The spectrum of (h)QCD in the Veneziano limit

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In this note we report on the analysis of the zero temperature spectra of glueballs and mesons for holographic QCD in the Veneziano limit. We work within a holographic bottom-up model named V-QCD which takes into account the full backreaction of the flavor degrees of freedom. By studying the fluctuations of this model we compute spectra of mesons and glueballs as a function of $x = N_f/N_c$. The spectra are discrete and gapped (modulo the pions) in the QCD regime, where x is below the critical value x_c at which the conformal transition takes place. The masses uniformly converge to zero in the walking region $x \to x_c$ following Miransky scaling. Moreover, all the ratios of masses asymptote to finite constants as $x \to x_c$. Therefore there is no "dilaton" in the spectrum. Finally, we compute the S-parameter, which is found to be of $\mathscr{O}(1)$ in the walking regime.

Proceedings of the Corfu Summer Institute 2012 "School and Workshops on Elementary Particle Physics and Gravity"
September 8-27, 2012
Corfu, Greece

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PoS(Corfu2012)129

1. Introduction

The gauge/gravity duality [1] is a powerful tool to study the dynamics of strongly coupled field theories and thus it has been extensively used to try and describe the physics of Strong interactions. Although a calculable string dual of QCD is far from our present understanding, it is possible to build models describing strong coupling physics bearing many similarities with QCD. Indeed, top-down models resulting from solutions of string theory describe qualitatively the low energy dynamics of QCD. However, these models are plagued by Kaluza-Klein modes that make difficult a quantitative matching to real QCD. A different approach is given by the bottom-up models, which are ad hoc holographic models inspired by string theory that use some QCD features as inputs. (See [2] and references therein for an overview on holographic duals of QCD). In this note we will be working with a class of bottom-up holographic theories that has been proposed recently, under the name of V-QCD [3], and has physics very close to QCD in the Veneziano limit.

The Veneziano limit [4] of QCD is given by:

$$N_c \to \infty\,, \qquad N_f \to \infty\,, \qquad x = \frac{N_f}{N_c} \quad \text{fixed}\,, \qquad \lambda = g_{YM}^2 N_c \quad \text{fixed}\,. \tag{1.1}$$

An interesting feature of the theory accessible in this limit is the conformal window in which the theory has an IR fixed point. This window extends from $x = 11/2$ to lower values of x, and it includes the Banks-Zaks weakly coupled region (as $x \to 11/2$) [5]. At a critical value x_c there is a phase transition from the conformal window to theories with chiral symmetry breaking in the IR. Interestingly, near and below x_c there is a transition region where the theory is expected to exhibit "walking" behavior. This "walking" regime has been conjectured to display Miransky scaling [6].

The dynamics of "walking" (or nearly conformal) quantum field theories, has been the subject of intensive study. It has been argued to be an important ingredient [7, 8, 9] in providing viable non-perturbative mechanisms for electroweak symmetry breaking like technicolor [10]. As we have said, this regime is expected to appear in standard QCD just below the boundary of the conformal window, $x \le x_c \simeq 4$; as well as in other quantum field theories [11].

The transition between the conformal window and QCD-like IR behavior has been called a conformal transition [12]. It has been suggested that in holographic theories this conformal transition is associated with a violation of the BF bound in the dual bulk theory [13]. Moreover, in QCD this correlates with the $\bar{\psi}\psi$ operators reaching a scaling dimension equal to two – another prerequisite of viable extended technicolor.

Apart from Miransky scaling, other phenomena have often been associated with the "walking regime" of QFT:

- The appearance of an anomalously light scalar state, the "dilaton", due to the almost unbroken scale invariance [8].
- The suppression of the electroweak S-parameter, a crucial ingredient for the viability of technicolor theories [14].

Both issues are controversial, especially since "walking regimes" appear at strong coupling, and therefore perturbative techniques do not apply. It is also difficult to study these phenomena on the lattice due to sizable finite size effects.

PoS(Corfu2012)129

In recently studied holographic models with walking behavior, the lightest state is often a scalar [15, 16]. Whether this state can be identified as the dilaton is, however, a difficult question, and the answer appears to depend on the model. The S-parameter has been studied in popular holographic bottom-up [17] as well as brane-antibrane models [18, 19] with a variety of answers found.

In this letter, to look into these and related issues we will study the spectra of mesons and glueballs for a bottom-up holographic model of QCD in the Veneziano limit (V-QCD [3]). This letter summarizes the results presented by one of the authors in the “XVIII European Workshop on String Theory”, and subsequently published in [20].

2. V-QCD

This model combines two sectors resulting from holographic models describing respectively the glue and flavor dynamics of QCD. The first one is improved holographic QCD (IHQCD), which is a holographic model for large-N Yang Mills in 4 dimensions [21]. The second one is a model for flavor inspired by tachyon condensation in string theory [22]. The relevant fields that are kept in these models in order to describe the vacuum structure are:

- The five-dimensional metric, the space-time components of which are dual to the energy-momentum tensor,
- A scalar (the dilaton, ϕ) that is dual to the YM 't Hooft coupling constant,
- A complex $N_f \times N_f$ matrix field (the tachyon, T_{ij}) transforming in the $(N_f, \bar{N}_f)$ of the $U(N_f) \times U(N_f)$ flavor group, and dual to the operator $\bar{\psi}_j \psi_i$.

The complete action for the V-QCD model can be written as

$$S = S_g + S_f + S_a\,, \tag{2.1}$$

where S_g, S_f, and S_a are the actions for the glue, flavor and CP-odd sectors, respectively. As discussed in [3], only the first two terms contribute to the vacuum structure of the theory. The CP-odd sector, whose physics contains the $U(1)_A$ anomaly, contributes to some sectors of the spectrum (flavor singlet pseudoscalars) that are not the subject of this letter; hence we will address its physics in a future publication [23]. The full structure of the flavor sector action (S_f) was not detailed in [3] since it is not necessary when studying the vacuum structure of the model. However, the extra terms do contribute to the spectrum of fluctuations, and will be discussed below.

2.1 The glue sector

The glue action was introduced in [21, 24, 25],

$$S_g = M^3 N_c^2 \int d^5x\,\sqrt{-g}\left(R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda)\right)\,. \tag{2.2}$$

Here $\lambda = e^{\phi}$ is the exponential of the dilaton. It is dual to the $\mathbb{T}rF^2$ operator, and its background value is identified as the 't Hooft coupling. Glue dynamics sets requirements on the dilaton potential: V_g asymptotes to a constant near $\lambda = 0$, and diverges as $V_g \sim \lambda^{4/3}\sqrt{\log\lambda}$ as $\lambda \to \infty$, generating confinement, a mass gap, discrete spectrum and asymptotically linear glueball trajectories [21, 25]. The Ansatz for the vacuum solution for the metric is

$$ds^2 = e^{2A(r)}(dx_{1,3}^2 + dr^2)\,, \tag{2.3}$$

where the warp factor A is identified as the logarithm of the energy scale in field theory. In our conventions the UV boundary lies at $r = 0$, and the bulk coordinate runs from zero to infinity. The metric will be close to the AdS one except near the IR singularity at $r = \infty$, and thus $A \sim -\log(r/\ell)$, where ℓ is the AdS radius. Therefore, r is identified roughly as the inverse of the energy scale of the dual field theory.

2.2 The flavor sector

The flavor sector consists of the generalized Sen's action [26]

$$S_f = -\frac{1}{2}M^3 N_c \mathbb{T}r \int d^4x\,dr\,\Big(V_f(\lambda, T^\dagger T)\sqrt{-\det \mathbf{A}_L} + V_f(\lambda, TT^\dagger)\sqrt{-\det \mathbf{A}_R}\Big)\,. \tag{2.4}$$

The quantities inside the square roots are defined as

$$\begin{aligned}\mathbf{A}_{LMN} &= g_{MN} + w(\lambda)F_{MN}^{(L)} + \frac{\kappa(\lambda)}{2}\left[(D_M T)^\dagger (D_N T) + (D_N T)^\dagger (D_M T)\right]\,,\\ \mathbf{A}_{RMN} &= g_{MN} + w(\lambda)F_{MN}^{(R)} + \frac{\kappa(\lambda)}{2}\left[(D_M T)(D_N T)^\dagger + (D_N T)(D_M T)^\dagger\right]\,,\end{aligned} \tag{2.5}$$

where the fields A_L, A_R, and T are $N_f \times N_f$ matrices in the flavor space. It is not known in general how the determinants over the Lorentz indices in (2.4) should be defined when the arguments (2.5) contain non-Abelian matrices in flavor space. However, for our purposes such definition is not required: our background solution will be proportional to the unit matrix $\mathbf{1}_{N_f}$, in which case the fluctuations of the Lagrangian are unambiguous up to quadratic order. The covariant derivative of the tachyon field is defined as

$$D_M T = \partial_M T + iTA_M^L - iA_M^R T\,. \tag{2.6}$$

And the class of tachyon potentials that we will consider is

$$V_f(\lambda, TT^\dagger) = V_{f0}(\lambda)e^{-a(\lambda)TT^\dagger}\,. \tag{2.7}$$

For the vacuum solutions (with flavor independent quark mass) we will have $T = \tau(r)\mathbf{1}_{N_f}$ where $\tau(r)$ is real, so that $V_f(\lambda, TT^\dagger)$ is replaced by

$$V_f(\lambda, \tau) = V_{f0}(\lambda)e^{-a(\lambda)\tau^2}\,. \tag{2.8}$$

The other undetermined functions in the flavor action must satisfy the requirements that we discuss below.

2.3 Background solutions and phase diagram

Agreement with the dynamics of QCD both in the IR and UV sets requirements on the undetermined functions appearing in the action of V-QCD. In the UV, agreement with the two-loop QCD beta-function and the one-loop anomalous dimension of the quark mass restricts the asymptotics of these potentials, [3]. Moreover, $V_g(\lambda)$ has already been fixed from glue dynamics [25], and the other undetermined functions in the flavor action ($V_{f0}(\lambda)$, $\kappa(\lambda)$, $a(\lambda)$, $w(\lambda)$) must satisfy the following generic requirements:

- There should be two extrema in the potential for τ: an unstable maximum at $\tau = 0$ (with chiral symmetry intact) and a minimum at $\tau = \infty$ (with chiral symmetry broken).
- The dilaton potential at $\tau = 0$, namely $V_{\rm eff}(\lambda) = V_g(\lambda) - xV_{f0}(\lambda)$, must have a nontrivial IR extremum at $\lambda = \lambda_*(x)$ that moves from $\lambda_* = 0$ at $x = 11/2$ to large values as x is lowered.

Notice that the Ansatz (2.7) for $V_f(\lambda,\tau)$ automatically satisfies the first requirement. On the other hand, the second requirement is necessary for the phase diagram to have the required structure as a function of $x = N_f/N_c$. Different classes of potentials have been studied in [3, 27] where they were classified according to the IR behavior of the tachyon as type I and type II models. These are the potentials that will be used in the analysis presented in this letter. The potentials I and II in [27] are completely determined up to a constant called W_0 which controls the flavor dependence of the UV AdS scale. We refer to [27] for the explicit form and a detailed explanation of the potentials used in this analysis[1].

The vacuum (zero temperature) solutions of [3] involve a Poincaré invariant metric, no vectors, and radially dependent scalars:

$$ds^2 = e^{2A(r)}(dx_{1,3}^2 + dr^2)\,, \quad \lambda(r)\,, \quad T = \tau(r)\,\mathbf{1}_{N_f}\,. \tag{2.9}$$

The background solutions corresponding to the aforementioned potentials result in a zero temperature phase diagram that is essentially universal. As a function of $0 < x < 11/2$, the standard phase diagram at zero quark mass presents two phases separated by a phase transition at some $x = x_c \simeq 4$ (the precise value depends on the potential chosen):

- In the region $0 < x < x_c$, the (massless) theory has chiral symmetry breaking, and flows to a massless $SU(N_f)$ pion theory in the IR. The IR dynamics is thus similar to the one of ordinary QCD. The corresponding background solution has nontrivial $\lambda(r)$, $A(r)$ and $\tau(r)$, with the tachyon diverging at the IR singularity of the geometry.
- In the conformal window, i.e., when $x_c < x < 11/2$, the theory flows to a nontrivial IR fixed point and there is no chiral symmetry breaking. The background solution has zero tachyon $\tau(r)$=0 and nontrivial $\lambda(r)$ and $A(r)$, giving rise to a geometry flowing to a nontrivial AdS fixed point in the IR.

Remarkably, in the region just below the conformal window ($x \lesssim x_c$) the theory exhibits a "walking" behavior. The phase transition at $x = x_c$ (which is only present at zero quark mass)

[1]A thorough study of the potentials, taking into account the constraints arising from the meson spectra, will be performed in [23].

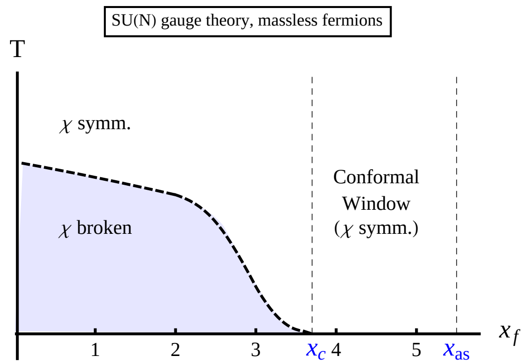

Figure 1: Qualitative behavior of the transition temperature between the low and high temperature phases of V-QCD matter [27].

involves BKT [28] or Miransky [6] scaling. Indeed, the order parameter of the transition, the chiral condensate $\sigma \sim \langle \bar{q}q \rangle$, vanishes exponentially:

$$\sigma \sim \exp\left(-\frac{2\hat{K}}{\sqrt{x_c - x}}\right) , \tag{2.10}$$

as $x \to x_c$ from below. (The constant $\hat{K}$ is positive [3]). And for $x_c \geq x$, σ is identically zero as chiral symmetry is intact. The Miransky scaling is linked to the "walking" behavior of the coupling constant: the field $\lambda(r)$ takes an approximately constant value λ_* for a wide range of r (the length of this region scales as the square root of the condensate in (2.10)). Hence, the coupling stays approximately constant for many decades in the RG time, until the deep IR where the non-zero tachyon drives the theory away from the nontrivial fixed point and towards $\lambda = \infty$.

At finite temperature a rich structure of black holes, with one or two scalar hairs, was found in [27]. The general structure of their phase diagram is depicted in figure 1. The chiral restoration transition is first order at low values of x, but typically becomes second order as we approach x_c. When this happens, a separate first order deconfinement transition still exists, so that an extra chirally broken phase appears for temperatures between the two transitions. There can also be two first-order transitions, depending on the details of the potentials. The transition temperatures obey Miransky scaling [27].

3. Quadratic fluctuations and spectra

In this section we present some results for the spectra of mesons and glueballs of V-QCD, restricting to the case of zero quark mass.

In order to compute the spectrum of mesons and glueballs we need to study the fluctuations of all the fields of V-QCD. In the glue sector the relevant fields are the metric g_{mn}, the dilaton ϕ and the QCD axion a (which belongs to the CP-odd sector). Their normalizable fluctuations correspond to glueballs with $J^{PC} = 0^{++}$, 0^{-+}, 2^{++}, where J stands for the spin and P and C for the field properties under parity and charge conjugation respectively. In the meson sector one has the tachyon T, and the gauge fields $A_\mu^{L/R}$; their normalizable fluctuations corresponding to mesons with $J^{PC} = 1^{++}$, 1^{--}, 0^{++}, 0^{-+}.

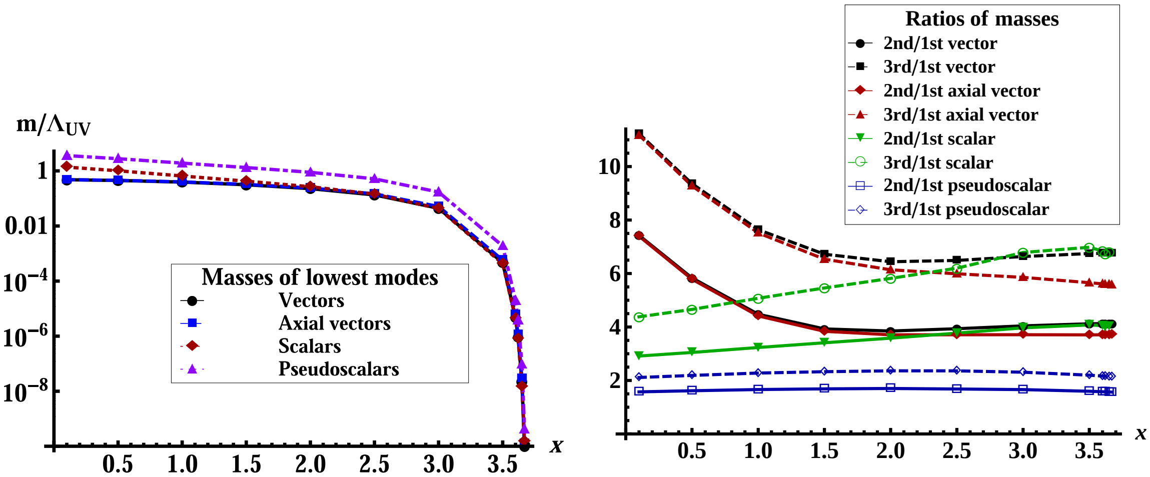

Figure 2: Nonsinglet meson spectra in the potential II class with Stefan-Boltzmann (SB) normalization for W_0 (see [27]), with $x_c \simeq 3.7001$. Left: the lowest non-zero masses of all four towers of mesons, as a function of x, in units of $\Lambda_{\rm UV}$, below the conformal window. Right, the ratios of masses of up to the fourth massive states in the same theory as a function of x.

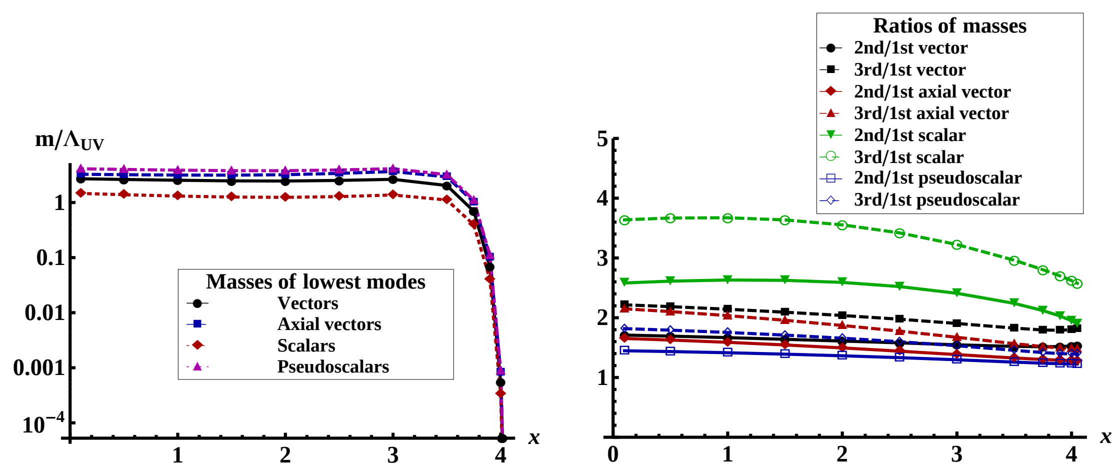

Figure 3: Nonsinglet meson spectra in the potential I class ($W_0 = 3/11$), with $x_c \simeq 4.0830$. Left: the lowest non-zero masses of all four towers of mesons, as a function of x, in units of $\Lambda_{\rm UV}$, below the conformal window. Right, the ratios of masses of up to the fourth massive states in the same theory as a function of x.

The fluctuations fall into two classes according to their transformation properties under the flavor group: flavor nonsinglet (transforming in the adjoint representation of $SU(N_f)$) modes and flavor singlet modes. The glue sector contains only flavor singlet modes, whereas each fluctuation in the meson sector can be divided into flavor singlet and nonsinglet terms. Those (flavor singlet) modes which are present in both sectors will mix. Since we are in the Veneziano limit the mixing takes place at leading order in $1/N_c$: the 0^{++} glueball mixes with the 0^{++} flavor singlet σ-meson, and the pseudoscalar 0^{-+} flavor singlet meson mixes with the 0^{-+} glueball due to the axial anomaly

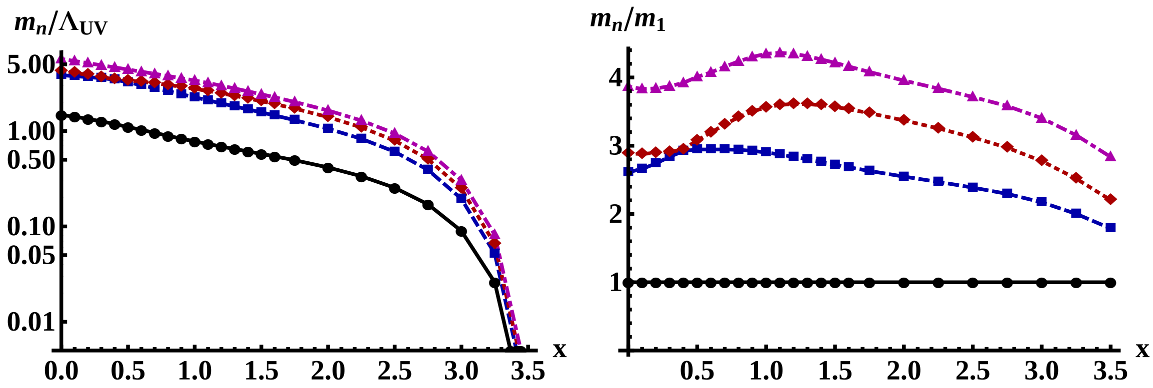

Figure 4: Singlet scalar meson spectra for the potential II class with SB normalization for W_0. They contain the 0^{++} glueballs and the singlet 0^{++} mesons that mix here at leading order. Left: the four lowest masses as a function of x in units of $\Lambda_{\rm UV}$. Right: the ratios of masses of up to the fourth massive states as a function of x.

(realized by the CP-odd sector which we will not study here, see [23]). All classes, with various J^{PC} and tranformation properties under the $U(N_f)$ group, contain an infinite discrete tower of excited states.

We start by defining the vector and axial vector combinations of the gauge fields:

$$V_M = \frac{A_M^L + A_M^R}{2}\,, \qquad A_M = \frac{A_M^L - A_M^R}{2}\,, \tag{3.1}$$

appearing both in the singlet and nonsinglet flavor sectors that we describe in the following. We work in the axial gauge with $V_r = 0 = A_r$. We can take the vector fluctuation to be transverse, $\partial^\mu V_\mu = 0$, and separate the axial vectors in transverse and longitudinal parts as $A_\mu(x^\mu, r) = A_\mu^\perp(x^\mu, r) + A_\mu^\parallel(x^\mu, r)$ with $\partial^\mu A_\mu^\perp = 0$. We also write the complex tachyon field as

$$T(x^\mu, r) = [\tau(r) + s(x^\mu, r) + \mathfrak{s}^a(x^\mu, r) t^a] \exp[i\theta(x^\mu, r) + i\pi^a(x^\mu, r) t^a]\,, \tag{3.2}$$

where t^a are the generators of $SU(N_f)$, τ is the background solution, s (θ) is the scalar (pseudoscalar) flavor singlet fluctuation, and $\mathfrak{s}^a$ (π^a) are the scalar (pseudoscalar) flavor nonsinglet fluctuations.

3.1 Nonsinglet fluctuations

The nonsinglet fluctuations include the vector and axial vector meson fluctuations (3.1), the pseudoscalar mesons (including the massless pions), and the scalar mesons. Their second order equations are relatively simple, and we present those of the vectors below. We use the standard factorized Ansatz $V_\mu(x^\mu, r) = \psi^V(r)\, \mathcal{V}_\mu(x^\mu)$. The radial wave function satisfies the following equation:

$$\frac{\partial_r \left(V_f(\lambda,\tau) w(\lambda)^2 e^A G^{-1} \partial_r \psi^V\right)}{V_f(\lambda,\tau) w(\lambda)^2 e^A G} + m_V^2 \psi^V = 0\,, \quad G \equiv \sqrt{1 + e^{-2A} \kappa(\lambda) (\partial_r \tau)^2}\,. \tag{3.3}$$

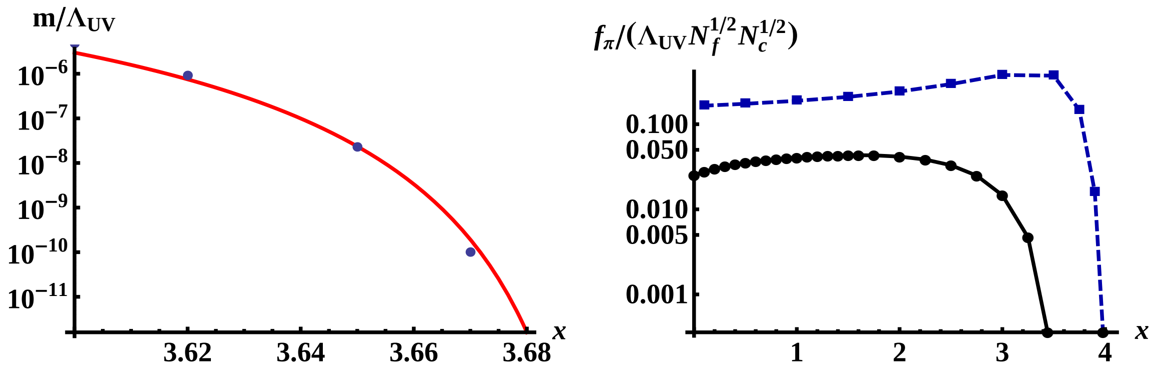

Figure 5: Left: A fit of the ρ mass to the Miransky scaling factor for Potentials II with SB normalization for W_0. Right: f_π as a function of x in units of $\Lambda_{\rm UV}\sqrt{N_c N_f}$. Again, it vanishes near x_c following Miransky scaling. The dashed blue curve is the result for potentials I with $W_0 = 3/11$, while the continuous black curve is for potentials II with SB normalized W_0.

The radial wave function for the transverse axial fluctuations can be defined by $A_\mu^\perp(x^\mu, r) = \psi^A(r)\,\mathscr{A}_\mu(x^\mu)$, and it satisfies

$$\frac{\partial_r\left(V_f(\lambda,\tau)w(\lambda)^2 e^A G^{-1}\,\partial_r\psi^A\right)}{V_f(\lambda,\tau)w(\lambda)^2 e^A G} - \frac{4\tau^2 e^{2A}\kappa(\lambda)}{w(\lambda)^2}\psi^A + m_A^2\psi^A = 0\,. \tag{3.4}$$

The nonsinglet scalar and pseudoscalar fluctuation equations are more complicated and we will present them in [23].

The general behavior of the spectra in the nonsinglet sector is as follows:

- In the conformal window all spectra are continuous.
- Below the conformal window, $x < x_c$, the spectra are discrete and gapped. The only exception being the $SU(N_f)$ pseudoscalar pions that are massless due to chiral symmetry breaking.

In the "walking regime", i.e., $x_c - x \ll 1$, we find the following specific features:

- All masses obey Miransky scaling: $m_n \sim \Lambda_{\rm UV}\exp(-\frac{\kappa}{\sqrt{x_c - x}})$. This is explicitly seen in the case of the ρ mass in figure 5 left.
- All nonsinglet mass ratios asymptote to non-zero constants as $x \to x_c$.

We present the results of our numerical analysis of the nonsinglet meson spectra in figures 2 and 3 (note that the plots on the left of those figures are in logarithmic scale). These results reflect the properties of the spectra listed above. The lowest masses of the mesons vary little with x until we reach the walking region. There, Miransky scaling takes over and the lowest masses dip down exponentially fast. The Λ_{UV} scale is extracted as usual from the logarithmic running of λ in the UV.

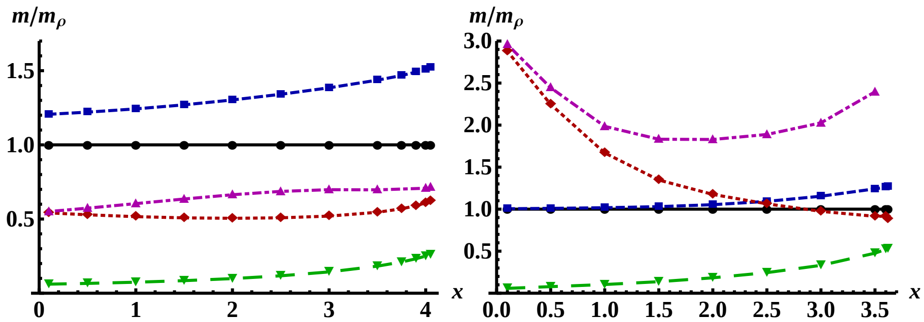

Figure 6: The masses of the lightest states of various towers, and $f_\pi/\sqrt{N_f N_c}$ as a function of x in units of the ρ mass. Left: potentials I with $W_0 = 3/11$. Right: potentials II with SB normalization for W_0. Solid black, dashed blue, dotted red, and dotdashed magenta curves show the masses of the lightest vector, axial, flavor nonsinglet scalar, and flavor singlet scalar states, respectively, while the long-dashed green curve is $f_\pi/\sqrt{N_f N_c}$ (see section 4).

3.2 Singlet fluctuations

The singlet fluctuations consist of the 2^{++} glueballs, the 0^{++} glueballs and scalar mesons that mix to leading order in $1/N$ in the Veneziano limit, the 0^{-+} glueballs, and the η' pseudoscalar tower. The spin-two fluctuation equations are simple and can be summarized by the appropriate Laplacian (see for instance [29]). The scalar and pseudoscalar equations are, however, very involved and will be presented in detail in [23]. Here we show results of the numerical analysis for the 0^{++} singlet scalars in figure 4.

The general properties of the singlet spectra are similar as in the nonsinglet sector: below the conformal window, $x < x_c$, the singlet spectra are discrete and gapped, and there is again Miransky scaling as $x \to x_c$ from below (see figure 4).

There are some specific properties related to the mixing of the glueball and meson states:

- The $U(1)_A$ anomaly appears at leading order in the Veneziano limit, and consequently the mixture of the 0^{-+} glueball and the η' has a mass of $\mathcal{O}(1)$.

- In the scalar sector, for small x, where the mixing between glueballs and mesons is small, the lightest state is a meson, the next lightest state is a glueball, the next a meson and so on. However, with increasing x, nontrivial mixing sets in and level-crossing seems to be generic. This can be seen in the right hand plot of figure 4.

All singlet mass ratios asymptote to constants as $x \to x_c$. The same holds for mass ratios between the flavor singlet and nonsinglet sectors, as confirmed numerically in figure 6. There seems to be no unusually light state (termed the "dilaton") that reflects the nearly broken scale invariance in the walking region. The reason is a posteriori simple: the nearly broken scale invariance is reflected in the *whole* spectrum of bound states scaling exponentially to zero due to Miransky scaling.

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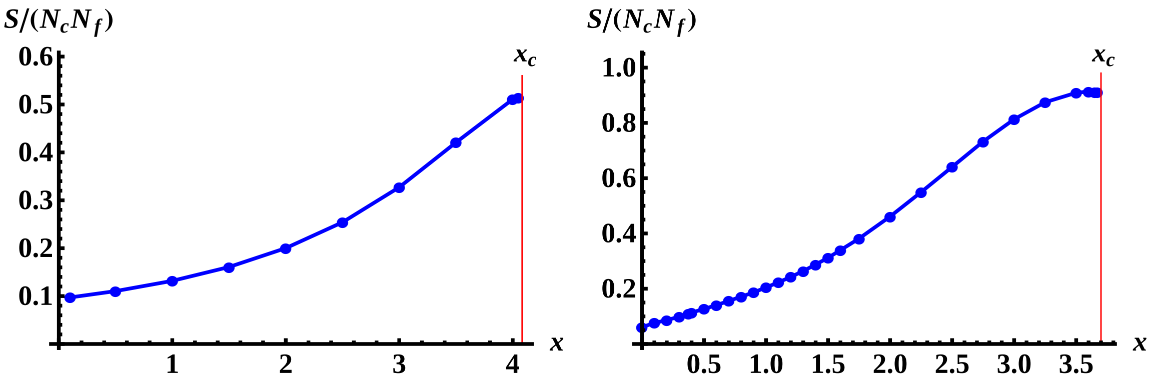

Figure 7: Left: The S-parameter as a function of x for potential class I with $W_0 = 3/11$. Right: The S-parameter as a function of x for potential class II with SB normalization for W_0. In both cases S asymptotes to a finite value as $x \to x_c$.

3.3 Asymptotics of the spectra

The asymptotics of the spectra at high masses is in general a power-law with logarithmic corrections, with the powers depending on the potentials. The trajectories are approximately linear ($m_n^2 \sim cn$) for type I potentials and quadratic ($m_n^2 \sim cn^2$) for type II potentials. There is the possibility, first seen in [22] that the proportionality coefficient c in the linear case is different between axial and vector mesons[2]. These possibilities do not affect substantially the issues of the dilaton and the S-parameter.

Finally, let us comment on the possibility of using as background the nontrivial saddle points, found in [3], where the tachyon solution has at least one zero (analogous to the Efimov minima). We have verified explicitly that such saddle points are unstable, as the scalar meson equation has a single mode with a negative mass squared, both in the singlet and nonsinglet channels. This mass is small for small x, but becomes large as $x \to x_c$ [16]. Therefore, the Efimov minima are strongly unstable in the walking regime.

4. Two-point functions and the S-parameter

We have computed the two-point functions of several operators including the axial and vector currents as well as the scalar mass operator. We will focus here on the two-point functions of the vector and axial currents which can be written in momentum space as

$$\langle V_\mu^a(q) V_\nu^b(p)\rangle = \Pi_{\mu\nu,V}^{ab}(q,p) = -(2\pi)^4 \delta^4(p+q)\, \delta^{ab} \left(q^2 \eta_{\mu\nu} - q_\mu q_\nu\right) \Pi_V(q)\,, \tag{4.1}$$

and similarly for the axial vector. Here we have used that

$$V_\mu(x) = \int \frac{d^4 q}{(2\pi)^4}\, e^{iqx}\, V_\mu^a(q)\, t^a\, \psi_V(r)\,, \tag{4.2}$$

where t^a, $a = 1, \ldots, N_f^2 - 1$ are the flavor group generators.

[2]A careful analysis of the effects of different potentials on the asymptotics of the spectra will be presented in [23].

Next, using the expansions

$$\Pi_A = \frac{f_\pi^2}{q^2} + \sum_n \frac{f_n^2}{q^2 + m_n^2 - i\varepsilon}\,, \qquad \Pi_V = \sum_n \frac{F_n^2}{q^2 + M_n^2 - i\varepsilon}\,, \tag{4.3}$$

we determine f_π as

$$f_\pi^2 = -\frac{N_c N_f}{12\pi^2}\, \frac{\partial_r \psi^A}{r}\bigg|_{r=0,\,q=0}\,, \tag{4.4}$$

where the normalization is fixed by matching the UV limit of the two point functions to QCD. Here the normalization of the radial wave function was fixed in the UV by $\psi^A(r=0)=1$, and it is required to be normalizable in the IR.

Typical results for f_π are plotted in figure 5, right. Notice that the pion scale changes smoothly for most of the range of x, but as $x \to x_c$ Miransky scaling sets in such that it vanishes exponentially.

Finally, we shall now compute the S-parameter for V-QCD. It is given by:

$$\begin{aligned} S = 4\pi \frac{d}{dq^2}\left[q^2(\Pi_V - \Pi_A)\right]_{q=0} &= -\frac{N_c N_f}{3\pi}\frac{d}{dq^2}\left(\frac{\partial_r \psi^V(r)}{r} - \frac{\partial_r \psi^A(r)}{r}\right)\bigg|_{r=0,\,q=0} \\ &= 4\pi \sum_n \left(\frac{F_n^2}{M_n^2} - \frac{f_n^2}{m_n^2}\right)\,. \end{aligned} \tag{4.5}$$

As both masses and decay constants in (4.5) obey Miransky scaling, the S-parameter is insensitive to it. Therefore subleading terms determine its scaling behavior as $x \to x_c$. Our results show that generically the S-parameter (in units of $N_f N_c$) remains finite in the QCD regime, $0 < x < x_c$, and asymptotes to a finite constant at x_c (see figure 7). The S-parameter is identically zero inside the conformal window (massless quarks) because of unbroken chiral symmetry. This suggests a subtle discontinuity of correlators across the conformal transition, which will be analyzed in detail in [23]. In [19] similar conclusions are reached in a different context (probe tachyon-flavor dynamics in AdS). We find that in the walking region of V-QCD the backreaction of flavor to matter (that is fully implemented here) is important, among other things, for the spectra, and therefore the two results are not directly comparable.

This behavior of S is in qualitative agreement with recent estimates based on analysis of the BZ limit in field theory [30]. We have also found choices of potentials where the S-parameter becomes very large as we approach x_c. Our most important result is that generically the S-parameter is an increasing function of x, and reaches its highest value at or near x_c contrary to previous expectations [30].

5. Conclusions

In this letter we have studied the zero temperature spectra of glueballs and mesons in a class of holographic theories (V-QCD) that is in the universality class of QCD in the Veneziano limit. This model takes into account the backreaction of the flavor degrees of freedom and therefore allows us to analyze the spectra as a function of $x = N_f/N_c$.

V-QCD was formulated in [3] where the zero temperature phase diagram was studied and shown to agree with expectations for QCD in the Veneziano limit. It is for these solutions that we have computed the spectra of fluctuations. We have found that the main features of the spectra are shared by various choices of the potentials, although some important issues like the large mass asymptotics do depend on their specific details. A thorough study of the spectra for a wider class of potentials is underway and we expect to report its results in the near future [23]. The study presented here has nevertheless allowed us to outline the main characteristics of the spectra of fluctuations and also to address issues relevant for the use of this setup as a holographic model of walking technicolor. We will now summarize our main results.

In the conformal window, $x_c < x < 11/2$ where the theory flows to an IR nontrivial fixed point, all the spectra are continuous. For $x < x_c$ the massless theory spontaneously breaks chiral symmetry in the IR. The spectra in this region are discrete and gapped (except for the pions). For $x \lesssim x_c$ the model is in its "walking region", where the coupling stays almost constant for many decades of energy and it only diverges in the deep IR. As for the spectra, in the "walking region" all masses obey Miransky scaling: $m_n \sim \Lambda_{\rm UV} \exp(-\frac{\kappa}{\sqrt{x_c - x}})$, and the same applies to other mass parameters like f_π.

Remarkably, as $x \to x_c$ all flavor singlet and nonsinglet mass ratios asymptote to non-zero constants. Therefore, there is no unusually light state (also called "dilaton") in the spectrum. Such a state was expected as a consequence of the approximate conformal symmetry in the "walking region". In this scenario this approximate conformal symmetry is instead correlated with Miransky scaling of all masses. One should also notice that for finite values of x there is strong mixing between singlet mesons and glueballs, and occasional level crossing as x is varied.

Finally, by computing the two-point functions of vector and axial vector mesons, we have been able to determine the S-parameter for our setup. In units of $N_f N_c$ the S-parameter is generically of $\mathscr{O}(1)$. Additionally, it is an increasing function of x and asymptotes to a finite constant as $x \to x_c$. Inside the conformal window the S-parameter is identically zero and therefore it is discontinuous at the conformal transition (at $x = x_c$).

These results for the S-parameter suggest that making S arbitrarily small in a walking theory may be more difficult than expected before. Moreover, our results indicate that this is probably not the case for QCD in the Veneziano limit.

Acknowledgments

We thank the organizers of the "XVIII European Workshop on String Theory" held at the Corfu Summer Institute for the oportunity to present our work there.

This work was in part supported by grants PERG07-GA-2010-268246, PIEF-GA-2011-300984, the EU program "Thales" ESF/NSRF 2007-2013, and by the European Science Foundation "Holograv" (Holographic methods for strongly coupled systems) network. It has also been co-financed by the European Union (European Social Fund, ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) under "Funding of proposals that have received a positive evaluation in the 3rd and 4th Call of ERC Grant Schemes". D.A. would like to thank the Crete Center for Theoretical Physics for hospitality and the FRont Of pro-Galician Scientists for unconditional support. I. Iatrakis' work was supported by the project "HERAKLEITOS II - University of Crete" of the Operational

Programme for Education and Lifelong Learning 2007 - 2013 (E.P.E.D.V.M.) of the NSRF (2007 - 2013), which is co-funded by the European Union (European Social Fund) and National Resources.

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