

Discrete symmetries in quantum electrodynamics with electric and magnetic sources

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QED with magnetic monopoles gives Maxwell's equations with dual symmetry and leads to the quantization of electric charge. However the transformation of parity P and time inversion T are no longer the symmetries of theory. Also the CP transformation is broken. The symmetry is restored for PT and CPT transformations. These conclusions follow from the classical Maxwell's equations and the quantum field analysis leading to the 2-point Wightman functions in Zwanziger's model of QED.

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Quantum electrodynamics with electric and magnetic currents begins from Maxwell's equations with two kinds of external sources

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_e}{\epsilon_0}, \quad \vec{\nabla} \cdot \vec{B} = \mu_0 \rho_m, \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \mu_0 \vec{K}, \quad \vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}. \quad (1)$$

Zwanziger's model of QED [1] introduces two independent gauge potentials A_μ and B_μ , which describe the electromagnetic field tensor as

$$F^{\mu\nu} = -n^\mu (\partial_3 A^\nu - \partial^\nu A_3) + n^\nu (\partial_3 A^\mu - \partial^\mu A_3) - \epsilon^{\mu\nu 3\rho} (\partial_3 C_\rho - \partial_\rho C_3). \quad (2)$$

We make fix a space-like 4-vector $n^\mu = (0, 0, 0, 1)$, $\epsilon^{0123} = 1$ and $g_{\mu\nu} = (+, -, -, -)$. This leads to the electric and magnetic fields: $E_i = -\epsilon^{0ij3} (\partial_3 C_j - \partial_j C_3)$, $E_3 = \partial_0 A_3 - \partial_3 A_0$, $B_i = -\epsilon^{0ij3} (\partial_3 A_j - \partial_j A_3)$, $B_3 = \partial_0 C_3 - \partial_3 C_0$, with $i, j \in \{1, 2\}$. The gauge potentials A_μ and C_μ transform under gauge transformations as $A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \chi_e(x)$, $C_\mu(x) \rightarrow C_\mu(x) + \partial_\mu \chi_g(x)$. They also transform under the electromagnetic duality: $A_\mu \rightarrow C_\mu \rightarrow -A_\mu$.

We want study how Maxwell's equations (1) behave under discrete transformation of the parity, the time reversal and the charge conjugation. First, the parity transformation P acts on the spatial coordinates as $x = (\vec{r}, t) \xrightarrow{P} x^p = (-\vec{r}, t)$, which leads to

$$\rho_a(x) \xrightarrow{P} {}^p\rho_a(x) = \rho_a(x^p), \quad \vec{J}_a(x) \xrightarrow{P} {}^p\vec{J}_a(x) = -\vec{J}_a(x^p), \quad (3a)$$

$$\vec{E}(x) \xrightarrow{P} {}^p\vec{E}(x) = e_p \vec{E}(x^p), \quad \vec{B}(x) \xrightarrow{P} {}^p\vec{B}(x) = b_p \vec{B}(x^p), \quad (3b)$$

with $a \in \{e, m\}$. We observe that

$$\vec{\nabla} \times {}^p\vec{B}(x) = \mu_0 \epsilon_0 \frac{\partial}{{}^p\partial t} {}^p\vec{E}(x) + \mu_0 {}^p\vec{J}_e(x) \quad \Longrightarrow \quad b_p = -e_p = +1 \quad (4a)$$

$$\vec{\nabla} \times {}^p\vec{E}(x) = -\frac{\partial}{{}^p\partial t} {}^p\vec{B}(x) - \mu_0 {}^p\vec{K}(x) \quad \Longrightarrow \quad b_p = -e_p = -1 \quad (4b)$$

Therefore equations (1) do not have the parity symmetry. Similarly we proceed with the time reversal transformation T which is $x = (\vec{r}, t) \xrightarrow{T} x^t = (\vec{r}, -t)$. Now we have new coefficients b_t and e_t . We find that

$$\vec{\nabla} \times {}^t\vec{B}(x) = \mu_0 \epsilon_0 \frac{\partial}{{}^t\partial t} {}^t\vec{E}(x) + \mu_0 {}^t\vec{J}_e(x) \quad \Longrightarrow \quad b_t = -e_t = -1 \quad (5a)$$

$$\vec{\nabla} \times {}^t\vec{E}(x) = -\frac{\partial}{{}^t\partial t} {}^t\vec{B}(x) - \mu_0 {}^t\vec{K}(x) \quad \Longrightarrow \quad b_t = -e_t = +1. \quad (5b)$$

So we do not have time reversal symmetry either. At last we consider the charge conjugation transformation C , where electric and magnetic charges take their opposite values. In this case we find $b_c = e_c = -1$ for all Maxwell's equations, which means that the charge conjugation symmetry holds. We make check that PT and PCT transformations are the symmetries of Maxwell's equations QED with electric and magnetic sources (1) with electric and magnetic sources.

In the quantum case discrete transformations are generated by the (anti-)unitary operators: for parity we have \mathcal{P} , for time reversal - \mathcal{T} and for charge conjugation - \mathcal{C} . This leads to the transformation of gauge potentials

$$\mathcal{P}\mathcal{C}\mathcal{T}A_\mu(x)(\mathcal{P}\mathcal{C}\mathcal{T})^\dagger = -A_\mu(-x), \quad \mathcal{P}\mathcal{C}\mathcal{T}C_\mu(x)(\mathcal{P}\mathcal{C}\mathcal{T})^\dagger = -C_\mu(-x). \quad (6)$$

The vacuum state $|0\rangle$ stays invariant under PCT transformation, which is described as $\mathcal{P}\mathcal{C}\mathcal{T}|0\rangle = \langle 0|$, due to the anti-unitary operator T . This leads to the relation for the mixed Wightman function

$$\begin{aligned}\langle 0|A_\mu(x)C_\nu(0)|0\rangle &= \langle 0|\mathcal{P}\mathcal{C}\mathcal{T}A_\mu(x)C_\nu(0)(\mathcal{P}\mathcal{C}\mathcal{T})^\dagger|0\rangle = \\ &= \langle 0|C_\nu(x)A_\mu(0)|0\rangle = -\langle 0|A_\nu(x)C_\mu(0)|0\rangle,\end{aligned}\quad (7)$$

where in the last step we use the electromagnetic duality transformation. The definition (2) of $F_{\mu\nu}(x)$ in terms of gauge potentials A_μ and C_μ , after some algebraic manipulations, gives the following relation

$$\begin{aligned}(n \cdot \partial)^2 \varepsilon^{\lambda\alpha\beta\rho} \partial_\lambda \langle 0|A_\alpha(x)C_\beta(0)|0\rangle &= (n \cdot \partial) \langle 0|(n \cdot F)_\beta(x)F^{\rho\beta}(0)|0\rangle + \\ &+ n^\rho \langle 0|(n \cdot F)_\beta(x) \partial_\lambda F^{\lambda\beta}(0)|0\rangle = 2(n \cdot \partial)^2 \partial^\rho D_+(x).\end{aligned}\quad (8)$$

The last expression follows from the commutator function $\langle 0|[F_{\mu\nu}(x), F_{\lambda\rho}(0)]|0\rangle$ given in [2], where we take only the positive frequency part D_+ of the massless Jordan-Pauli function $D(x)$. Since $n^2 < 0$, then we can integrate out $(n \cdot \partial)^2$ operator and we obtain a simple equation

$$\frac{1}{2} \varepsilon^{\mu\nu\lambda\rho} \partial_\lambda \langle 0|A_\mu(x)C_\nu(0)|0\rangle = \partial^\rho D_+(x).\quad (9)$$

We can prove that there is no Lorentz covariant solutions to this equation [3]. However we can find a special solution which is spherically symmetric.

$$\langle 0|A_\mu(x)B_\nu(0)|0\rangle_0 = -\varepsilon_{\mu\nu\alpha\beta} \bar{\partial}^\alpha \partial^\beta \Delta^{-1} \star D_+(x).\quad (10)$$

where $\bar{\partial}^\mu = \partial^\mu - t^\mu \partial_0$, $t^\mu = (1, 0, 0, 0)$ and

$$\Delta^{-1}(\vec{x}) = -\frac{1}{4\pi} \frac{1}{|\vec{x}|}.\quad (11)$$

Then the general solution of (9) has the following form

$$\langle 0|A_\mu(x)B_\nu(0)|0\rangle = \langle 0|A_\mu(x)B_\nu(0)|0\rangle_0 + \partial_\mu \phi_\nu - \partial_\nu \phi_\mu.\quad (12)$$

Here ϕ_μ may depend on a gauge fixing condition and a fixed vector n_μ , so this is a general form of the mixed Wightman functions for an arbitrary gauge.

We conclude that discrete symmetries lead to the strong constrains on the allowed mixed Wightman function. Though our analysis has been limited to a free field case in (8), then we may extend it also to an interacting case by means of the spectral form of the commutator functions $\langle 0|[F_{\mu\nu}(x), F_{\lambda\rho}(0)]|0\rangle$.

References

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