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Constraints on QCD order parameters from $\eta \to 3\pi$

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Quark condensate and pseudoscalar decay constant in the chiral limit are the principal order parameters of spontaneous chiral symmetry breaking (SBχS) in QCD. Yet their three flavor values are still only weakly constrained by analyses using experimental data. We try to obtain such constraints by statistical methods from the decay width of the $\eta \to \pi^+\pi^-\pi^0$ decay in the framework of resummed chiral perturbation theory . We rely on recent estimates of the isospin violating parameter R, which is proportional to the difference of the u and d quark masses. Alternatively, by the same methods, we try to extract information on R.

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Quark condensate and pseudoscalar decay constant in the chiral limit are the principal order parameters of spontaneous chiral symmetry breaking in QCD [1]. Yet their three flavor values are still only weakly constrained by analyses using experimental data [2, 3]. We try to obtain such constraints by statistical methods from the decay width of the $\eta \to \pi^+\pi^-\pi^0$ decay [4, 5, 6] in the framework of resummed chiral perturbation theory [2]. We rely on recent estimates of the isospin violating parameter R [7], which is proportional to the difference of the u and d quark masses. Alternatively, by the same methods, we try to extract information on R.

Our calculation closely follows the procedure outlined in [8]. In accord with the method, leading order low energy constants (LECs) are expressed in terms of convenient free parameters

$$Z = \frac{F_0^2}{F_\pi^2}\,, \;\; X = \frac{2\hat{m}\Sigma}{F_\pi^2 M_\pi^2}\,, \;\; r = \frac{m_s}{\hat{m}}\,, \;\; R = \frac{(m_s - \hat{m})}{(m_d - m_u)}\,,$$

where F_0 is the pseudoscalar decay constant in the chiral limit, Σ is the chiral condensate and $\hat{m}$=$(m_u + m_d)/2$. We fix $r = 27.8 \pm 1.0$, an averaged value extracted from lattice results [9].

We use two approaches to deal with R. In the first one we assume it to be a known quantity. We use the value $R = 37.8 \pm 3.3$, obtained from a dispersive analysis of $\eta \to 3\pi$ [7]. Alternatively, we leave R free, or more precisely, assume it to be in a wide range $R \in (20, 60)$.

At next-to-leading order, the LECs L_4-L_8 are algebraically reparametrized using chiral expansions of two point Green functions. For L_1-L_3 we use the estimate described in [10]. The $O(p^6)$ and higher order LECs, notorious for their abundance, are collected in a relatively smaller number of higher order remainders.

We use a statistical analysis based on Bayes' theorem [2]

$$P(X_i|\Gamma_{\text{exp}}) = \frac{P(\Gamma_{\text{exp}}|X_i)P(X_i)}{\int \mathrm{d}X_i\, P(\Gamma_{\text{exp}}|X_i)P(X_i)}\,,$$

where $P(X_i|\Gamma_{\text{exp}})$ is the probability density of the parameters and remainders, denoted as X_i, having a specific value given the observed experimental width $\Gamma_{\text{exp}} = 296 \pm 16\,\text{eV}$ [4]. $P(\Gamma_{\text{exp}}|X_i)$ is the known probability density of observing Γ_{exp} in an experiment under the assumption that the values of X_i are known

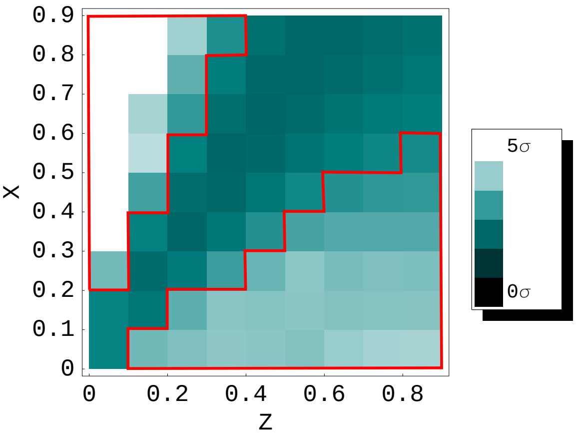
$$P(\Gamma_{\text{exp}}|X_i) = \frac{1}{\sigma_{\text{exp}}\sqrt{2\pi}} \exp\left[-\frac{(\Gamma_{\text{exp}} - \Gamma(X_i))^2}{\sigma_{\text{exp}}^2} \right].$$

$P(X_i)$ is the prior probability distribution of X_i. We use it to implement the theoretical uncertainties connected with our parameters and remainders. The treatment of remainders is based on general arguments about the convergence of the chiral series, leading to

$$G \;=\; G^{(2)} + G^{(4)} + \Delta_G^{(6)}, \quad \Delta_G^{(6)} \;\sim \pm 0.1 G,$$

where G stands for any of our 2- or 4-point Green functions, which generate the remainders. This we statistically implement as a normal probability distribution. We use Monte Carlo sampling with 10^4 samples per grid element for the integration, the total number of samples being $\sim 10^6$.

The obtained probability density distributions can be found in figures 1 and 2. As can be seen, our first results have shown that the $\eta \to \pi^+\pi^-\pi^0$ decay width is sensitive to X and Z. A large portion of the parameter space can be excluded at $>2.0\,\sigma$ C.L., given information about R. It seems $Y = X/Z \geq 1$ is preferred, therefore we have a specific test for Y in preparation.



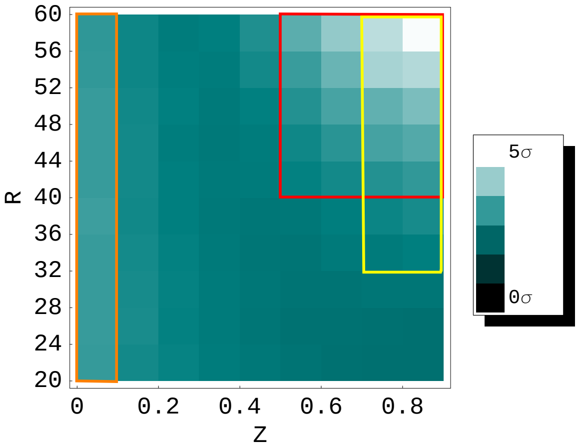


Figure 1: Probability density $P(X,Z|\Gamma_{\text{exp}})$ for $R = 37.8 \pm 3.3$
highlighted area - regions with $Y_{\text{max}} \leq 0.75$ and $Y_{\text{min}} \geq 2$ $(Y = X/Z)$: excluded at p=96.4%=2.1σ C.L.

Figure 2: Probability density $P(R,Z|\Gamma_{\text{exp}})$ for $X \in (0,0.9)$
highlighted areas - $Z>0.5$ and $R>40$: excluded at p=95.0%=2.0σ C.L.
- $Z>0.7$ and $R>32$: excluded at p=94.6%=1.9σ C.L.
- $Z<0.1$: excluded at p=97.8%=2.3σ C.L.

As expected, it's hard to constrain R without information on X and Z, we have thus obtained conditional constraints. Assuming $Z>0.5$ excludes the region $R>40$ at 2.0σ C.L. and $Z>0.7$ excludes $R>32$ at 1.9σ C.L. $Z<0.1$ can be excluded at 2.3σ C.L.

As an outlook, we work on an in depth statistical stability test of the Monte Carlo sampling and plan to extend the analysis to more parameters and include a wider range of experimental data.

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