On the pair correlations of neutral $K$, $D$, $B$ and $B_s$ mesons with close momenta produced in inclusive multiparticle processes

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The phenomenological structure of inclusive cross-sections of the production of two neutral $K$ mesons in collisions of hadrons and nuclei is investigated taking into account the strangeness conservation in strong and electromagnetic interactions. Relations describing the dependence of the correlations of two short-lived and two long-lived neutral kaons $K_S^0 K_S^0$, $K_L^0 K_L^0$ and the correlations of "mixed" pairs $K_S^0 K_L^0$ at small relative momenta upon the space-time parameters of the generation region of $K^0$ and $\bar{K}^0$ mesons have been obtained. It is shown that under the strangeness conservation the correlation functions of the pairs $K_S^0 K_S^0$ and $K_L^0 K_L^0$, produced in the same inclusive process, coincide, and the difference between the correlation functions of the pairs $K_S^0 K_S^0$ and $K_L^0 K_L^0$ is conditioned exclusively by the production of the pairs of non-identical neutral kaons $K^0 \bar{K}^0$. Analogous correlations for the pairs of neutral heavy mesons $D^0$, $B^0$ and $B_s^0$, generated in multiple processes with the charm (beauty) conservation, are analyzed, and differences from the case of neutral $K$ mesons are discussed.
1. Consequences of the strangeness conservation for neutral kaons

In the work [1] the properties of the density matrix of two neutral $K$ mesons, following from the strangeness conservation in strong and electromagnetic interactions, have been investigated. By definition, the diagonal elements of the non-normalized two-particle density matrix coincide with the two-particle structure functions, which are proportional to the double inclusive cross-sections.

Due to the strangeness conservation, the pairs of neutral kaons $K^0K^0$, $\bar{K}^0\bar{K}^0$ and $K^0\bar{K}^0$ are produced incoherently, i.e. the non-diagonal elements of the two-kaon density matrix between the respective states are equal to zero. However, the non-diagonal elements of the density matrix between the two states $|K^0\rangle(\mathbf{p}_1)|K^0\rangle(\mathbf{p}_2)$ and $|\bar{K}^0\rangle(\mathbf{p}_1)|\bar{K}^0\rangle(\mathbf{p}_2)$ with the zero strangeness ( $\mathbf{p}_1$ and $\mathbf{p}_2$ are the kaon momenta ) are not equal to zero, in general.

The internal states of $K^0$ meson (strangeness $S = 1$) and $\bar{K}^0$ meson ($S = -1$) have the form: $|K^0\rangle = \frac{1}{\sqrt{2}} (|K^0_S\rangle + |K^0_L\rangle)$, $|\bar{K}^0\rangle = \frac{1}{\sqrt{2}} (|K^0_S\rangle - |K^0_L\rangle)$, where $K^0_S$ is the short-lived neutral kaon ( $CP$-even state ) and $K^0_L$ is the long-lived one ( $CP$-odd state ) – neglecting the small effect of $CP$ non-invariance. Meantime, the $CP$ parity of the system $K^0\bar{K}^0$ is always positive [2].

The system $K^0\bar{K}^0$ in the symmetric internal state ( even orbital momenta ) is decomposed into the schemes $|K^0_S\rangle|K^0_S\rangle$ and $|K^0_L\rangle|K^0_L\rangle$, and in the antisymmetric internal state ( odd orbital momenta ) – into the scheme $|K^0_S\rangle|K^0_L\rangle$ [2].

The strangeness conservation leads to the fact that all the double inclusive cross-sections of production of pairs $K^0_S K^0_S$, $K^0_L K^0_L$ and $K^0_S K^0_L$ (two-particle structure functions) prove to be invariant with respect to the replacement of the short-lived state $K^0_S$ by the long-lived state $K^0_L$, and vice versa [1]: $f_{SS}(\mathbf{p}_1,\mathbf{p}_2) = f_{LL}(\mathbf{p}_1,\mathbf{p}_2)$, $f_{SL}(\mathbf{p}_1,\mathbf{p}_2) = f_{LS}(\mathbf{p}_1,\mathbf{p}_2)$. In doing so, we have:

$f_{SS}(\mathbf{p}_1,\mathbf{p}_2) - f_{SL}(\mathbf{p}_1,\mathbf{p}_2) = \text{Re} \rho_{K^0\bar{K}^0} K^0\bar{K}^0(\mathbf{p}_1,\mathbf{p}_2)$, where $\rho_{K^0\bar{K}^0} K^0\bar{K}^0(\mathbf{p}_1,\mathbf{p}_2)$ is the non-zero non-diagonal element of the two-kaon density matrix.

2. Pair correlations of identical and non-identical neutral kaons with close momenta

Now let us consider, within the model of one-particle sources [2-7], the correlations of pairs of neutral $K$ mesons with close momenta ( see also [8-10] ). In the case of the identical states $K^0_S K^0_S$ and $K^0_L K^0_L$ we obtain the following expressions for the correlation functions $R_{SS}$, $R_{LL}$ (proportional to the structure functions), normalized to unity at large relative momenta:

\[
R_{SS}(\mathbf{k}) = R_{LL}(\mathbf{k}) = \lambda_{K^0K^0} [1 + F_{K^0}(2\mathbf{k}) + 2 b_{int}(\mathbf{k})] + \\
+ \lambda_{\bar{K}^0\bar{K}^0} \left[ 1 + F_{\bar{K}^0}(2\mathbf{k}) + 2 \tilde{b}_{int}(\mathbf{k}) \right] + \lambda_{K^0\bar{K}^0} [1 + F_{K^0\bar{K}^0}(2\mathbf{k}) + 2 B_{int}(\mathbf{k})].
\] (2.1)

Here $\mathbf{k}$ is the momentum of one of the kaons in the c.m. frame of the pair, and the quantities $\lambda_{K^0K^0}$, $\lambda_{\bar{K}^0\bar{K}^0}$ and $\lambda_{K^0\bar{K}^0}$ are the relative fractions of the average numbers of produced pairs $K^0K^0$, $\bar{K}^0\bar{K}^0$ and $K^0\bar{K}^0$, respectively ($\lambda_{K^0K^0} + \lambda_{\bar{K}^0\bar{K}^0} + \lambda_{K^0\bar{K}^0} = 1$). The “formfactors” $F_{K^0}(2\mathbf{k})$, $F_{\bar{K}^0}(2\mathbf{k})$ and $F_{K^0\bar{K}^0}(2\mathbf{k})$ appear due to the contribution of Bose statistics, and they are determined by averaging $\cos(2\mathbf{k} \mathbf{r})$ over the probability distributions of distances between the sources of emission of two $K^0$ mesons, two $\bar{K}^0$ mesons and the $K^0$ and $\bar{K}^0$ mesons, respectively, in the c.m. frame of the kaon pair – $W_{K^0}(\mathbf{r})$, $W_{\bar{K}^0}(\mathbf{r})$ and $W_{K^0\bar{K}^0}(\mathbf{r})$. Meantime, the quantities $b_{int}(\mathbf{k})$, $\tilde{b}_{int}(\mathbf{k})$ and $B_{int}(\mathbf{k})$ describe the respective contributions of the $S$-wave interaction of two $K^0$ mesons, two
\(\bar{K}^0\) mesons and of the \(K^0\) and \(\bar{K}^0\) mesons. Due to the \(CP\) invariance, the quantities \(b_{\text{int}}(k)\) and \(\tilde{b}_{\text{int}}(k)\) have the structure: \(b_{\text{int}}(k) = \int W_{K^0}(r)b(k,r)d^3r\), \(\tilde{b}_{\text{int}}(k) = \int W_{\bar{K}^0}(r)b(k,r)d^3r\) (the function \(b(k,r)\) is the same). But: \(B_{\text{int}}(k) = \int W_{K^0}(r)b(k,r)d^3r\), where \(B(k,r) \neq b(k,r)\). The function \(B(k,r)\) is expressed through the parameters of low-energy \(K^0\bar{K}^0\) scattering [10].

Let us emphasize that when the pair \(K^0\bar{K}^0\) is produced but the pair of identical quasistationary states \(K^0_sK^0_s\) (or \(K^0_LK^0_L\)) is registered over decays, the two-particle correlations at small relative momenta have the same character as in the case of usual identical bosons with zero spin [2].

For the pairs of non-identical kaon states \(K^0_sK^0_L\) the correlation functions have the form:

\[
R_{SL}(k) = R_{LS}(k) = \lambda_{K^0\bar{K}^0}\left[1 + F_{K^0}(2k) + 2b_{\text{int}}(k)\right] + \lambda_{K^0\bar{K}^0}\left[1 + F_{\bar{K}^0}(2k) + 2\tilde{b}_{\text{int}}(k)\right] + \lambda_{K^0\bar{K}^0}\left[1 - F_{K^0\bar{K}^0}(2k)\right].
\]

(2.2)

In accordance with Eqs. (2.1) and (2.2), the following relation holds:

\[
R_{SS}(k) - R_{SL}(k) = 2\lambda_{K^0\bar{K}^0}\left[F_{K^0\bar{K}^0}(2k) + B_{\text{int}}(k)\right].
\]

(2.3)

So, the difference between the correlation functions of the pairs of identical and non-identical neutral kaons \(K^0_sK^0_s, K^0_sK^0_L\) is conditioned exclusively by the generation of \(K^0\bar{K}^0\)-pairs.

3. Correlations of neutral heavy mesons

Formally, analogous relations are valid also for the neutral heavy mesons \(D^0, B^0\) and \(B_s^0\). Here the role of strangeness conservation is played, respectively, by the conservation of charm and beauty in inclusive multiple processes with production of these mesons, and the quasistationary states are also states with definite \(CP\) parity (neglecting the effects of \(CP\) nonconservation), e.g., \(|B_s^0\rangle = \frac{1}{\sqrt{2}}(|B^0\rangle + |\bar{B}^0\rangle)\), \(CP\) parity \((+1)\); \(|B_s^0\rangle = \frac{1}{\sqrt{2}}(|B^0\rangle - |\bar{B}^0\rangle)\), \(CP\) parity \((-1)\).

The difference of masses between the respective \(CP\)-odd and \(CP\)-even states is very insignificant in all the cases, ranging from \(10^{-12}\) MeV for \(K^0\) mesons up to \(10^{-8}\) MeV for \(B_s^0\) mesons. Concerning the lifetimes of these states, for \(K^0\) mesons they differ by 600 times, but for \(D^0, B^0\) and \(B_s^0\) mesons the respective lifetimes are almost the same. Thus, it is practically impossible to distinguish the states of \(D^0, B^0\) and \(B_s^0\) mesons with definite \(CP\) parity by the difference in their lifetimes. These states, in principle, can be identified through the purely \(CP\)-even and \(CP\)-odd decay channels; however, in fact the branching ratio for such decays is very small. For example, \(Br(D^0 \to \pi^+\pi^-) = 1.62 \times 10^{-3}\) \((CP = +1)\); \(Br(D^0 \to K^+K^-) = 4.25 \times 10^{-3}\) \((CP = +1)\); \(Br(B^0 \to J/\Psi \pi^0) < 1.2 \times 10^{-3}\) \((CP = +1)\); \(Br(B^0 \to J/\Psi K^0_s) = 9 \times 10^{-4}\) \((CP = -1)\).

Just as for neutral \(K\) mesons, the correlation functions for the pairs of states of neutral \(D, B\) and \(B_s\) mesons with the same \(CP\) parity \((R_{SS} = R_{LL})\) and for the pairs of states with different \(CP\) parity \((R_{SL})\) do not coincide, and the difference between them is conditioned exclusively by the production of pairs \(D^0\bar{D}^0, B_s^0\bar{B}_s^0\) and \(B_s^0\bar{B}_s^0\), respectively; e.g., for \(B_s^0\) mesons we have:

\[
R_{SS}(k) - R_{SL}(k) = 2\lambda_{B_s^0\bar{B}_s^0}\left[F_{B_s^0\bar{B}_s^0}(2k) + B_{\text{int}}(k)\right];
\]

(3.1)

here \(\lambda_{B_s^0\bar{B}_s^0}\) is the relative fraction of generated pairs \(B_s^0\bar{B}_s^0\), and the contributions of Bose statistics \(F_{B_s^0\bar{B}_s^0}(2k)\) and the \(S\)-wave \(B_s^0\bar{B}_s^0\)-interaction \(B_{\text{int}}(k)\) are determined analogously to the case of \(K^0\bar{K}^0\)-pairs (see Section 2).
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References