Minimal Adjoint-$SU(5) \times Z_4$ GUT Model

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We review the minimal adjoint $SU(5) \times Z_4$ model. The aim for the flavour discrete symmetry $Z_4$ is to force the up- and down-quark mass matrices to have the Nearest-Neighbour-Interaction form, upon spontaneous breaking chain of the adjoint-$SU(5)$ GUT group. Consequently the charged lepton mass matrix also gets the same form. Three copies of adjoint fermionic 24 fields are needed in order to account for neutrino masses. Thus, the light neutrinos get their masses through type-I, type-III and one-loop radiative seesaw mechanisms, implemented, respectively, via a singlet, a triplet and an octet from the adjoint fermionic 24 fields. The symmetry $SU(5) \times Z_4$ allows only two viable zero textures for the effective neutrino mass matrix, with one texture only compatible with normal hierarchy and the second with inverted hierarchy in the light neutrino mass spectrum. Finally, $SU(5) \times Z_4$ freezes out the possibility of proton decay through exchange of coloured Higgs triplets at tree-level.

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The Grand Unified Theories (GUT) [1] are well motivated Physics beyond the Standard Model (SM) and provide an appealing framework for the search for the theory of flavour. This is sustained by the one-loop evolution of the SM gauge couplings that rather unify around $10^{15-17}$ GeV. One gets also the unification of the SM fermions in a few multiplets, possibiliting the implementation of a flavour symmetry and leading to the proton decay, not yet observed.

In ref. [2] a discrete flavour symmetry $Z_4$ was constructed within a small extension of the adjoint-SU(5) model [3],following ref. [4]. The purpose of $Z_4$ is to force the up- and down-quark mass matrices, $M_u$ and $M_d$ to have Nearest-Neighbour-Interaction (NNI) form, i.e.

$$M_u = \begin{pmatrix} 0 & A_u & 0 \\ A_u' & 0 & B_u \\ 0 & B_u' & C_u \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & A_d & 0 \\ A_d' & 0 & B_d \\ 0 & B_d' & C_d \end{pmatrix}. \quad (1)$$

This parallel structure is an exact weak basis in the context of pure SM and therefore no physical predictions can be made. Instead, in the adjoint-SU(5) assuming NNI form for $M_u$ and $M_d$ leads now to new physical implications.

The adjoint-SU(5) $\times Z_4$ model [2] contains three generations of $5^*$ and 10 fermionic multiplets to include the SM fermions. To generate for light neutrino masses and mixing, three adjoint fermionic fields $\rho(24)$ are added to account for the neutrino data. One gets then three non-vanishing light neutrino masses arising from three different seesaw mechanisms [5]. An adjoint multiplet, $\Sigma(24)$ to break spontaneously the GUT gauge group down to the SM group is used. A quintet $5_H$ and a 45-dimensional representation $45_H$ further break the SM group to SU(3)$_C \times U(1)_{e.m.}$. The 45$_H$ explains the mismatch between the down-type quark and charged lepton mass matrices. The masses of the doublet and the triplets components of $5_H$ and 45$_H$ should be of order $10^{-13}$ to prevent rapid proton decay - the doublet-triplet-splitting problem.

The Gauge coupling unification at one-loop level can be achieved [2] within a large range of the parameter space compatible with the proton lifetime limits, implying that the unification scale must be greater than $(4.9 - 5.7) \times 10^{15}$ GeV.

The most general $Z_4$-charges that lead to NNI for the quark mass matrices $M_u, M_d$ are

$$\mathcal{D}(10_j) = (3q_3 + \phi, -q_3 - \phi, q_3), \quad \mathcal{D}(5^*_j) = (q_3 + 2\phi, -3q_3, -q_3 + \phi), \quad \mathcal{D}(5_H) = -2q_3 \quad (2)$$

where $\phi \equiv \mathcal{D}(45_H)$ and $q_3 \equiv \mathcal{D}(10_3)$. In addition one must verify that the bilinears $10, 10_j$ and $10, 5^*_j$:

$$\begin{pmatrix} 6q_3 + 2\phi & 2q_3 & 4q_3 + \phi \\ 2q_3 & -2\phi - 2q_3 & -\phi \\ 4q_3 + \phi & -\phi & 2q_3 \end{pmatrix} \begin{pmatrix} 4q_3 + 3\phi & \phi & 2\phi + 2q_3 \\ \phi & -4q_3 - \phi & -2q_3 \\ 2\phi + 2q_3 & -2q_3 & \phi \end{pmatrix}, \quad (3)$$

do not get contributions in the vanishing entries of the NNI form. The charges in eq. (2) are incompatible with $Z_2$ or $Z_3$ and therefore $Z_4$ is the minimal group. The charges assignement also imply the charged lepton mass matrix to be NNI and proton decay via the representations $(3, 1, -1/3)$ from the $5_H$ and $(3, 1, -1/3), (3^*, 1, 4/3), (3, 3, -1/3)$ from $45_H$ is freezed [2].

Also terms involving simultaneously the fields $5_H, \Sigma$ and $45_H$ are forbidden. This gives rise to an accidental global continuous symmetry in the scalar potential, which upon spontaneous elec-
troweak symmetry breaking would lead to a massless Nambu-Goldstone boson at tree-level. A soft-
breaking term as \((5^* H 45_H \Sigma + \text{h.c.})\), the inclusion of extra a singlet field \(S_0\), \((5^* H 45_H \Sigma S_0 + \text{h.c.})\), or a \(75\)-dimensional scalar field \(S_{75}\), \((5^* H 45_H S_{75} + \text{h.c.})\) can cure such problem in the potential.

Finally, the charges for the \(\rho_{24}\) are left free and only some combination lead to realistic neu-
trino masses and mixing. Only two possible textures are allowed for the light neutrino mass matrix \(m_\nu\), which coincide with the matrices found in ref. [4].

\[
\begin{align*}
m_\nu & \mid_{\mathcal{L}(5_H)=2} = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix} \\
& \quad \text{and} \\
m_\nu & \mid_{\mathcal{L}(5_H)=0} = \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}.
\end{align*}
\]  

As reported in ref. [4], the first texture is compatible only with normal hierarchy (NH) while the second texture turns to be compatible only with inverted hierarchy (IH) in the light neutrino mass spectrum. The allowed ranges for the lightest neutrino masses are \(m_1 = [0.22, 2.2] \times 10^{-3}\) eV for NH and \(m_3 = [2.4, 16] \times 10^{-3}\) eV for IH. The presence of a massless neutrino as well as a quasi-
degenerate neutrino mass spectrum are excluded. This is particularly important since the neutrino
mass spectrum predicted can be used to prove or disprove the model in the near future.

The effective Majorana neutrino mass \(m_{ee} \equiv |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2|\) characterising the
neutrinoless double beta decay amplitude are in this model within the range \(m_{ee} = [0.83, 3.3]\) meV for NH and \(m_{ee} = [12, 26]\) meV for the IH case. The latter range will be experimentally covered
in the next years, a sensitivity of around 10-30 meV while the former range will be even tougher,
since a sensitivity of the order of 1 meV would be required, far away from the more optimistic
scenarios. Neutrino masses have an impact in the cosmic microwave background (CMB) power
spectrum, and therefore CMB observations can be used to constrain the total neutrino mass \(\sum m_\nu\).
Dependent on the model assumptions and the data samples considered, different bounds on the sum
of neutrino masses are derived. The corresponding upper bounds on the lightest neutrino mass \(m\) lie in the range between 0.06 and 0.33 eV either for normal and inverted neutrino mass orderings.

A positive signal of neutrinoless double beta decay in the next years as well as a cosmological
measurement of the sum of neutrino masses of the order of 0.1 eV would further constrain this type
of model. Therefore, future experimental improvements in the neutrino physics can be decisive for
testing the viability of the \(SU(5) \times Z_4\) model.

References