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Mass of the b Quark from QCD Sum Rules for $f_{B_{(s)}}$

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We demonstrate that Borel QCD sum rules for heavy–light currents entail a very strong correlation between the b-quark mass m_b and the B-meson's decay constant f_B, that is, $\delta f_B/f_B \approx -8\,\delta m_b/m_b$. By starting from f_B as input, this observation allows for an accurate sum-rule determination of m_b. Employing precise lattice QCD results for f_B in our sum-rule study based on the three-loop $O(\alpha_s^2)$ heavy–light correlation function implies $\overline{m}_b(\overline{m}_b) = (4.247 \pm 0.034)$ GeV for the b-quark $\overline{\rm MS}$ mass.

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1. Introduction

Within the "standard model of elementary particle physics," the mass of the b quark constitutes a fundamental parameter of the theory. Therefore, the knowledge of its numerical value as precisely as possible is of utmost importance. Lattice QCD provides a framework to determine this parameter by direct albeit purely numerical procedures; unfortunately, the b quark is too heavy to be dealt with by current lattice setups: lattice-QCD computations of its mass require either an extrapolation of the lattice-QCD findings from lighter simulated masses or the use of the "heavy-quark effective theory" (HQET) formulated on the lattice. The actual value of a quark mass depends on the renormalization scheme employed for the rigorous definition of this quantity; for the b quark, usually the predictions for its pole mass, for its $\overline{\mathrm{MS}}$ running mass at renormalization scale ν, $\overline{m}_b(\nu)$, or for $m_b \equiv \overline{m}_b(\overline{m}_b)$ are compared. Using unquenched gauge configurations and $N_f = 2$ dynamical sea-quark flavours gives:

- $m_b = (4.29 \pm 0.14)$ GeV [1] and $m_b = (4.35 \pm 0.12)$ GeV [2] when confiding in extrapolation;
- $m_b = (4.26 \pm 0.09)$ GeV [3], $m_b = (4.25 \pm 0.11)$ GeV [4] and $m_b = (4.22 \pm 0.11)$ GeV [5], for instance, if one is willing to accept the expansions involved in the HQET-based computations.

Moment sum rules for two-point functions of heavy–heavy currents entail more accurate m_b values:

- Low-n moment sum rules adopting three-loop $O(\alpha_s^2)$ [6] and four-loop $O(\alpha_s^3)$ [7] fixed-order perturbative-QCD results combined with experiment yield $m_b = (4.209 \pm 0.050)$ GeV [6] and $m_b = (4.163 \pm 0.016)$ GeV [7], respectively, where the latter result is supported by combining perturbative QCD and lattice-QCD efforts using $N_f = 2+1$ dynamical sea-quark flavours [8].
- Large-n moment renormalization-group-improved next-to-next-to-leading-logarithmic-order Υ sum rules, underpinned by experiment, give $m_b = (4.235 \pm 0.055_{(\mathrm{pert})} \pm 0.03_{(\mathrm{exp})})$ GeV [9].

Our recent study of m_b by means of heavy–light QCD sum rules reveals that m_b may be found with comparable accuracy if a precise input value of the $B_{(s)}$-meson decay constant $f_{B_{(s)}}$ is available [10].

2. Anticorrelation Between Beauty-Meson Decay Constant and Bottom-Quark Mass

Quantum theory allows for easy exploration of the sensitivity of any $B_{(s)}$-meson decay constant $f_{B_{(s)}}$ to the b-quark mass m_b: in any nonrelativistic potential model where the potential involves only one coupling constant (e.g., pure Coulomb or pure harmonic-oscillator potentials), the ground-state wave function at the origin $\psi(0)$ and binding energy ε are related by $|\psi(0)| \propto \varepsilon^{3/2}$; for any potential that is a sum of confining and Coulomb interactions, this relation is only a good approximation [11].

Recalling that a decay constant is the analogue of the wave function at the origin and exploiting the (well-known) scaling behaviour of the decay constant of a heavy meson in the heavy-quark limit entails, as approximate relation between B-meson mass M_B and pole mass m_Q of the heavy quark Q,

$$f_B \sqrt{M_B} = \kappa \, (M_B - m_Q)^{3/2} \ .$$

Now, keeping M_B fixed and equal to its experimental value $M_B = 5.27$ GeV, we can easily derive the dependence of f_B on small variations δm_Q around some given value of m_Q. Taking into account that

$f_B \approx 200$ MeV for $m_Q \approx 4.6$–4.7 GeV, we obtain $\kappa \approx 0.9$–1.0 and $\delta f_B \approx -0.5\,\delta m_Q$ or, equivalently,

$$\frac{\delta f_B}{f_B} \approx -(11\text{–}12)\,\frac{\delta m_Q}{m_Q}\ .$$

From this example, we expect a rather high sensitivity of f_B to m_Q: Varying m_Q by $+100$ MeV gives $\delta f_B \approx -50$ MeV. Similar effects should be observable in the predictions of QCD sum rules [12, 13].

3. QCD Sum-Rule Extractions of Beauty-Meson Decay Constants in the Literature

More or less recently, several QCD sum-rule results [14 – 17] of beauty-meson decay constants using three-loop heavy–light correlators [18], all of them deriving, in fact, from essentially the same analytical expression for the correlator, have been published; Table 1 summarizes the corresponding findings for f_B. At first glance, the predictions appear to be rather stable and practically independent of the m_b input value. However, this may not be put forward as argument in support of the reliability of all the extractions, since, evidently, the figures in Table 1 do not follow our above general pattern. For instance, the central values of m_b found by Ref. [14] and Ref. [17] differ by almost 200 MeV but the corresponding decay constants are nearly identical. This forces us to suspect that not all findings are equally trustable. Recall that the ground-state parameters in Table 1 are subject to two decisions:

- How is the three-loop perturbative result organized in terms of pole or $\overline{\rm MS}$ heavy-quark mass?
- How are the *auxiliary* sum-rule quantities, such as the *effective* onset of the continuum, fixed?

Table 1: QCD sum-rule extractions of the B-meson decay constant f_B from heavy–light two-point correlator.

	Reference [14]	Reference [15]	Reference [16]	Reference [17]
m_b (GeV)	4.05 ± 0.06	4.21 ± 0.05	4.245 ± 0.025	4.236 ± 0.069
f_B (MeV)	203 ± 23	210 ± 19	193 ± 15	206 ± 7

In a recent critical detailed analysis of the sum-rule extraction of f_B [10], we demonstrated that,

- if the correlator is expressed in terms of the $\overline{\rm MS}$ running instead of the pole b-quark mass and
- if consistent procedures for the extraction of the bound-state properties of interest are applied,

the QCD sum-rule extractions of f_B exhibit excellent agreement with the behaviour expected, on the general grounds detailed above, from quantum mechanics: the decay constant f_B predicted by QCD sum rules is strongly correlated with the value of the heavy-quark mass m_b used as input. If all input parameters of the correlator except for m_b—renormalization scales, α_s, quark condensate, etc.—are kept fixed, we obtain a linear dependence of f_B on m_b with negative slope, that is, an anticorrelation:

$$f_B(m_b) = \left(192.0 - 37\,\frac{m_b - 4.247\ \text{GeV}}{0.1\ \text{GeV}} \pm 3_{\rm (syst)}\right) \text{MeV}\ .$$

The above strong (anti-) correlation between f_B and m_b enables us to deduce an accurate value of m_b from f_B as starting point. Feeding our average $f_B^{\rm LQCD} = (191.5 \pm 7.3)$ MeV of recent findings for f_B by some lattice-QCD collaborations [1, 2, 5, 19 – 21] into our QCD sum-rule investigation adopting the heavy–light correlator at $O(\alpha_s^2)$ accuracy yields the precise estimate $m_b = (4.247 \pm 0.034)$ GeV.

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4. Heavy–Light Two-Point Correlation Function and (Borelized) QCD Sum Rule

This sum-rule study of the heavy pseudoscalar $B_{(s)}$ mesons starts from the correlator [12, 13] of two pseudoscalar currents $j_5(x) \equiv (m_b+m)\,\bar{q}(x)\,\mathrm{i}\,\gamma_5\,b(x)$ of a b quark and a light quark q of mass m:

$$\Pi(p^2) \equiv \mathrm{i}\int \mathrm{d}^4x \exp(\mathrm{i}\,p\,x)\left\langle 0\left|\mathrm{T}\left(j_5(x)\,j_5^\dagger(0)\right)\right|0\right\rangle .$$

Upon Borel transformation $\Pi(p^2) \to \Pi(\tau)$ to a new "Borel" variable τ, the QCD sum rule sought is obtained by equating the results of evaluating this correlator at *QCD level*, with the help of Wilson's operator product expansion (OPE), and at *hadronic level*, by insertion of intermediate hadron states:

$$\begin{aligned}\Pi(\tau) &= f_B^2\,M_B^4 \exp\left(-M_B^2\,\tau\right) + \int\limits_{s_{\mathrm{phys}}}^{\infty} \mathrm{d}s \exp(-s\,\tau)\,\rho_{\mathrm{hadr}}(s) \\ &= \int\limits_{(m_b+m)^2}^{\infty} \mathrm{d}s \exp(-s\,\tau)\,\rho_{\mathrm{pert}}(s,\mu) + \Pi_{\mathrm{power}}(\tau,\mu)\ ,\end{aligned}$$

with the $B_{(s)}$ meson's mass M_B and decay constant f_B defined by $(m_b+m)\,\langle 0|\bar{q}\,\mathrm{i}\,\gamma_5\,b|B\rangle = f_B\,M_B^2$; the physical continuum threshold, $s_{\mathrm{phys}} = (M_{B^*}+M_P)^2$, is fixed by the beauty vector meson's mass M_{B^*} and the mass M_P of the lightest pseudoscalar meson with appropriate quantum numbers, i.e., π or K.

For large τ, the contributions of the excited states to $\Pi(\tau)$ decrease faster than the ground-state contribution, so $\Pi(\tau)$ becomes saturated by the lowest state: the large-τ behaviour of $\Pi(\tau)$ provides direct access to ground-state features. However, analytic results for $\Pi(\tau)$ are found from a truncated OPE approximating $\Pi(\tau)$ well only for τ not too large, where excited states still contribute sizeably.

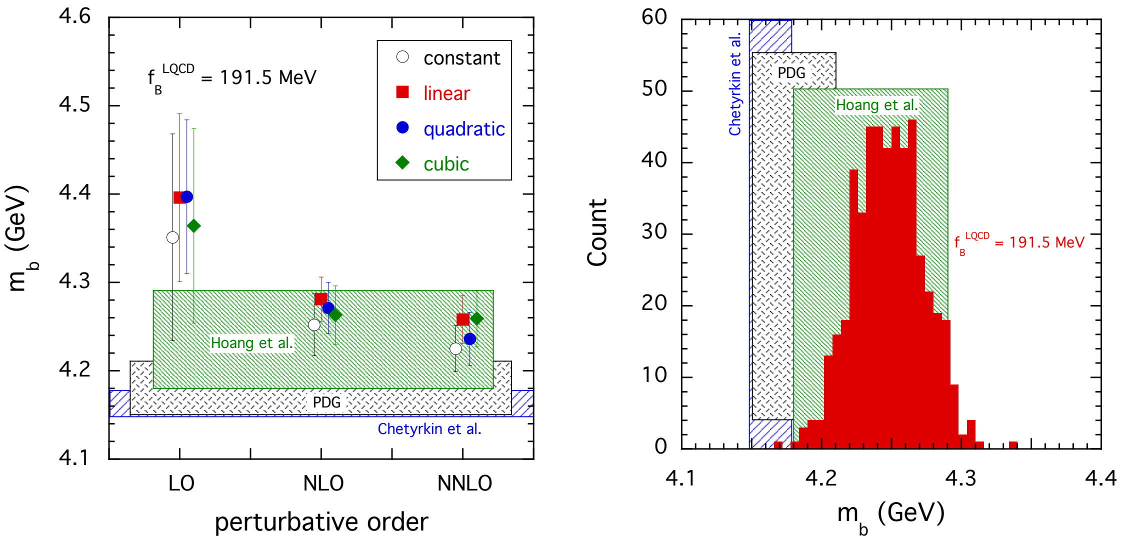
Excited-state contributions may be banished from $\Pi(\tau)$ by assuming *quark–hadron duality*: all excited states' contributions are counterbalanced by the perturbative contribution above an *effective continuum threshold* $s_{\mathrm{eff}}(\tau)$, not to be confused with the physical continuum threshold: the constant physical continuum threshold, s_{phys}, is determined by the masses of the lightest hadrons that may be produced from the vacuum by the interpolating current whereas the effective continuum threshold is a quantity intrinsic to the sum-rule technique, with a lot of interesting and nontrivial properties [22]. Specifically, we have unambiguously shown that the true effective threshold, defined by requiring it to reproduce correctly the ground-state parameters, will exhibit a dependence on the variable τ [23]. Applying duality results in a relation, a *QCD sum rule*, between ground-state observables and OPE:

$$f_B^2\,M_B^4 \exp\left(-M_B^2\,\tau\right) = \int\limits_{(m_b+m)^2}^{s_{\mathrm{eff}}(\tau)} \mathrm{d}s \exp(-s\,\tau)\,\rho_{\mathrm{pert}}(s,\mu) + \Pi_{\mathrm{power}}(\tau,\mu)\ . \tag{4.1}$$

Clearly, any evaluation of this sum rule does not only require the knowledge of both spectral density $\rho_{\mathrm{pert}}(s,\mu)$ and nonperturbative power corrections $\Pi_{\mathrm{power}}(\tau,\mu)$: in addition, we have to formulate or develop a criterion for determining $s_{\mathrm{eff}}(\tau)$. Furthermore, we have to make sure that the OPE exhibits a reasonable convergence; to this end, following Ref. [15] we reorganize the perturbative expansion of $\rho_{\mathrm{pert}}(s,\mu)$, derived in Ref. [18] in terms of the heavy quark's pole mass, in terms of the associated $\overline{\mathrm{MS}}$ mass. The explicit results for ρ_{pert} at three-loop level and Π_{power} may be found in Refs. [18, 15].

5. Anticorrelation as Serendipity: Extracting the $\overline{\mathrm{MS}}$ Mass m_b of the Bottom Quark

The strong sensitivity of f_B and f_{B_s} on the precise value of m_b resulting from the QCD sum-rule approach allows us to invert our line of thought and to derive an accurate prediction of $m_b \equiv \overline{m}_b(\overline{m}_b)$ from (reasonably accurate) lattice-QCD outcomes for f_B and f_{B_s}. Figure 1 summarizes our findings, obtained from the QCD sum rule (4.1) by applying our algorithms for fixing the effective continuum threshold $s_{\rm eff}(\tau)$, which adopt a polynomial *Ansatz* for $s_{\rm eff}(\tau)$ up to third order (i.e., constant, linear, quadratic, or cubic dependence on τ). Figure 1(a) depicts the resulting m_b values for different orders taken into account in the perturbative expansion of the correlator: Increasing its accuracy from $O(1)$ leading order (LO) to $O(\alpha_s)$ next-to-leading order (NLO) diminishes central value and OPE error of m_b from $m_b^{\rm LO} = (4.38 \pm 0.1_{\rm (OPE)} \pm 0.020_{\rm (syst)})$ GeV to $m_b^{\rm NLO} = (4.27 \pm 0.04_{\rm (OPE)} \pm 0.015_{\rm (syst)})$ GeV. Considering also the $O(\alpha_s^2)$ next-to-next-to-leading order (NNLO) has very little numerical impact: $m_b^{\rm NNLO} = (4.247 \pm 0.027_{\rm (OPE)} \pm 0.011_{\rm (syst)})$ GeV. Anyway, the extracted values of m_b nicely show a kind of convergence for increasing perturbative accuracy. The OPE error is estimated by varying all OPE parameters in their "usual" intervals and both renormalization scales μ, ν independently in the range $3\,\mathrm{GeV} < \mu, \nu < 6\,\mathrm{GeV}$. For our final result $m_b^{\rm NNLO}$, these quantities contribute 14 MeV (μ, ν), 20 MeV (quark condensate), 7 MeV (gluon condensate), 8 MeV (α_s) and 4 MeV (light-quark mass), respectively, to the total OPE error of 27 MeV, obtained by adding all the individual contributions in quadrature. The spread of m_b values for different $s_{\rm eff}(\tau)$ *Ansätze* is regarded as systematic error [25] and amounts to 11 MeV; the lattice input $f_B = (191.5 \pm 7.3)$ MeV adds a Gaussian error of 18 MeV.



(a) (b)

Figure 1: Our findings for the b-quark mass $m_b \equiv \overline{m}_b(\overline{m}_b)$, inferred from the heavy–light QCD sum rule (4.1) by a bootstrap analysis of all OPE errors for central value $f_B = 191.5$ MeV of the B-meson decay constant f_B. (a) Dependence of m_b on the order of the perturbative expansion of the correlator, indicated by "LO," "NLO," and "NNLO," respectively. For comparison, the ($\pm 1\,\sigma$) ranges of the results found by the Particle Data Group (PDG) [24], by Chetyrkin *et al.* [7], and by Hoang *et al.* [9], for example, are represented by the shaded areas. (b) Distribution of m_b from bootstrapping, adopting Gaussian distributions for the OPE parameters except for the renormalization scales μ and ν and uniform distributions in the range $3\,\mathrm{GeV} < \mu, \nu < 6\,\mathrm{GeV}$ for μ and ν.

6. Summary of Main Results and Conclusions

This application of QCD sum rules to the beauty-meson system provides some pivotal insights:

1. Accepting the dependence of the effective continuum threshold introduced when applying the notion of quark–hadron duality on variables entering when performing Borel transformations significantly improves the determination of hadronic properties, by increasing the accuracy of the duality approximation and probing the intrinsic uncertainty of the QCD sum-rule method.

2. For beauty mesons, the sum-rule prediction for f_B is strongly correlated to the exact m_b value:
$$\frac{\delta f_B}{f_B} \approx -8\,\frac{\delta m_b}{m_b}\ .$$
Realizing this behaviour, we use precise lattice-QCD results for $f_{B_{(s)}}$ to extract the value of m_b by combining the most recent lattice-QCD findings for f_B and f_{B_s} with our sum-rule analysis:
$$m_b = (4.247 \pm 0.027_{(\mathrm{OPE})} \pm 0.018_{(\mathrm{exp})} \pm 0.011_{(\mathrm{syst})})\ \mathrm{GeV}\ . \tag{6.1}$$
Here, the OPE error arises from the errors of all the OPE input parameters, the "exp" error is a consequence of the error in the QCD-lattice determination of $f_{B_{(s)}}$, and the systematic error of the QCD sum-rule method inferred from the spread of results when varying the *Ansatz* for the effective continuum threshold is under control. Finally, adding the errors in quadrature yields
$$m_b = (4.247 \pm 0.034)\ \mathrm{GeV}\ .$$
This implies, by the sum rule (4.1) from heavy–light correlators evaluated at $O(\alpha_s^2)$ accuracy,
$$f_B = \left(192.0 \pm 14.3_{(\mathrm{OPE})} \pm 3.0_{(\mathrm{syst})}\right)\ \mathrm{MeV}\ , \qquad f_{B_s} = \left(228.0 \pm 19.4_{(\mathrm{OPE})} \pm 4_{(\mathrm{syst})}\right)\ \mathrm{MeV}\ .$$
In view of the fact that the predicted value of m_b changes only marginally when increasing the correlator's perturbative accuracy from $O(\alpha_s)$ to $O(\alpha_s^2)$, we do not expect that an inclusion of the at present unknown $O(\alpha_s^3)$ corrections will modify the extracted value of m_b substantially.

3. Comparing our prediction in Eq. (6.1) with the other findings for m_b available in the literature, we note agreement with $m_b = (4.209 \pm 0.050)$ GeV from moment sum rules for heavy–heavy correlators also at $O(\alpha_s^2)$ accuracy [6], with $m_b = (4.235 \pm 0.055_{(\mathrm{pert})} \pm 0.003_{(\mathrm{exp})})$ GeV from a renormalization-group-improved next-to-next-to-leading-logarithmic-order discussion of Υ sum rules [9] as well as, within two standard deviations, with the Particle Data Group average $m_b = (4.18 \pm 0.03)$ GeV [24] but an evident disagreement with $m_b = (4.163 \pm 0.016)$ GeV [7] and $m_b = (4.171 \pm 0.009)$ GeV [26] due to sum rules using heavy–heavy correlators at $O(\alpha_s^3)$ accuracy; we doubt that $O(\alpha_s^3)$ corrections to the heavy–light sum rule can restore agreement.

In conclusion, let us emphasize that *properly formulated* Borel QCD sum rules for heavy–light correlators form competitive tools both for reliable determinations of heavy-meson observables and for the extraction of basic QCD parameters by exploiting the results of lattice QCD and experiment.

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