phenomenology of the minimal inflation scenario: inflationary trajectories and particle production*, 1110.3984.

[18] A. Achucarro, S. Mooij, P. Ortiz and M. Postma, *Sgoldstino inflation*, 1203.1907.

[19] R. Kallosh, A. Linde and T. Rube, *General inflaton potentials in supergravity*, Phys.Rev. **D83** (2011) 043507 [1011.5945].

[20] R. Kallosh, A. Linde, K. A. Olive and T. Rube, *Chaotic inflation and supersymmetry breaking*, Phys.Rev. **D84** (2011) 083519 [1106.6025].

[21] D. H. Lyth, *What would we learn by detecting a gravitational wave signal in the cosmic microwave background anisotropy?*, Phys.Rev.Lett. **78** (1997) 1861–1863 [hep-ph/9606387].

[22] J. Garriga and V. F. Mukhanov, *Perturbations in k-inflation*, Phys. Lett. **B458** (1999) 219–225 [hep-th/9904176].

[23] M. Alishahiha, E. Silverstein and D. Tong, *DBI in the sky*, Phys. Rev. **D70** (2004) 123505 [hep-th/0404084].

[24] X. Chen, M.-x. Huang, S. Kachru and G. Shiu, *Observational signatures and non-Gaussianities of general single field inflation*, JCAP **0701** (2007) 002 [hep-th/0605045].

[25] P. Creminelli and M. Zaldarriaga, *Single field consistency relation for the 3-point function*, JCAP **0410** (2004) 006 [astro-ph/0407059].

[26] C. T. Byrnes and K.-Y. Choi, *Review of local non-Gaussianity from multi-field inflation*, Adv. Astron. **2010** (2010) 724525 [1002.3110].