

## Measurements of $\phi_3$ at Belle

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The Belle experiment, running at the KEKB  $e^+e^-$  asymmetric energy collider during 1999-2010, has recorded  $711 \text{ fb}^{-1}$  of data at the  $\Upsilon(4S)$  resonance. A combined fit for  $\phi_3$  with the latest Belle results of the ADS, GLW and GGSZ method is performed. We use  $B^\pm \rightarrow DK^\pm$  and  $B^\pm \rightarrow D^*K^\pm$  decays where the  $D$  meson ( $D^0$  or  $\bar{D}^0$ ) decays into  $K_S^0\pi\pi$ ,  $K\pi$ ,  $KK$ ,  $\pi\pi$ ,  $K_S^0\pi^0$  and  $K_S^0\eta$  final states and  $D^*$  decays into  $D\pi^0$  and  $D\gamma$ . Belle obtains its most precise measurement to date,  $\phi_3 = (68_{-14}^{+15})^\circ$ . In addition a new ADS analysis of  $B^\pm \rightarrow DK^\pm$  following by  $D \rightarrow K\pi\pi^0$ , is reported.

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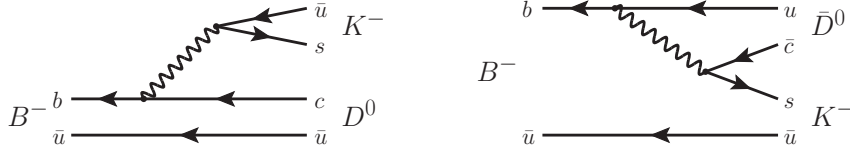
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## 1. Introduction

In the Standard Model (SM), the Cabbibo-Kobayashi-Maskawa (CKM) mechanism [1] has been well established, but some measurements of the mechanism warrant further investigation. For example, two angles of the CKM unitarity triangle,  $\phi_1$  and  $\phi_2$ , have now been measured with high precision [2]. The  $\phi_3$  determination from data is much less accurate. Its determination is however theoretically clean due to no loop contributions;  $\phi_3$  can be determined using tree-level processes only, exploiting the interference between  $b \rightarrow c\bar{u}d$  and  $b \rightarrow u\bar{c}d$  transitions that occurs when the  $D^0$  and  $\bar{D}^0$  decay to the same final state (Fig. 1). Therefore, the angle  $\phi_3$  provides a SM benchmark, and



**Figure 1:** Feynman diagrams for  $B^- \rightarrow D^0 K^-$  and  $B^- \rightarrow \bar{D}^0 K^-$ .

its precise measurement is crucial in order to disentangle non-SM contributions to other processes, via global CKM fits. The size of the interference depends on the ratio ( $r_B$ ) of the magnitudes of the two tree diagrams and  $\delta_B$ , the strong phase difference between them. These hadronic parameters will be extracted from data together with the angle  $\phi_3$ . The value of  $r_B$  is expected to be 0.1 from the ratio of CKM matrix elements  $|V_{ub}^* V_{cs}|/|V_{cb}^* V_{us}| \sim 0.38$  and the color suppression, whereas  $\delta_B$  can not be calculated precisely from theory. Note that  $r_B$  and  $\delta_B$  can take different values for different  $B$  decays: the values of  $B^\pm \rightarrow DK^\pm$  and  $B^\pm \rightarrow D^* K^\pm$  are not the same. Several different  $D$  decays have been studied in order to maximize the sensitivity to  $\phi_3$ . One of the approaches is the use of  $D$  decays to  $CP$  eigenstates, a method proposed by M. Gronau, D. London, and D. Wyler, which is referred to as the GLW method [3]. An alternative approach was proposed by D. Atwood, I. Dunietz, and A. Soni, which is referred to as the ADS method [4]. Instead of using  $D^0$  decays to  $CP$  eigenstates, the ADS method uses Cabibbo-favored and doubly Cabibbo-suppressed  $D$  decays. In the decays  $B^\pm \rightarrow [K^\mp \pi^\pm]_D K^\pm$ , the suppressed  $B$  decay is followed by a Cabibbo-favored  $D^0$  decay. Therefore, the interfering amplitudes are of similar magnitude, and one can expect a large  $CP$  asymmetry. But those decay mode branching fractions tend to be small. Another approach, proposed by Giri, Grossman, Soffer and Zupan, [5] that is referred to as the GGSZ method, is to use three-body decays of the  $D$ . The advantage is that  $\phi_3$ ,  $\delta_B$  and  $r_B$  can be obtained from a single decay. At present the GGSZ method using  $D \rightarrow K_S \pi^+ \pi^-$  decays gives the best sensitivity to  $\phi_3$ . Latest Belle results using the full data sample taken at the  $\Upsilon(4S)$  (corresponding to  $772 \times 10^6 B\bar{B}$  pairs) are described in these proceedings and the value of  $\phi_3$  obtained by combining these results is presented.

## 2. GGSZ results

Assuming no  $CP$  asymmetry in neutral  $D$  decays, the amplitude for  $B^+ \rightarrow [K_S^0 \pi^+ \pi^-]_D K^+$

decay as a function of Dalitz plot variables of  $m_+^2 = m_{K_S^0\pi^+}^2$  and  $m_-^2 = m_{K_S^0\pi^-}^2$  is

$$f_{B^+} = f_D(m_+^2, m_-^2) + r_B e^{i\phi_3 + i\delta_B} f_D(m_-^2, m_+^2), \quad (2.1)$$

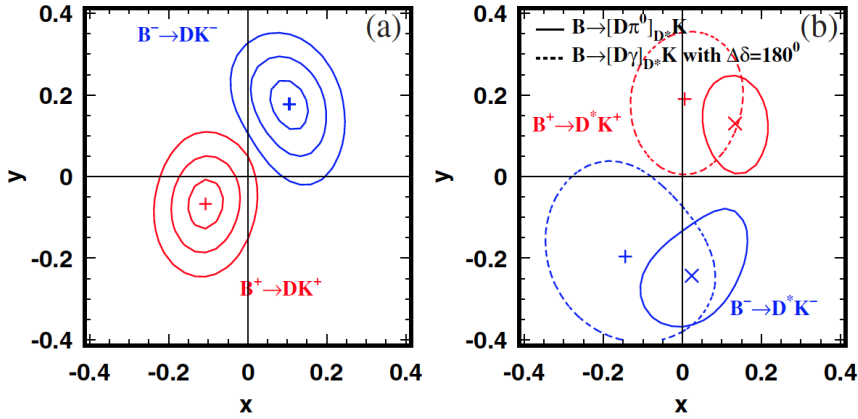
where  $f_D(m_+^2, m_-^2)$  is the amplitude of the  $\bar{D}^0 \rightarrow K_S^0\pi^+\pi^-$  decay. Similarly, the amplitude for  $B^- \rightarrow [K_S^0\pi^+\pi^-]_D K^-$  decay is

$$f_{B^-} = f_D(m_-^2, m_+^2) + r_B e^{-i\phi_3 + i\delta_B} f_D(m_+^2, m_-^2). \quad (2.2)$$

With a large sample of flavor-tagged  $\bar{D}^0 \rightarrow K_S^0\pi^+\pi^-$  decays produced in continuum  $c\bar{c}$  production from  $e^+e^-$  annihilation events, the  $\bar{D}^0 \rightarrow K_S^0\pi^+\pi^-$  decay amplitude  $f_D$  can be determined. Once  $f_D$  is known, a simultaneous fit to  $B^+$  and  $B^-$  data allows the contributions of  $r_B$ , and  $\delta_B$  to be separated. This method has a two-fold ambiguity in the solutions:  $(\phi_3, \delta_B)$  and  $(\phi_3 + 180^\circ, \delta_B + 180^\circ)$ . Due to the fact that  $r_B$  is bounded to be positive, the direct extraction of  $r_B$ ,  $\delta_B$  and  $\phi_3$  can be biased. To avoid these biases, the Cartesian coordinates have been introduced,  $x_\pm = r_B \cos(\delta_B \pm \phi_3)$  and  $y_\pm = r_B \sin(\delta_B \pm \phi_3)$ . A combined GGSZ analysis of  $B^+$  and  $B^-$  samples results in the  $(x_\pm, y_\pm)$  values given in Table 1 and Fig. 2.

Observables	$B \rightarrow DK$	$B \rightarrow D^*K$
$x_+$	$-0.107 \pm 0.043 \pm 0.011$	$+0.083 \pm 0.022 \pm 0.011$
$y_+$	$-0.067 \pm 0.059 \pm 0.018$	$+0.157 \pm 0.109 \pm 0.018$
$x_-$	$+0.105 \pm 0.047 \pm 0.011$	$-0.036 \pm 0.127 \pm 0.011$
$y_-$	$+0.177 \pm 0.060 \pm 0.018$	$-0.249 \pm 0.118 \pm 0.018$

**Table 1:** Results of Belle GGSZ analyses.



**Figure 2:** Results of Belle GGSZ analyses.

The combined result of  $\phi_3$  from  $B^\pm \rightarrow DK^\pm$  and  $B^\pm \rightarrow D^*K^\pm$  is  $\phi_3 = (78_{-12}^{+11} \pm 4 \pm 9)^\circ$  [6], where the three uncertainties are statistical, systematic and that due to the  $D \rightarrow K_S^0\pi^+\pi^-$  decay model assumptions, respectively. The last source of uncertainty can be eliminated by binning the

Dalitz plot (Refs. [5, 7]), using information on the averaged strong phase difference  $\delta_D$  between  $D^0$  and  $\bar{D}^0$  decays in bin; this information can be determined using the quantum-correlated  $\psi(3770)$  data. The results have been published by CLEO-c [8]. The measured strong phase difference is used to obtain the model-independent result [9],  $\phi_3 = (77 \pm 15 \pm 4 \pm 4)^\circ$  where the last uncertainty is due to the statistical precision of the CLEO-c results.

### 3. ADS results

For the ADS method, Belle has studied the  $B \rightarrow D^{(*)}K$  decays where  $D \rightarrow K^- \pi^+$ . The observables measured in the ADS method are the ratio of the suppressed and allowed branching fractions:

$$\begin{aligned} \mathcal{R}_{\text{ADS}} &= \frac{\Gamma(B^\pm \rightarrow [K^\mp \pi^\pm]_D K^\pm)}{\Gamma(B^\pm \rightarrow [K^\pm \pi^\mp]_D K^\pm)} \\ &= r_B^2 + r_D^2 + 2r_B r_D \cos \phi_3 \cos \delta, \end{aligned} \quad (3.1)$$

and

$$\begin{aligned} \mathcal{A}_{\text{ADS}} &= \frac{\Gamma(B^- \rightarrow [K^+ \pi^-]_D K^-) - \Gamma(B^+ \rightarrow [K^- \pi^+]_D K^+)}{\Gamma(B^- \rightarrow [K^+ \pi^-]_D K^-) + \Gamma(B^+ \rightarrow [K^- \pi^+]_D K^+)} \\ &= 2r_B r_D \sin \phi_3 \sin \delta / \mathcal{R}_{\text{ADS}}, \end{aligned} \quad (3.2)$$

where  $r_D$  is the ratio of the doubly Cabibbo-suppressed and Cabibbo-allowed  $D^0$  decay amplitudes and  $\delta$  is the sum of strong phase differences in  $B$  and  $D$  decays:  $\delta = \delta_B + \delta_D$ . Recently, Belle reported the ADS analysis using their final data sample of  $772 \times 10^6 B\bar{B}$  pairs [10], and these results are summarized in Table 2. This analysis includes the two  $B$  decays:  $B^\pm \rightarrow DK^\pm$  with  $D$

Mode	$\mathcal{R}_{\text{ADS}} [\times 10^{-2}]$	$\mathcal{A}_{\text{ADS}}$
$B^\pm \rightarrow DK^\pm$	$1.63^{+0.44+0.07}_{-0.41-0.13}$	$-0.39^{+0.26+0.04}_{-0.28-0.03}$
$B^\pm \rightarrow D^* K^\pm, D^* \rightarrow D\pi^0$	$1.0^{+0.8+0.1}_{-0.7-0.2}$	$+0.4^{+1.1+0.2}_{-0.7-0.1}$
$B^\pm \rightarrow D^* K^\pm, D^* \rightarrow D\gamma$	$3.6^{+1.4}_{-1.2} \pm 0.2$	$-0.51^{+0.33}_{-0.29} \pm 0.08$

**Table 2:** Results of Belle ADS analyses.

decaying to  $K^+ \pi^-$  and  $K^- \pi^+$ . The use of two additional decay modes,  $D^* \rightarrow D\pi^0$  and  $D^* \rightarrow D\gamma$ , provides an extra handle on the extraction of  $\phi_3$  as explained in Ref. [11]. This effect (larger ratio for  $B^\pm \rightarrow D^* K^\pm$  with  $D^* \rightarrow D\gamma$  and opposite asymmetry between both  $B^\pm \rightarrow D^* K^\pm$  channels) is becoming visible in the results from Belle [13].

### 4. GLW results

In the study of the GLW method, the modes of interest are  $D$  decays into  $CP$  eigenstates [3] such as  $K^+ K^-$ ,  $\pi^+ \pi^-$  ( $CP$ -even) and  $K_S^0 \pi^0$ ,  $K_S^0 \eta^0$  ( $CP$ -odd). To extract  $\phi_3$  using the GLW method, the following observables sensitive to  $CP$  violation are used: the asymmetries

$$\begin{aligned} \mathcal{A}_{CP^\pm} &= \frac{\Gamma(B^- \rightarrow D_{CP^\pm} K^-) - \Gamma(B^+ \rightarrow D_{CP^\pm} K^+)}{\Gamma(B^- \rightarrow D_{CP^\pm} K^-) + \Gamma(B^+ \rightarrow D_{CP^\pm} K^+)} \\ &= \pm \frac{2r_B \sin \phi_3 \sin \delta_B}{1 + r_B^2 \pm 2r_B \cos \phi_3 \cos \delta_B}, \end{aligned} \quad (4.1)$$

and

$$\begin{aligned}\mathcal{R}_{CP\pm} &= 2 \frac{\Gamma(B^- \rightarrow D_{CP\pm} K^-) + \Gamma(B^+ \rightarrow D_{CP\pm} K^+)}{\Gamma(B^- \rightarrow \bar{D}^0 K^-) + \Gamma(B^+ \rightarrow D^0 K^+)} \\ &= 1 + r_B^2 \pm 2r_B \cos \phi_3 \cos \delta_B.\end{aligned}\quad (4.2)$$

Among these four observables,  $\mathcal{A}_{CP\pm}$  and  $\mathcal{R}_{CP\pm}$ , only three are independent (since  $\mathcal{A}_{CP+}\mathcal{R}_{CP+} = -\mathcal{A}_{CP-}\mathcal{R}_{CP-}$ ). Recently, Belle updated the GLW analysis using their final data sample of  $772 \times 10^6 B\bar{B}$  pairs [13]. These results include the two  $B$  decays:  $B^\pm \rightarrow D^0 K^\pm$  and  $B^\pm \rightarrow D^{*0} K^\pm$ , where  $D^{*0} \rightarrow D^0 \pi^0$  and  $D^0 \gamma$ . The signs of the  $\mathcal{A}_{CP+}$  and  $\mathcal{A}_{CP-}$  asymmetries should be opposite which is now confirmed by the Belle experiment (Table 3).

Observables	$B \rightarrow DK$	$B \rightarrow D^*K$
$\mathcal{R}_{CP+}$	$1.03 \pm 0.07 \pm 0.03$	$1.19 \pm 0.13 \pm 0.03$
$\mathcal{R}_{CP-}$	$1.13 \pm 0.09 \pm 0.05$	$1.03 \pm 0.13 \pm 0.03$
$\mathcal{A}_{CP+}$	$+0.29 \pm 0.06 \pm 0.02$	$-0.14 \pm 0.10 \pm 0.01$
$\mathcal{A}_{CP-}$	$-0.12 \pm 0.06 \pm 0.01$	$+0.22 \pm 0.11 \pm 0.01$

**Table 3:** Results of Belle GLW analyses.

## 5. $\phi_3$ combination from Belle measurements

We combine the available Belle observables of the  $D^{(*)}K$  system obtained from the GGSZ method (model-dependent results shown in Table 1), the ADS method (Table 2) and the GLW method (Table 3) using the frequentist procedure described in Ref. [12]. The 1-CL curves obtained for the angle  $\phi_3$ , as well as for the hadronic parameters ( $\delta_B$  and  $r_B$ ) of  $B \rightarrow DK$  mode, are shown in Fig. 3 and the 68% C.L. intervals are summarized in Table 4. Belle obtains its most precise  $\phi_3$  measurement to date:  $\phi_3 = (68_{-14}^{+15})^\circ$ .

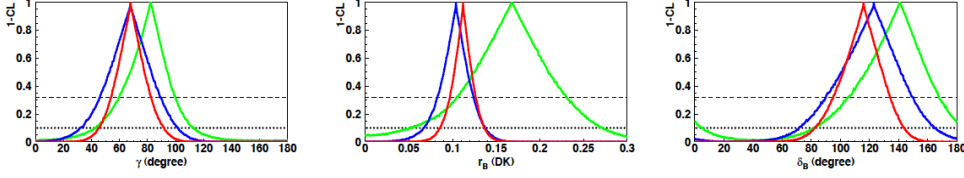
Method	$\phi_3$ [ $^\circ$ ]	$\delta_B$ [ $^\circ$ ]	$r_B$
GGSZ	$82_{-23}^{+18}$	$141_{-36}^{+27}$	$0.168_{-0.064}^{+0.063}$
GGSZ+ADS	$68 \pm 22$	$123_{-33}^{+27}$	$0.104_{-0.021}^{+0.020}$
GGSZ+ADS+GLW	$68_{-24}^{+15}$	$116_{-21}^{+18}$	$0.112_{-0.015}^{+0.014}$

**Table 4:** Confidence intervals (68% C.L.) for the angle  $\phi_3$  and the hadronic parameters of  $DK$  ( $\delta_B$  and  $r_B$ ) obtained by the combination of the  $D \rightarrow D^{(*)}K$  results of the Belle collaboration.

## 6. New Belle results

We have a new result related to  $\phi_3$ ,  $B^\pm \rightarrow [K\pi\pi^0]_D K^\pm$ , with ADS method. In  $B^\pm \rightarrow [K\pi\pi^0]_D K^\pm$  decay, observables  $\mathcal{R}_{\text{ADS}}$  (Eq. 3.1) and  $\mathcal{A}_{\text{ADS}}$  (Eq. 3.2) are written in

$$\begin{aligned}\mathcal{R}_{\text{ADS}} &= \frac{\Gamma(B^- \rightarrow [K^+\pi^-\pi^0]_D K^-) + \Gamma(B^+ \rightarrow [K^-\pi^+\pi^0]_D K^+)}{\Gamma(B^- \rightarrow [K^-\pi^+\pi^0]_D K^-) + \Gamma(B^+ \rightarrow [K^+\pi^-\pi^0]_D K^+)} \\ &= r_B^2 + r_D^2 + 2R_{K\pi\pi^0} r_B r_D \cos \phi_3 \cos(\delta_B + \delta_D^{K\pi\pi^0}),\end{aligned}\quad (6.1)$$



**Figure 3:** 1 – CL curves for  $\phi_3$  (left),  $r_B$  (center) and  $\delta_B$  (right) from the Belle  $D^{(*)}K$  results. The green curve is for the GGSZ results, the blue for GGSZ and ADS results using  $\delta_D$  from mixing and CLEO-c measurements, the red for GGSZ, ADS and GLW results.

and

$$\begin{aligned} \mathcal{A}_{\text{ADS}} &= \frac{\Gamma(B^- \rightarrow [K^+\pi^-\pi^0]_D K^-) - \Gamma(B^+ \rightarrow [K^-\pi^+\pi^0]_D K^+)}{\Gamma(B^- \rightarrow [K^+\pi^-\pi^0]_D K^-) + \Gamma(B^+ \rightarrow [K^-\pi^+\pi^0]_D K^+)} \\ &= 2R_{K\pi\pi^0} r_B r_D \sin\phi_3 \sin(\delta_B + \delta_D^{K\pi\pi^0}) / \mathcal{R}_{\text{ADS}}, \end{aligned} \quad (6.2)$$

where  $R_{K\pi\pi^0}$  is called the coherence factor,  $\delta_D^{K\pi\pi^0}$  is  $D$  decay strong-phase difference averaged over  $K\pi\pi^0$  phase space, and  $r_D = \Gamma(D^0 \rightarrow K^+\pi^-\pi^0) / \Gamma(D^0 \rightarrow K^-\pi^+\pi^0) = (2.20 \pm 0.10) \times 10^{-3}$ . The CLEO collaboration report values of  $R_{K\pi\pi^0}$  and  $\delta_D^{K\pi\pi^0}$  [14], which can be used to constrain  $\phi_3$  in conjunction with these measurements.

This analysis [15] uses the full  $\Upsilon(4S)$  data sample recorded by the Belle experiment. The suppressed mode signal yield obtained is  $77 \pm 24$  events, which corresponds to the first evidence for an ADS signal (with a significance of  $3.2\sigma$ ). The ratio of the suppressed and allowed modes and asymmetry are summarized in Table 5 and the signal yield fit result are shown in the combined fit, the results of which are reported in Fig. 4. Note that This result are not included in Table 4.

Observables	$B^\pm \rightarrow [K\pi\pi^0]_D K^\pm$
$\mathcal{R}_{\text{ADS}}$	$(1.98 \pm 0.62 \pm 0.23) \times 10^{-2}$
$\mathcal{A}_{\text{ADS}}$	$0.41 \pm 0.30 \pm 0.05$

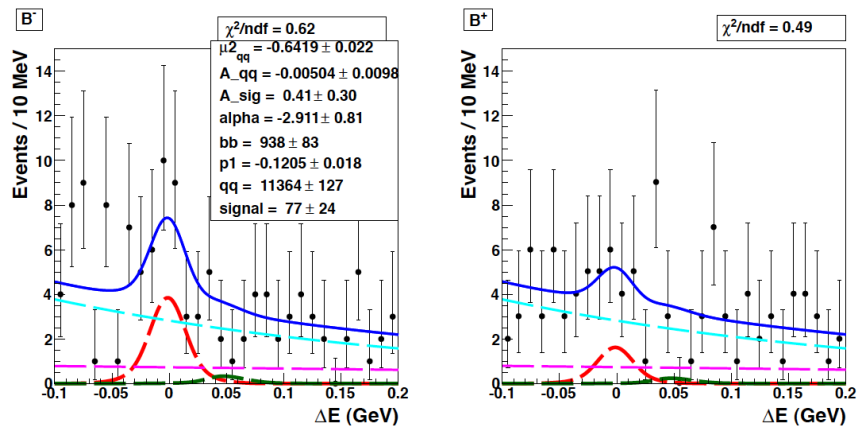
**Table 5:** Results of Belle  $B^\pm \rightarrow [K\pi\pi^0]_D K^\pm$  ADS analysis.

## 7. Conclusion

Belle is a good environment for  $B$  physics because it has the largest  $\Upsilon(4S)$  data of the world. The  $\phi_3$  determination is needed as a SM benchmark.  $\phi_3 = (68_{-14}^{+15})^\circ$  is the combined result of Belle, and a new  $B^\pm \rightarrow [K\pi\pi^0]_D K^\pm$  ADS result has been presented. Moreover many other analyses for  $\phi_3$  measurement using the full Belle data sample are ongoing.

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**Figure 4:** Results of Belle  $B^\pm \rightarrow [K\pi\pi^0]_D K^\pm$  analysis. The plotted variable  $\Delta E$  peaks at zero for signal decays.

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