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Lattice QCD - Progress and Prospects

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I review recent progress in lattice computations contributing to precision flavour physics. By presenting recent compilations of results for quantities such as quark masses, hadronic matrix elements and CKM matrix elements, I emphasise the very impressive precision which can now be reached, approaching $O(1\%)$ for some quantities. I stress that the answer to the frequently posed question "can we trust the lattice results?" is now a resounding *yes*. I illustrate progress in extending the range of physical processes which can be studied using lattice simulation by presenting recent results on non-leptonic kaon decays which are providing an emerging explanation of the long-standing $\Delta I = 1/2$ rule. Prospects for further improvement both in the precision, e.g. by including isospin breaking effects, and in the extension of simulations to new processes are also discussed.

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1.	[QCD at] Non-zero temperature and density	7.	Hadron spectroscopy and interactions
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Table 1: Titles of parallel sessions at Lattice 2013, held the week after the EPS HEPP Conference.

1. Introduction

In this talk I review the status and prospects of lattice computations contributing to precision flavour physics. I had to be selective in making this choice because the lattice QCD community is a large one with a very broad scientific programme, for example in Table 1 I list the titles of the parallel sessions at next week's annual symposium on Lattice Field Theory.

Flavour physics is a key tool in exploring the limits if the Standard Model and in searches for new physics. It is complementary to high-energy experiments, most notably those on the large hadron collider. If, as expected (or at least hoped), ATLAS and CMS discover new elementary particles not present in the Standard Model then precision flavour physics will be necessary to help determine the underlying framework. We should however, not underestimate the discovery potential of precision flavour physics; for this we need an unambiguous discrepancy between a weak decay rate or CP-asymmetry and the theoretical Standard Model prediction. Such calculations require a quantitative control of hadronic effects for which lattice QCD simulations are essential.

The question "can we trust the lattice results?" is frequently asked; indeed it was even raised at this conference. This is not surprising, since for about 20 years lattice simulations were being performed in the *quenched* approximation in which vacuum polarisation effects are neglected. It is only in the last few years that simulations with physical u and d quark masses have become possible and the answer to the question at the start of this paragraph is an emphatic YES. Indeed, the main message of this talk is the remarkable precision with which many physical quantities can now be evaluated.

I presented a review talk on lattice phenomenology at an EPS HEPP conference once before, in Marseille in 1993 [1]. It has been fascinating to compare the 20-year old results to current ones (see below for some examples).

1.1 Lattice QCD

Lattice phenomenology starts with the evaluation of correlation functions of the form:

$$\langle 0|\, O(x_1,x_2,\cdots,x_n)\,|0\rangle = \frac{1}{Z}\int [dA_\mu]\,[d\psi]\,[d\bar\psi]\,e^{-S}\,O(x_1,x_2,\cdots,x_n)\;, \tag{1.1}$$

where $O(x_1,x_2,\cdots,x_n)$ is a multilocal operator composed of quark and gluon fields and Z is the partition function. The functional integral is performed by discretising space-time and using Monte-Carlo Integration.

The physics which can be studied depends on the choice of the operator O. Consider for example the two-point correlation function sketched in Fig. 1(a). Operators with the quantum numbers

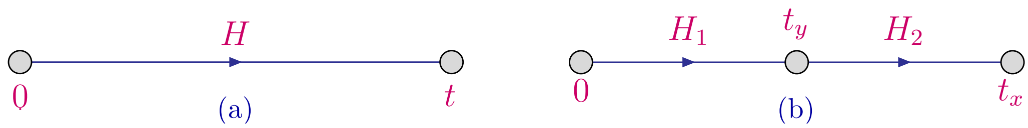

Figure 1: (a) Two-point correlation function in which the hadron H is created at time 0 and annihilated at time t. (b) Three-point correlation function in which hadron H_1 is created at time 0, undergoes a transition to hadron H_2 at time t_y which is subsequently annihilated at t_x.

to create and annihilate the hadron H are inserted at time 0 and t respectively. It is convenient to integrate over the space coordinates of the operators to ensure that the spatial momentum is zero (other momenta are also possible by taking the corresponding Fourier transform). By studying the time dependence of the correlation function, which is given by $\exp(-m_H t)$, the mass of the hadron is determined and from the prefactor the matrix element of the interpolating operator between the vacuum and H is obtained. For example if the operator is the (local) axial current then the matrix element is the corresponding pseudoscalar decay constant. Fig. 1(b) contains a sketch of a three point function, in which hadron H_1 is created at time 0, a local operator (Q say) is inserted at time t_y which leads to the transition $H_1 \to H_2$ and H_2 is subsequently annihilated at t_x. By evaluating such 3-point functions, for example the amplitudes for semileptonic decays can be evaluated.

1.2 The Flavour Physics Lattice Averaging Group (FLAG)

To help answer the question "what is currently the best value from lattice QCD for the quantity X?", the *Flavour Physics Lattice Averaging Group* (FLAG) was formed in which experts critically review the results and present the *best values*. The compilations quoted in this talk were taken from the current FLAG averages [2], which are an update of its first publication [3] [1].

2. Selected results from the physics of light hadrons

I now review the current status of results for some quantities in the physics of light hadrons.

2.1 Quark masses

Quark masses are fundamental parameters of the Standard Model. Because of confinement they are not directly measurable, but have to be inferred from the measurement of other physical quantities. Moreover they depend on the renormalization procedure used to define them. Lattice determinations proceed as follows: (i) for simulations with N_f non-degenerate quark flavours we compute $N_f + 1$ known physical quantities as a function of the quark masses. The physical values of the bare quark masses in the discretised theory and the value of the lattice spacing are those which reproduce the physical values. As an example, the RBC-UKQCD collaboration, for its simulations in the isospin symmetric limit (i.e. with $m_u = m_d$) use the pion, kaon and Ω-baryon masses to calibrate the lattices [4]. Having obtained the bare quark masses, non-perturbative renormalization can be used to obtain the values in a scheme which can be simulated, circumventing the need for a perturbative calculation in the lattice theory. The $\overline{\text{MS}}$-scheme, which is most often used in perturbative calculations, and which is based on calculations in $4+2\varepsilon$ dimensions, is not one which

[1] Since this conference, FLAG have also posted an updated review [2].

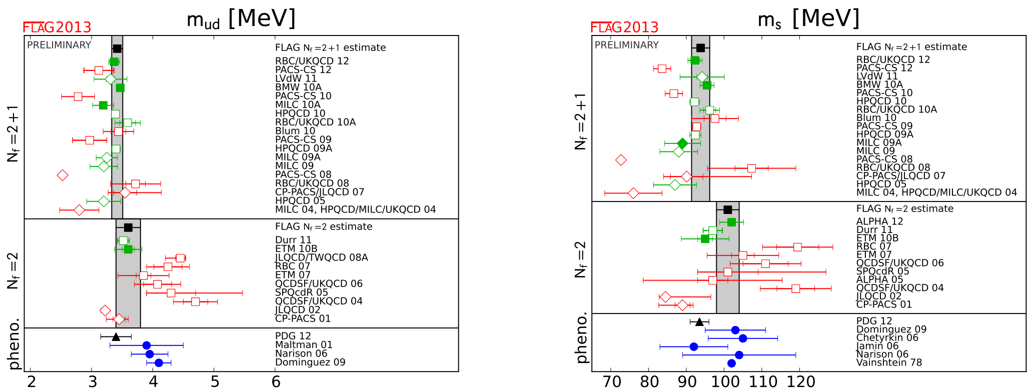

Figure 2: Results obtained in the isospin limit with $m_u = m_d$ with 2 and 2+1 flavours of sea quarks [2]. The squares (diamonds) refer to non-perturbative (perturbative) renormalization. References to the original papers can be found in [2].

can be used for lattice simulations. At some point therefore, a calculation in continuum perturbation theory is unavoidable.

For the current values of the quark masses in the $\overline{\text{MS}}$ scheme at a renormalisation scale of 2 GeV obtained from $N_f = 2+1$ simulations, FLAG quote [2]

$$m_{ud} \equiv \frac{m_u + m_d}{2} = (3.42 \pm 0.09)\,\text{MeV} \qquad m_s = (93.8 \pm 2.4)\,\text{MeV} \qquad \frac{m_s}{m_{ud}} = 27.5 \pm 0.4\,. \quad (2.1)$$

These values were obtained by critically reviewing the results plotted in Fig. 2 [2]. As an illustration of the ten-fold improvement in precision over the last decade, in the 2002 review for the Particle Data Group, with A. Manohar I quoted $m_{ud} = 4.2 \pm 1.0$ MeV and $m_s = 105 \pm 25$ MeV [5].

2.2 ε_K and neutral kaon mixing

The dominant contribution to the indirect CP-violating parameter ε_K is conventionally written in terms of the B_K parameter defined by:

$$\langle \bar{K}^0 \,|H_W^{\Delta S=2}\,|K^0\rangle \propto \langle \bar{K}^0\,|(\bar{s}\gamma^\mu(1-\gamma^5)d)\;(\bar{s}\gamma_\mu(1-\gamma^5)d)|K^0\rangle \equiv \frac{8}{3} f_K^2 m_K^2 B_K(\mu)\,, \qquad (2.2)$$

where the explicit μ dependence reminds us that the B_K is renormalization scheme and scale dependent. Lattice calculations of B_K have been performed since the mid 1980s and the precision is now such that the $O(5\%)$ long-distance (LD) effects have to be considered [6,7]. It should also be noted that the dominant contribution to $\varepsilon_K \propto |V_{cb}|^4$ and PDG(2012) [8] quote $|V_{cb}| = (40.9 \pm 1.1) \times 10^{-3}$ so that the error on B_K is no longer the dominant one.

From a critical review of the results in Fig. 3, FLAG quotes from simulations with $N_f = 2+1$ [2]:

$$\hat{B}_K = 0.766(10) \quad \text{corresponding to} \quad B_K^{\overline{\text{MS}}}(2\,\text{GeV}) = 0.560(7)\,. \qquad (2.3)$$

To illustrate the recent progress the 2011 FLAG review [3] quoted $\hat{B}_K = 0.738(20)$ (at EPS in 1993 I quoted $\hat{B}_K = 0.8(2)$ [1]).

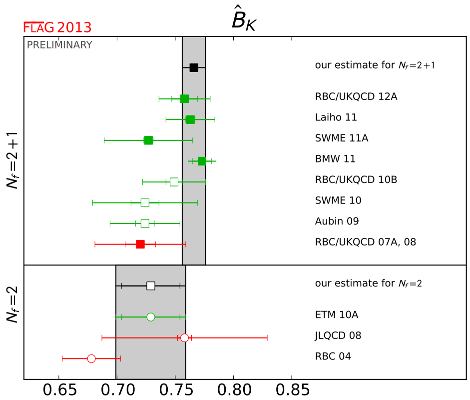

Figure 3: Compilation of results for $\hat{B}_K$ from simulations with 2 and $2+1$ flavours of sea quarks.

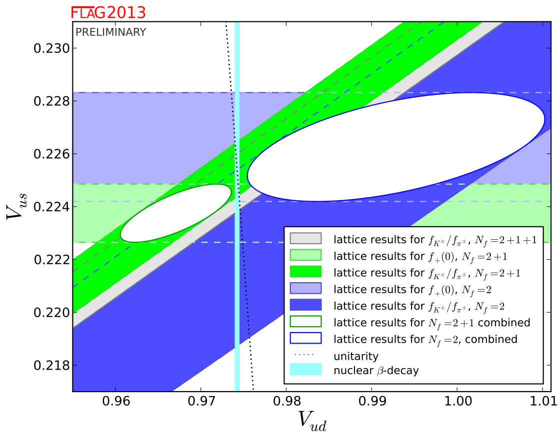

Figure 4: Summary of values of V_{ud} and V_{ud} obtained by combining the experimental results in Eq. (2.5) with lattice calculations of f_K/f_π and $f_+(0)$ [2].

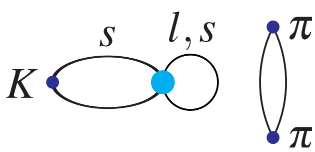

Figure 5: An example of a disconnected diagram contributing to $K \to (\pi\pi)_{I=0}$ decays. l,s label light and strange quarks respectively.

2.3 Determination of V_{ud} and V_{us}

For leptonic decays of a pseudoscalar meson P, all QCD effects are contained in a single constant, f_P, the (leptonic) decay constant: $\langle 0|\,\bar{s}(\bar{d})\gamma^\mu\gamma^5 u\,|P(p)\rangle \equiv i\,f_P\,p^\mu$. For $K_{\ell 3}$ decays QCD effects are contained in form factors e.g. for $K \to \pi$ decays:

$$\langle\,\pi(p_\pi)\,|\bar{s}\gamma_\mu u\,|\,K(p_K)\,\rangle = f_0(q^2)\,\frac{m_K^2-m_\pi^2}{q^2}q_\mu + f_+(q^2)\,\left[(p_\pi+p_K)_\mu - \frac{m_K^2-m_\pi^2}{q^2}q_\mu\right], \qquad (2.4)$$

where $q \equiv p_B - p_\pi$. In order to extract the CKM matrix elements V_{ud} and V_{us} we start with the very precise experimental results:

$$\frac{|V_{us}|}{|V_{ud}|}\,\frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2758(5) \qquad \text{and} \qquad |V_{us}|\,f_+(0) = 0.2163(5)\,. \qquad (2.5)$$

The job of the lattice community is therefore to calculate f_K/f_π and $f_+(0)$. Fig. 4 contains the FLAG summary [2] for V_{ud} and V_{us} obtained from $N_f = 2$ and $N_f = 2+1$ simulations. From

$N_f = 2+1$ simulations, FLAG quotes

$$\frac{f_{K^\pm}}{f_{\pi^\pm}} = 1.192(5) \Rightarrow \frac{|V_{us}|}{|V_{ud}|} = 0.2314(11) \quad \text{and} \quad f_+(0) = 0.9667(23)(33) \Rightarrow |V_{us}| = 0.2238(7)(8)\,. \tag{2.6}$$

These results show that there is very little room for any discrepancy of the unitarity relation $|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \simeq |V_{ud}|^2 + |V_{us}|^2 = 1$. The results in Eq.(2.6) by themselves imply $|V_u|^2 = 0.985(13)$ and when the lattice results for f_K/f_π and $f_+(0)$ are combined with $V_{ud} = 0.97425(22)$ obtained from super-allowed nuclear β-decays [9], the constraint becomes even tighter, $|V_u|^2 = 1.0000(6)$ and $0.9992(6)$ respectively.

2.4 Isospin Breaking Effects

As we approach 1% precision in QCD simulations it is becoming necessary to include isospin breaking (IB) effects in order to make further progress [10]. This involves setting $m_u \neq m_d$ and including electromagnetic effects. For example, defining $\Delta_\pi \equiv m^2_{\pi^+} - m^2_{\pi^0}$ and $\Delta^\gamma_P \equiv m^2_P - \hat{m}^2_P$ (where $\hat{m}_p$ is the mass of pseudoscalar meson P in QCD alone), the violations of Dashen's Theorem can be parametrized by:

$$\Delta^\gamma_{K^+} - \Delta^\gamma_{K^0} - \Delta^\gamma_{\pi^+} + \Delta^\gamma_{\pi^0} = \varepsilon\,\Delta_\pi\,. \tag{2.7}$$

Some groups take ε from phenomenological studies, others introduce quenched electromagnetism as a $U(1)$ degree of freedom and some by performing perturbation theory in α. The FLAG averages [2] for ε and the quark masses are:

$$\varepsilon = 0.7(3),\;\; m_u^{\overline{\rm MS}}(2\,{\rm GeV}) = 2.16(9)(7)\,{\rm MeV},\;\; m_d^{\overline{\rm MS}}(2\,{\rm GeV}) = 4.68(14)(7)\,{\rm MeV},\;\; \frac{m_u}{m_d} = 0.46(2)(2)\,. \tag{2.8}$$

3. $K \to \pi\pi$ Decays

As an example of the extension of lattice calculations to non-standard quantities, ones which have required considerable theoretical developments to achieve, I now discuss non-leptonic kaon decays. The RBC-UKQCD collaboration has performed the first calculation of the $K \to (\pi\pi)_{I=2}$ amplitude A_2 [11, 12] finding:

$$\mathrm{Re}A_2 = (1.381 \pm 0.046_{\rm stat} \pm 0.258_{\rm syst})\,10^{-8}\,{\rm GeV} \tag{3.1}$$

$$\mathrm{Im}A_2 = -(6.54 \pm 0.46_{\rm stat} \pm 1.20_{\rm syst})\,10^{-13}\,{\rm GeV}. \tag{3.2}$$

The result for $\mathrm{Re}\,A_2$ agrees well with the experimental value of $1.479(4)\times 10^{-8}$ GeV obtained from K^+ decays. $\mathrm{Im}\,A_2$ is unknown so that our result provides its first direct determination. Combining our result for $\mathrm{Im}\,A_2$ with the experimental results for $\mathrm{Re}\,A_2$, $\mathrm{Re}\,A_0 = 3.3201(18)\cdot 10^{-7}$ GeV and ε'/ε we obtain: $\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} = -1.61(19)_{\rm stat}(20)_{\rm syst}\times 10^{-4}$. The error in Eqs. (3.1) and (3.2) is dominated by *lattice artefacts*, since the calculation was performed at a single, rather coarse, lattice spacing. Preliminary results at two finer lattice spacings will be presented at Lattice 2013 next week which will reduce this uncertainty very significantly.

Among the theoretical developments required to make the evaluation of A_2 possible is the control of finite-volume and rescattering effects [13–15]. This is possible because for each isospin channel (and assuming that only a finite number of partial waves contribute) only elastic channels

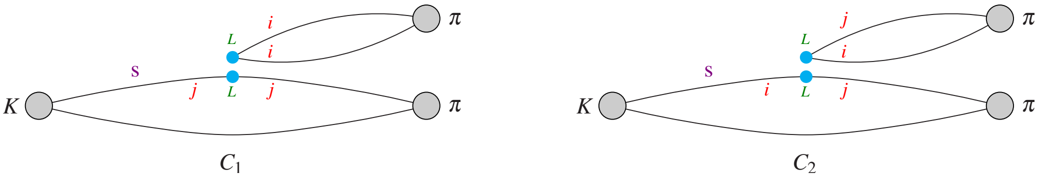

Figure 6: The two diagrams (contractions) contributing to the correlation function for the calculation of Re A_2. The two small (blue) circles denote the left-handed currents in the effective Hamiltonian and i, j are colour labels. The strange-quark propagator is explicitly labelled by s; the remaining propagators correspond to the u and d quarks.

propagate. I mention in passing that for hadronic B-decays, for which many inelastic intermediate states are important, we do not even know how to formulate a possible lattice computation.

The calculation of the $K \to (\pi\pi)_{I=0}$ amplitude A_0 is much more difficult. The presence of disconnected diagrams, such as that in Fig. 5 leads to noisy results; the efficient evaluation of disconnected diagrams in general is a major area of research. A more serious challenge is to find boundary conditions such that the two pions are in the ground state and to this end RBC-UKQCD are developing G-parity boundary conditions, which preserve isospin but for which the conservation of flavour symmetry is subtle. I would estimate that a full calculation of ε'/ε will be done on a timescale of 2 years or so. In the meantime RBC-UKQCD have computed A_0 with the two pions at rest and with unphysical masses, finding e.g. [16, 17]

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 9.1 \pm 2.1 \qquad \text{877 MeV kaon decaying into two 422 MeV pions} \tag{3.3}$$

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 12.0 \pm 1.7 \qquad \text{662 MeV kaon decaying into two 329 MeV pions .} \tag{3.4}$$

Whilst both these results are obtained at unphysical kinematics and are different from the physical value of 22.5, it is nevertheless interesting to understand the origin of these enhancements (see Sec. 3.1). In this study it was found that 99% of the contribution to the real part of A_0 and A_2 come from the matrix elements of the current-current operators.

3.1 Emerging understanding of the $\Delta I = 1/2$ rule

The evaluation of A_2 contained a surprise, at least to the authors and much of the community (see however [18]), which we interpret as a significant contribution to the large observed value of Re A_0/Re $A_2 \simeq 22.5$. The surprise is a significant cancellation of the two contributions to Re A_2 [19], as I now briefly explain. Re A_2 is dominated by the contribution from a single operator $O^{3/2}_{(27,1)} = (\bar{s}^i d^i)_L \left\{(\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L\right\} + (\bar{s}^i u^i)_L (\bar{u}^j d^j)_L$ where the L denotes *Left* and i, j are colour indices. The simple property of L ⊗ L operators under Fierz transformations, means that there are two diagrams contributing to the correlation function; these are illustrated in Fig. 6. The correlation function for Re A_2 is proportional to $C_1 + C_2$. Colour counting and the vacuum insertion approximation might suggest that $C_2 \simeq \frac{1}{3} C_1$. We find instead that in QCD C_1 and C_2 have the opposite sign and that there is a significant cancellation between the two contributions. This is illustrated in Fig. 7 where the kaon source is at time 0, the two-pion sink is at time 24 and t labels the time at which the operator is inserted. We see a nice plateau in $C_1 + C_2$ confirming that we have tuned the volume such that the two pion energy $E_{\pi\pi}$ indeed satisfies $E_{\pi\pi} = m_K$.

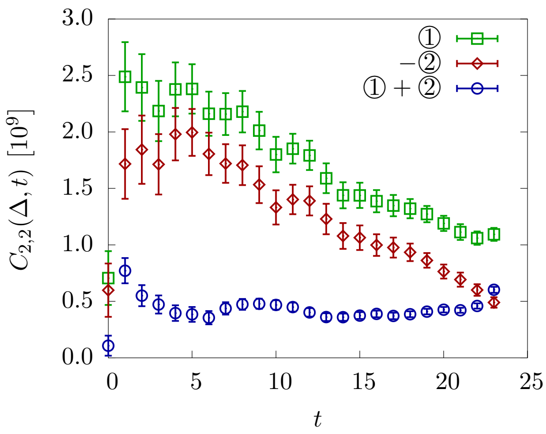

Figure 7: The two contractions contributing to the correlation function from which $\mathrm{Re}\,A_2$ is determined. ①, ② label contractions $C_{1,2}$ respectively.

Having established the cancellation between the two contributions to $\mathrm{Re}\,A_2$ we can ask about $\mathrm{Re}\,A_0$. The contribution to $\mathrm{Re}\,A_0$ from the current-current operator Q_2 is proportional to $2C_1 - C_2$ and that from Q_1 is proportional to $C_1 - 2C_2$ with the same overall sign. Thus the fact that C_1 and C_2 have opposite signs also leads to an enhancement of $\mathrm{Re}\,A_0$. Of course before claiming a quantitative understanding of the $\Delta I = 1/2$ rule we need to compute $\mathrm{Re}\,A_0$ at physical kinematics and reproduce the experimental value of 22.5. We believe however, that the strong suppression of $\mathrm{Re}\,A_2$ and the (less-strong) enhancement of $\mathrm{Re}\,A_0$ is a major factor in the $\Delta I = 1/2$ rule.

4. *B*-physics

B-physics is a particularly important area of flavour physics. The *b*-quark is light-enough to be produced copiously and yet heavy enough to have a very large number of possible decay channels so that subtle features of the standard model can be explored. Lattice computations in *B*-physics have to deal with the fact that $m_b a \gtrsim 1$, where a is the lattice spacing, so that the Compton wavelength of the *b*-quark is smaller than a. Most approaches rely on effective theories and invest a considerable effort in the matching to QCD; this includes the use of (i) the Heavy Quark Effective Theory which is an expansion in $\frac{\Lambda_{\mathrm{QCD}}}{m_B}$ (ii) nonrelativistic QCD (an expansion in the quark's velocity) and (iii) the relativistic heavy quark approach of the Fermilab group [20] and its extensions. Some groups also extrapolate results from the charm to the bottom region, using scaling laws where applicable and possibly also using results obtained in the static limit. The number of collaborations working in heavy-quark physics has recently increased, which will provide an opportunity for us to check for consistency of different approaches as has been the case for light quarks.

4.1 f_B, f_{B_s} and neutral *B*-meson mixing.

For the *B*-meson decay constants, the FLAG compilation for $N_f = 2+1$ simulations gives [2]:

$$f_B = (190.5 \pm 4.2)\,\mathrm{MeV}\,, \quad f_{B_s} = (227.7 \pm 4.5)\,\mathrm{MeV}\,, \quad \frac{f_{B_s}}{f_B} = 1.202 \pm 0.022\,. \tag{4.1}$$

(At EPS HEPP in 1993 I quoted $f_B = 180 \pm 40\,\mathrm{MeV}$.) In principle, V_{ub} can be obtained from experimental measurements of $\mathrm{B}(B \to \tau \nu_\tau)$ by Belle and BaBar together with lattice determina-

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tions of f_B. However, the current uncertainty in $\mathrm{B}(B \to \tau \nu_\tau)$ means that this is not the best way to determine V_{ub}.

For the SU(3)-breaking parameter ξ, FLAG take the result from [21] as the best value:

$$\xi^2 \equiv \frac{\langle \bar{B}^0_s \,|\, (\bar{b}\gamma^\mu(1-\gamma^5)s)(\bar{b}\gamma_\mu(1-\gamma^5)s)\,|B^0\rangle}{\langle \bar{B}^0_s \,|\, (\bar{b}\gamma^\mu(1-\gamma^5)s)(\bar{b}\gamma_\mu(1-\gamma^5)s)\,|B^0\rangle} = 1.268(63)\,. \tag{4.2}$$

Combining this result with experimental values of Δm_d and Δm_s, yields $\left|\frac{V_{td}}{V_{ts}}\right| = 0.216 \pm 0.011$. I also mention in passing that for generic BSM theories, there are 5 $\Delta B = 2$ operators (and 5 $\Delta S = 2$ operators for neutral kaon mixing) whose matrix elements can be computed in a similar way.

4.2 Semileptonic *B*-decays

For semileptonic $B \to \pi, \rho$ decays, in order to avoid lattice artefacts, the momentum of the π or ρ is limited so that we only get results at large q^2 (q is the momentum transfer). Thus V_{ub} can be obtained directly by combining the lattice results with a subset of the experimental data:

$$\Delta\zeta(q_1^2, q_2^2) = \frac{1}{|V_{ub}|^2} \int_{q_1^2}^{q_2^2} dq^2\, \frac{d\Gamma}{dq^2}\,. \tag{4.3}$$

FLAG quote [2] $\Delta\zeta(16\,\mathrm{GeV}^2, q^2_{\mathrm{max}}) = 2.16(50)\,\mathrm{ps}^{-1}$ and $|V_{ub}| = 3.37(20) \times 10^{-3}$ and $|V_{ub}| = 3.47(22) \times 10^{-3}$ based on the results from the FNAL/MILC [22] and HPQCD [23] collaborations. Assuming (not assuming) unitarity, PDG [8] quote $|V_{ub}| = 3.51^{+0.15}_{-0.14} \times 10^{-3}$ ($|V_{ub}| = (4.15 \pm 0.49) \times 10^{-3}$). The issue is the tension with the inclusive determination $|V_{ub}| = (4.41 \pm 0.15^{+0.15}_{-0.19}) \times 10^{-3}$, which has different systematics and cannot be studied in lattice simulations. The results in [22, 23] are relatively old and the evaluation of the form-factors and determination of V_{ub} is a major priority for lattice simulations, with several collaborations having made conference presentations with preliminary results.

V_{cb} is known more precisely that V_{ub}, because the experimental cuts are at higher energies where the OPE is more reliable and heavy-quark symmetry implies that the form factors are close to 1. It is not known well enough however! (Recall for example that $\varepsilon_K \propto |V_{cb}|^4$.) Up to now the evaluation of the $B \to D, D^*$ form factors has been led by the FNAL/MILC collaboration, using the Fermilab approach and staggered fermions [24, 25] although several new computations are under way. Defining $\omega = v_B \cdot v_{D^*}$, where v represents the four-velocity,

$$\frac{d\Gamma_{B^- \to D^{0*}\ell\bar{\nu}}}{d\omega} \propto |V_{cb}|^2 |\mathscr{F}(\omega)|^2\,, \tag{4.4}$$

FNAL/MILC find $\mathscr{F}(1) = 0.9017(51)(156)$ corresponding to $|V_{cb}| = 39.55(72)(50) \times 10^{-3}$. For inclusive $B \to X_c \ell \nu$ decays, PDG [8] quote $|V_{cb}|^{\mathrm{incl}} = (41.9 \pm 0.7) \times 10^{-3}$.

5. Charm Physics

The charm quark is between *light* and *heavy*; typically $am_c \lesssim 1$. Some calculations are performed with (improved) light-quark actions (increasingly this is the case) and some using one of the heavy-quark approaches. A wide program of research in charm physics is now a major priority for the lattice community.

The FLAG averages for decay constants and semileptonic form factors from simulations with $N_f = 2+1$ sea quarks are:

$$f_D = (209.2 \pm 3.3)\,\text{MeV}, \quad f_{D_s} = (248.6 \pm 2.7)\,\text{MeV}, \quad \frac{f_{D_s}}{f_D} = 1.187 \pm 0.012 \qquad (5.1)$$

$$f_+^{D\pi}(0) = 0.666(29) \qquad f_+^{DK}(0) = 0.747(19) \qquad (5.2)$$

The results obtained from leptonic D decays are $|V_{cd}| = 0.2218(35)(95)$ and $|V_{cs}| = 1.018(11)(21)$ and from semileptonic decays $|V_{cd}| = 0.2192(95)(45)$ and $|V_{cs}| = 0.9746(248)(67)$. There was some excitement a few years ago when the HPQCD result $f_{D_s} = (241 \pm 3)\,\text{MeV}$ [26] appeared to be very different from the compilation of experimental results in [27], $f_{D_s} = (277 \pm 9)\,\text{MeV}$. More recently however, there have been updates to both the HPQCD result, now $f_{D_s} = (248.0 \pm 2.5)\,\text{MeV}$ [28] and to the experimental average $f_{D_s} = (257.2 \pm 4.5)\,\text{MeV}$ [29] so that any discrepancy is much less significant.

6. Conclusions

Before presenting my conclusions, let me say that our community is mourning the founder of Lattice QCD, the 1982 Nobel Laureate, Kenneth Wilson, who died earlier this year.

The main message of this talk is that Lattice QCD has matured into the quantitative ab-initio method for computing non-perturbative strong-interaction effects in an increasing range of processes. We have now reached the era of $O(1\%)$ precision for many important physical quantities. This was illustrated by a number of examples. For the future,

• The improvement in precision will continue and the computations will increasingly include electromagnetism and other isospin-breaking effects.

• I expect a major expansion of results in heavy-quark physics in the near future, generating competition among different approaches and leading to increased confidence in the existing results and underlying frameworks.

• The technology for calculating *disconnected diagrams*, necessary e.g. for flavour singlet channels, will be fully developed.

• The range of quantities being studied will continue to be extended (e.g. ε'/ε and long-distance contributions to rare kaon decay amplitudes.)

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