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Enhanced $B_d^0 \to \mu^+\mu^-$ Decay: What if?

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The very rare $B_d^0 \to \mu^+\mu^-$ decay may be the last chance for New Physics in flavor sector at the LHC, before the 13 TeV run in 2015. Partially motivated by the known tension in $\sin 2\beta/\phi_1$, enhancement beyond (3–4)$\times 10^{-10}$ would likely imply the effect of a fourth generation of quarks. If observed at this level, the 126 GeV boson may not be the actual Higgs boson, while the $b \to d$ quadrangle (modulo $m_{t'}$) would jump out. The 2011-2012 data is likely not sensitive to values below 3×10^{-10}, and the mode should continue to be pursued with the 13 TeV run. We comment on implications of new LHC data reported during this conference.

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1. Introduction

So far, no New Physics beyond the Standard Model (SM) has been found at the Large Hadron Collider (LHC). Although a new 126 GeV boson was discovered at the LHC in 2012, its property is consistent with the Higgs boson predicted in SM. As for flavor sector, there was much anticipation for finding New Physics in $b \to s$ transitions at the LHC, however, it has not come true. One such example is the rare decay $B_s^0 \to \mu^+\mu^-$, for which the LHCb reported the first evidence with a SM-like decay rate in 2012 [1], excluding a possible large enhancement by New Physics effects.

On the other hand, the $B_d^0 \to \mu^+\mu^-$ decay still offers much room for enhancement. The combined LHC bound for the decay rate is within a factor of 8 [2] of the SM prediction, which is 30 times lower than $B_s^0 \to \mu^+\mu^-$. Indeed, the $B_d^0 \to \mu^+\mu^-$ decay may be the last chance for discovering New Physics in flavor sector at the LHC, before 13 TeV run in 2015. There is some motivation for enhancement, from the well known [3] mild (of order 2σ) but lingering tension between direct measurement of CP violation (CPV) phase of $\bar{B}_d$–B_d mixing, versus extraction by indirect means.

As pictorialized by the "Straub plot" [4], while most models of enhancement for $B_d^0 \to \mu^+\mu^-$ have now been eliminated by the SM-like $B_s^0 \to \mu^+\mu^-$ rate measured by LHCb, the 4th generation (4G) still allows $B_d^0 \to \mu^+\mu^-$ to be enhanced up to the current bound, since $B_d^0 \to \mu^+\mu^-$ and $B_s^0 \to \mu^+\mu^-$ decays are governed by different Cabibbo-Kobayashi-Maskawa (CKM) products $V_{t'd}^* V_{t'b}$ and $V_{t's}^* V_{t'b}$ (subject to constraint from Kaon physics via $V_{t'd}^* V_{t's}$). As discussed recently by Stone [5], however, conventional wisdom is that 4G has been "eliminated by the Higgs discovery", because it "would cause the Higgs production cross-section to be nine times larger ... ".There are two catches in this pessimism, however. First of all, despite the recent Nobel prize, the observed 126 GeV object might still be something else. For example, a dilaton might mimic [6] the Higgs with current data. Second, the Higgs boson of SM does not enter into the $B_d^0 \to \mu^+\mu^-$ process (the same holds for the B_d box diagram and $B^+ \to \pi^+\mu^+\mu^-$ processes we consider). To assume indirect arguments in the flavor pursuit is self-defeating, especially when there is still room for large enhancement; it actually highlights the potential impact of a discovery. If an enhanced $B_d^0 \to \mu^+\mu^-$ rate is discovered with 2011–2012 LHC data, with the likely explanation by 4G, this would cast doubt on the Higgs boson interpretation of the 126 GeV boson.

In the following, we review input parameters and constraints, then present our numerical study. We indeed find enhancement beyond 4×10^{-10} (4 times SM) is possible [7] within the parameter space indicated by the tension in $\sin 2\Phi_{B_d} \equiv \sin 2\beta/\phi_1$.

2. Constraints and Input Parameters

There is some motivation for New Physics in $b \to d$ transitions, from the well known [3] mild tension in $\sin 2\Phi_{B_d} \equiv \sin 2\beta/\phi_1$, between the directly measured value [8] of

$$\sin 2\beta/\phi_1 = 0.679 \pm 0.020, \tag{2.1}$$

and SM expectation estimated via $\beta/\phi_1 \cong \arg \lambda_t^{\rm SM}$, where

$$\lambda_t^{\rm SM} = -\lambda_u - \lambda_c \simeq -|V_{ud}||V_{ub}|e^{-i\phi_3} + |V_{cd}||V_{cb}|, \tag{2.2}$$

with $\lambda_i \equiv V_{id}^* V_{ib}$. The terms on right-hand side of Eq. (2.2) can be measured at the tree level. We adopt the central values by PDG [8], $|V_{ud}| = 0.974$, $|V_{cd}| = 0.23$, $|V_{cb}| = 0.041$, and $\phi_3 = 68°$. Variations in these values are not central to our discussion.

In contrast, $|V_{ub}|$ also has some tension in the measured values. Extraction via inclusive or exclusive semileptonic B decays yield approximately 4.41×10^{-3} and 3.23×10^{-3} [8], respectively, with the average value of 4.15×10^{-3} (the inclusive approach has better statistics). We use central values, as our purpose is only for illustration, hence we will treat the average (which is close to inclusive) and exclusive cases separately.

With these two values of $|V_{ub}|$ as inputs, the SM expectations are given by

$$\sin 2\beta/\phi_1 = \begin{cases} 0.76 & \text{for } |V_{ub}|^{\rm ave} \\ 0.63 & \text{for } |V_{ub}|^{\rm excl}, \end{cases} \tag{2.3}$$

which both deviate from Eq. (2.1) by more than 2σ. It could easily be due to the 4G quark t', where one simply augments Eq. (2.2) by $\lambda_t = \lambda_t^{\rm SM} - \lambda_{t'}$, and the $b \to d$ triangle becomes a quadrangle

$$\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0. \tag{2.4}$$

In our following study, we parameterize

$$\lambda_{t'} = r_{db}\, e^{i\phi_{db}}. \tag{2.5}$$

In our phase convention, $\lambda_c = V_{cd}^* V_{cb}$ is practically real, while $\lambda_u = V_{ud}^* V_{ub}$ is basically the same as in SM.

To study $\sin 2\Phi_{B_d}$ and $\mathscr{B}(B_d \to \mu^+\mu^-)$ in the r_{db}–ϕ_{db} plane, we take into account the constraints from the well-measured Δm_{B_d} and the $B^+ \to \pi^+\mu^+\mu^-$ decay, which was recently measured by LHCb [9]. We do not include $b \to d\gamma$ processes, as they are insensitive to virtual 4G effects. $B \to \pi\pi$ decays are quite well studied, but are not included in our study, as they suffer from hadronic effects and do not provide good constraints. We collect below the relevant formulas for our study.

The formulas for B_d mixing are given by

$$\Delta m_{B_d} \simeq \frac{G_F^2 M_W^2}{6\pi^2} m_{B_d} \hat{B}_{B_d} f_{B_d}^2 \eta_B |\Delta_{12}^d|, \quad \sin 2\Phi_{B_d} \simeq \sin(\arg \Delta_{12}^d), \tag{2.6}$$

where the short distance box functions are [10]

$$\Delta_{12}^d \equiv (\lambda_t^{\rm SM})^2 S_0(x_t) + 2\lambda_t^{\rm SM} \lambda_{t'} \Delta S_0^{(1)} + \lambda_{t'}^2 \Delta S_0^{(2)}, \tag{2.7}$$

$$\Delta S_0^{(1)} \equiv \tilde{S}_0(x_t, x_{t'}) - S_0(x_t), \quad \Delta S_0^{(2)} \equiv S_0(x_{t'}) - 2\tilde{S}_0(x_t, x_{t'}) + S_0(x_t), \tag{2.8}$$

with $x_i = m_i^2/M_W^2$. The hadronic uncertainty is in [11]

$$f_{B_d} \hat{B}_{B_d}^{1/2} = (227 \pm 19)\ \text{MeV}. \tag{2.9}$$

For the current bound [2] of

$$\mathscr{B}(B_d \to \mu^+\mu^-) < 8.1 \times 10^{-10}, \tag{2.10}$$

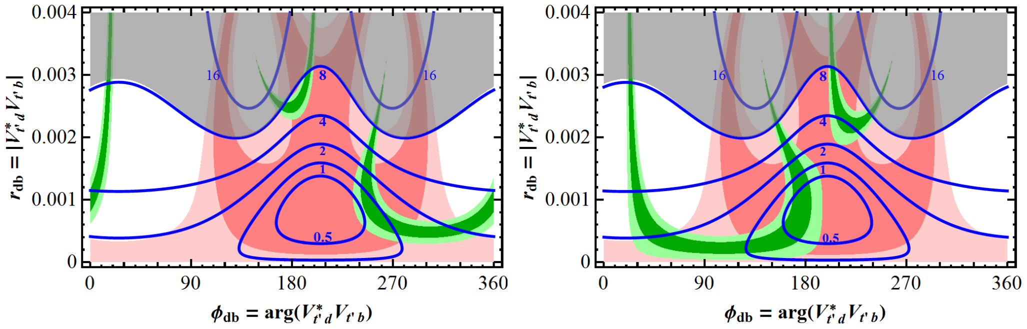

Figure 1: Allowed region in r_{db}–ϕ_{db} plane for (a) average (b) exclusive $|V_{ub}|$ values, for $m_{t'} = 700$ GeV. The solid-blue lines are labeled $10^{10}\mathscr{B}(B_d \to \mu^+\mu^-)$ contours, where above the value of 8 is excluded by the combined LHC constraint. The dark (light) green-shaded regions correspond to the 1(2)σ regions of $\sin 2\Phi_{B_d}$ (Eq. (2.1)), while the pink-shaded regions correspond to the 1(2)σ regions of Δm_{B_d} allowed by Eq. (2.9). $\cos 2\Phi_{B_d} > 0$ [8] is also imposed to eliminate some of the solution branches.

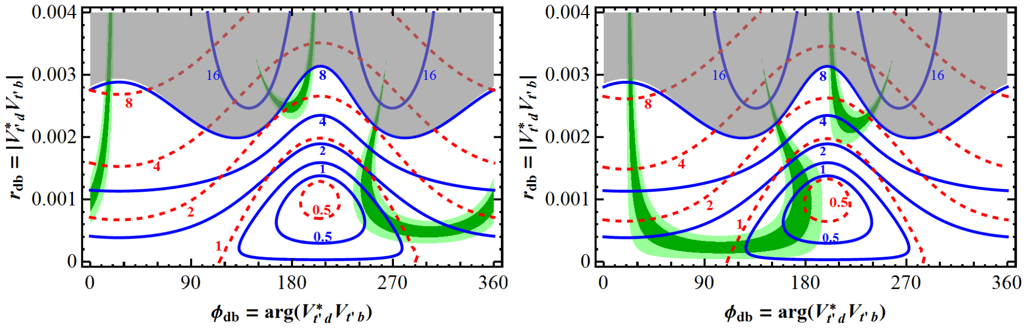

Figure 2: Same as Fig. 1, but with Δm_{B_d} allowed regions replaced by the contours (red-dashed) of $R_{\pi\mu\mu}$.

our purpose is to illustrate whether, and how, it could get enhanced to such values by 4G effect. Here, we use the usual trick [12] of “normalizing" the branching ratio,

$$\hat{\mathscr{B}}(B_d \to \mu^+\mu^-) \equiv \frac{\mathscr{B}(B_d \to \mu^+\mu^-)}{\Delta m_{B_d}}\Delta m_{B_d}^{\rm exp} = C\frac{\tau_{B_d}\Delta m_{B_d}^{\rm exp}}{\hat{B}_{B_d}}\frac{\eta_Y^2}{\eta_B}\frac{\left|\lambda_t^{\rm SM}Y_0(x_t)+\lambda_{t'}\Delta Y_0\right|^2}{|\Delta_{12}^d|} \qquad (2.11)$$

where $\Delta Y_0 = Y_0(x_{t'}) - Y_0(x_t)$ [10], and $C = 6\pi(\alpha/4\pi\sin^2\theta_W)^2 m_\mu^2/M_W^2$. Through the ratio of Eq. (2.11), one not only eliminates the hadronic parameter f_{B_d}, but the $\lambda_t^{\rm SM}$ factor also cancels in the SM case, and one recovers the SM result of 1.1×10^{-10}, with little sensitivity to $|V_{ub}|$.

The treatment of $B^+ \to \pi^+\mu^+\mu^-$ is given in the next section.

3. Phenomenological Study with Heavy t'

We plot in Fig. 1 for $m_{t'} = 700$ GeV the 2σ range in the r_{db}–ϕ_{db} plane, for $\sin 2\Phi_{B_d}$ (green) allowed by experimental measurement of Eq. (2.1), Δm_{B_d} (pink) allowed by lattice error in Eq. (2.9), and the exclusion by $B_d \to \mu^+\mu^-$ (gray) according to Eq. (2.10). We include labeled contours of 0.5, 1, 2, 4, 8 for $10^{10}\mathscr{B}(B_d \to \mu^+\mu^-)$. Fig. 1(a) and (b) are for taking $|V_{ub}|^{\rm ave} = 4.15\times 10^{-3}$ and $|V_{ub}|^{\rm excl} = 3.23\times 10^{-3}$, respectively.

Consider Fig. 1(a), i.e. for $|V_{ub}|^{\rm ave} = 4.15\times10^{-3}$, the average between inclusive and exclusive measurements (the inclusive case is qualitatively similar). The well measured CP phase $\sin 2\Phi_{B_d}$ is sensitive to t' effects, but free from hadronic uncertainties, hence the narrow (green) contour bands. In contrast, Δm_{B_d} is less sensitive to ϕ_{db}, and more accommodating because of hadronic uncertainty in $f_{B_d}\hat{B}_{B_d}^{1/2}$. The broad (pink) contour bands show the 1 and 2σ allowed region by Eq. (2.9), and rules out a branch of the $\sin 2\Phi_{B_d}$ contour (for ϕ_{db} between -10° to 15°), due to coherent enhancement of Δm_{B_d} from t' effects.

Consider now the gray excluded region from the combined LHC bound on $B_d \to \mu^+\mu^-$, Eq. (2.10). It is seen that there are two slivers of parameter space, around $(r_{db},\ \phi_{db}) \sim (0.0025,\ 180^\circ)$ (region A) and $(0.002,\ 252^\circ)$ (region B), where $\mathscr{B}(B_d \to \mu^+\mu^-)$ could be above 4×10^{-10}, or enhanced by 4 times over SM, which are discovery zones for 2011-2012 LHC data. Near region B, $\mathscr{B}(B_d \to \mu^+\mu^-)$ quickly drops below 4×10^{-10} as r_{db} becomes weaker than 0.002. For $\phi_{db} \sim 245^\circ$ and r_{db} varying from 0.0008 to 0.0015, $\mathscr{B}(B_d \to \mu^+\mu^-)$ hovers at $(1–2)\times10^{-10}$, while for $r_{db} \sim 0.0004$ to 0.0008 and ϕ_{db} varying from 240° to 330°, $\mathscr{B}(B_d \to \mu^+\mu^-)$ hovers at $(0.5–2)\times10^{-10}$, i.e. within a factor of two of SM expectations. These regions, combining to a broad crescent shape which we refer to as "region C", would likely need much more data to probe.

The LHCb experiment has recently measured [9]

$$\mathscr{B}(B^+ \to \pi^+\mu^+\mu^-) = (2.3 \pm 0.6 \pm 0.1)\times10^{-8}. \tag{3.1}$$

The result is consistent with SM expectations, but interpretation depends on form factor models. To reduce form factor dependence, we take the ratio

$$R_{\pi\mu\mu} \equiv \frac{\mathscr{B}(B^+ \to \pi^+\mu^+\mu^-)|_{\rm 4G}}{\mathscr{B}(B^+ \to \pi^+\mu^+\mu^-)|_{\rm SM}}, \tag{3.2}$$

where both 4G and SM results are integrated from $q^2 = (1,\ 6)$ GeV^2, which is under better theoretical control [13]. Since this does not match what LHCb does, we draw contours in Fig. 2 (red-dashed), and view $R_{\pi\mu\mu} \sim 2–3$ as the range beyond which LHCb would have found inconsistency with SM expectations. Thus, we are interpreting LHCb's statement of consistency with SM, allowing for form factor uncertainties. For numerics, we combine Wilson coefficients at next-to-leading order with leading order decay amplitude based on the QCD factorization approach [13]. For dealing with New Physics, and as we take a ratio, this should suffice for our purpose.

If we now compared Fig. 1(a) with Fig. 2(a), we see that Δm_{B_d} is more powerful than $\mathscr{B}(B^+ \to \pi^+\mu^+\mu^-)$ in excluding the $\sin 2\Phi_{B_d}$-allowed branch near $\phi_{db} \sim 0$. It is, however, comforting to see that for region A, $R_{\pi\mu\mu}$ is not more than 2 (except the upper reach near $\phi_{db} \sim 190^\circ$), hence should be easy to accommodate by form factors, while for regions B and especially region C, $R_{\pi\mu\mu}$ is even less than 2 and closer to 1. Thus, the newly measured $B^+ \to \pi^+\mu^+\mu^-$ does provide a sanity check.

Turning to the case of exclusive $|V_{ub}|$ value, Fig. 1(b) and 2(b), we find that regions A and B basically switch roles. This is because for $|V_{ub}|^{\rm excl} = 3.23\times10^{-3}$, the expected $\sin 2\Phi_{B_d}$ value in SM falls below that of direct measurement, as seen in comparing Eq. (2.3) to Eq. (2.1). Calling it region A$'$, the sliver of region around $(r_{db},\ \phi_{db}) \sim (0.002,\ 160^\circ)$ could enhance $\mathscr{B}(B_d \to \mu^+\mu^-)$ more than 4 times above SM, and observable with present LHC data. Region A$'$ extends to the broad crescent region C$'$, where even r_{db} values as lower as 0.0002 could account for the measured $\sin 2\Phi_{B_d}$, but $\mathscr{B}(B_d \to \mu^+\mu^-)$ can be probed only beyond 2015. Again, Δm_{B_d} excludes the

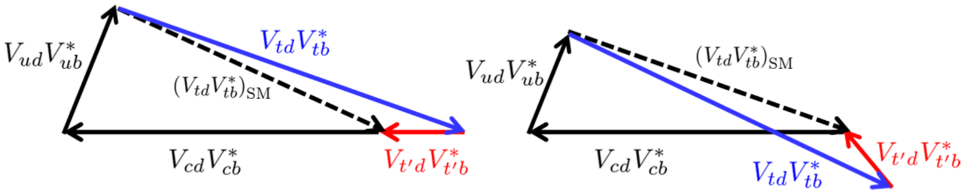

Figure 3: Sample $b \to d$ quadrangles for $\lambda_{t'} = V_{t'd}^* V_{t'b} = 0.0025\, e^{i180^\circ}$ with average $|V_{ub}| = 4.15 \times 10^{-3}$ (left), and for $\lambda_{t'} = V_{t'd}^* V_{t'b} = 0.0023\, e^{i230^\circ}$ with exclusive $|V_{ub}| = 3.23 \times 10^{-3}$ (right).

$\sin 2\Phi_{B_d}$-allowed branch around $\phi_{db} \sim 30^\circ$. Region B$'$ is now a considerably broader region in parameter space that allows enhancement of $\mathscr{B}(B_d \to \mu^+\mu^-)$ above 4×10^{-10}. For example, for r_{db} above 0.0023 and ϕ_{db} above 230°, $\mathscr{B}(B_d \to \mu^+\mu^-)$ can be greater than 6×10^{-10}, $f_{B_d}\hat{B}_{B_d}^{1/2}$ is within 2σ of Eq. (2.9), while $R_{\pi\mu\mu}$ is not more than 2. We also see that, for region B$'$, $R_{\pi\mu\mu}$ provides as good, perhaps better constraint, than Δm_{B_d}, disfavoring the region of r_{db} greater than 0.0025 around $\phi_{db} \sim 205^\circ$, that seems perfectly allowed by Δm_{B_d}.

For $m_{t'} = 1000$ GeV, which is far above the unitarity bound, we found the constraint from $\mathscr{B}(B_d \to \mu^+\mu^-)$ becomes stronger and $|\lambda_{t'}|$ values tend to drop by half [10].

4. Discussion and Conclusion

Let us illustrate an impact of a possible discovery for the $B_d \to \mu^+\mu^-$ decay in pictorial way, namely, drawing the $b \to d$ quadrangle given by Eq. (2.4). For this purpose, we take two representative values for $\lambda_{t'} = V_{t'd}^* V_{t'b}$ (for $m_{t'} = 700$ GeV): $\lambda_{t'} = 0.0025\, e^{i180^\circ}$ from region A of Fig. 1(a) (average $|V_{ub}| = 4.15 \times 10^{-3}$), and $\lambda_{t'} = 0.0023\, e^{i230^\circ}$ from region B' of Fig. 1(b) (exclusive $|V_{ub}| = 3.23 \times 10^{-3}$). The two corresponding quadrangles are plotted in Fig. 3 in the form to compare with the usual SM triangle [8]. These are relatively precise quadrangles, and illustrate how 4G accounts for a shift in $\sin 2\Phi_{B_d}$ away from SM expectation, where $\Phi_{B_d}^{\rm SM}$ is the angle between the dashed line, $\lambda_t^{\rm SM}$ and the real axis. Since t' is much heavier than t, a smaller $\lambda_{t'}$ could cause the shift.

The quadrangles of Fig. 3 reminds us of the possible [14] link to the baryon asymmetry of the Universe (BAU): 4G greatly enhances CPV from SM, and is seemingly sufficient for BAU (although a first order phase transition remains an issue), which boosts the merit of 4G. It does not depend much on the area of the quadrangle, as the enhancement rests in powers of $m_{t'}$ and $m_{b'}$. We note that $\lambda_{t'}$ in Fig. 3, though smaller in strength than λ_t and λ_c, is not that small compared with λ_u. Furthermore, we know that $|V_{t'b}|$ cannot be more than 0.1 [15], especially for our large $m_{t'}$ values. Hence, $|\lambda_{t'}|$ plotted in Fig. 3 correspond to $|V_{t'd}|$ that is larger than $|V_{td}|^{\rm SM} \simeq 0.0088$, which does not fit the usual CKM pattern of trickling off as one goes further off-diagonal. If t' is heavier, $|\lambda_{t'}|$ drops quickly and $V_{t'd}$ becomes more natural in the sense of CKM hierarchy.

We conclude that 2013 remains a pivotal year where one could discover the very rare $B_d^0 \to \mu^+\mu^-$ decay mode at over 4 times SM expectations. The chance is not large, but not zero either, with partial motivation from the (mild) $\sin 2\Phi_{B_d}$ discrepancy. If discovered with 2011–2012 data set, the implications would be quite huge [16]: uplifting the 4th generation (with prospect of CPV for BAU), casting some doubt on the SM Higgs interpretation of the 126 GeV boson, and perhaps the only New Physics (at least in flavor sector) uncovered at the 7 and 8 TeV runs at the LHC. But it is more likely that the LHC would once again push the limits down towards SM. If such is the case, the fate of the 4G would have to be determined elsewhere. But $B_d \to \mu^+\mu^-$ should certainly be pursued further at the 13 TeV run.

Note Added. During this conference, the CMS [17] and LHCb [18] reported updated results for $B_d \to \mu^+\mu^-$ using full 2011–2012 data set. Interestingly, both experiments have seen hints for the signals above the SM expectation, although significances are still 2σ level for both. The combined result is $\mathscr{B}(B_d \to \mu^+\mu^-) = (3.6^{+1.6}_{-1.4}) \times 10^{-10}$ [19]. If the central value stays with decreased errors in the future, a region spreading between B and C in Fig. 1(a) (for average $|V_{ub}|$), and a region spreading between A' and C' in Fig. 1(b) (for exclusive $|V_{ub}|$) would be favored for $m_{t'} = 700$ GeV. In order to clarify the situation, we look forward eagerly to the 13–14 TeV runs.

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[16] My collaborator has used the words, “Turn horse around and thrust” to describe this situation. See G.W.-S. Hou, arXiv:1307.2448 [hep-ph].

[17] U. Langenegger, talk at this conference.

[18] F. Archilli, talk at this conference.

[19] CMS-PAS-BPH-13-007 and LHCb-CONF-2013-012.